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SOME OBSERVATIONS ON CONFORMAL METRICAL N-LINEAR CONNECTIONS IN THE BUNDLE OF ACCELERATIONS

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Abstract. In the present paper we treat some special classes of conformal metrical N-linear connections on $E = Osc^2M$, which preserve the nonlinear connection N. We analyze the role of the torsion d-tensor fields $T_{(0)}$, $S_{(1)}$ and $S_{(2)}$ in this theory and we study the semi-symmetric conformal metrical N-linear connections, which preserve the nonlinear connection N.

1. Introduction

The geometry of k-osculator spaces presents not only a special theoretical interest, but also an applicative one. Motivated by concrete problems in variational calculation, higher order Lagrange geometry has witnessed a wide acknowledgment due to the papers [7 − 11] published by Acad.dr.R.Miron and Prof.dr.Gh.Atanasiu.

The various applications of the Lagrange geometry of order k in Physics and Mechanics are considerable [14].

In the present paper we introduce the conformal metrical d-structure notion on $E = Osc^2M$, we define the conformal metrical N-linear connection notion (§2), we analyze the role of the torsion d-tensor fields $T_{(0)}$, $S_{(1)}$ and $S_{(2)}$ in this theory, and we study the semi-symmetric conformal metrical N-linear cconnections, which preserve the nonlinear connection N(§3). As to the terminology and notations we use those from [12], which are essentially based on M.Matsumoto's book [4].

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2. The notion of conformal metrical N-linear connection

Let M be a real n-dimensional C^∞-differentiable manifold and (Osc^2M, π, M) its 2-osculator bundle, or the bundle of accelerations. The local coordinates on $E = Osc^2M$ are denoted by $(x^i, y^{(1)i}, y^{(2)i})$. If N is a nonlinear connection on E, with the coefficients $N_{(1)}{}^i{}_j$, $N_{(2)}{}^i{}_j$, then let $D\Gamma(N) = (L^i_{jk}, C_{(1)}{}^i_{jk}, C_{(2)}{}^i_{jk})$ be an N- linear connection on E. We consider a metric d-structure on E, defined by a d-tensor field of the type $(0, 2)$, let us say $g_{ij}(x^i, y^{(1)i}, y^{(2)i})$, symmetric and nondegenerate.

We associate to this d-structure Obata's operators:

$$\Omega^{ir}_{sj} = \frac{1}{2}(\delta^i_s\delta^r_j - g_{sj}g^{ir}),\ \Omega^{*ir}_{sj} = \frac{1}{2}(\delta^i_s\delta^r_j + g_{sj}g^{ir}), \tag{2.1}$$

where (g^{ij}) is the inverse matrix of (g_{ij}).

Obata's operators have the same properties as ones associated with the Finsler space [12]. Let $\mathcal{S}_2(E)$ be the set of all symmetric d-tensor fields of the type $(0, 2)$ on E. As is easily shown, the relation for a_{ij}, $b_{ij} \in \mathcal{S}_2(E)$ defined by:

$$a_{ij} \sim b_{ij} \Leftrightarrow \exists\rho(x, y^{(1)}, y^{(2)}) \in \mathcal{F}(E) \mid a_{ij} = e^{2\rho}b_{ij}, \tag{2.2}$$

is an equivalent relation on $\mathcal{S}_2(E)$.

Definition 2.1. [14] *The equivalence class $\hat{g}$ of $\mathcal{S}_2(E)/_\sim$, to which the metric d-structure g_{ij} belongs, is called conformal metrical d-structure on E.*

Definition 2.2. *An N-linear connection $D\Gamma(N) = (L^i_{jk}, C_{(1)}{}^i_{jk}, C_{(2)}{}^i_{jk})$ on E, is said to be compatible with the conformal metrical d-structure $\hat{g}$, or a conformal metrical N-linear connection, if the following relations are verified:*

$$g_{ij|k} = 2\omega_k g_{ij},\ g_{ij} \overset{(\alpha)}{|}_k = 2\lambda_{(\alpha)k}g_{ij},\ (\alpha = 1, 2), \tag{2.3}$$

where $\omega_k = \omega_k(x, y^{(1)}, y^{(2)})$, $\lambda_{(\alpha)k} = \lambda_{(\alpha)k}(x, y^{(1)}, y^{(2)}), (\alpha = 1, 2)$ are covariant d-vector fields and $|$ and $\overset{(\alpha)}{|}$ denote the h-and v_α-covariant derivatives $(\alpha = 1, 2)$ with respect to $D\Gamma(N)$.

Theorem 2.1. [14] *The set of all conformal metrical N-linear connections on E, which preserve the nonlinear connection N, $D\Gamma(N) = (L^i_{jk}, C_{(1)}{}^i_{jk}, C_{(2)}{}^i_{jk})$ is given by:*

$$L^i_{jk} = \overset{0}{L}{}^i_{jk} + \Omega^{ir}_{sj} X^s_{rk},\ C_{(\alpha)}{}^i_{jk} = \overset{0}{C}_{(\alpha)}{}^i_{jk} + \Omega^{ir}_{sj} Y_{(\alpha)}{}^s_{rk},\ (\alpha = 1, 2), \tag{2.4}$$

where X^i_{jk}, $Y_{(1)}{}^i_{jk}$, $Y_{(2)}{}^i_{jk}$ are arbitrary tensor fields of the type $(1, 2)$ and $D\overset{0}{\Gamma}(N) = (\overset{0}{L}{}^i_{jk}, \overset{0}{C}_{(1)}{}^i_{jk}, \overset{0}{C}_{(2)}{}^i_{jk})$ are the coefficients of an arbitrary fixed conformal metrical N-linear connection on E.

3. Some special classes of conformal metrical N-linear connections

We shall try to replace the arbitrary tensor fields X^i_{jk}, $Y_{(1)}{}^i_{jk}$, $Y_{(2)}{}^i_{jk}$ in Theorem 2.1 by the torsion d-tensor fields $T_{(0)}{}^i{}_{jk}$, $S_{(1)}{}^i{}_{jk}$, $S_{(2)}{}^i{}_{jk}$. We put:

$$\begin{cases} T^*_{(0)}{}^i{}_{jk} = \frac{1}{2} g^{il}(g_{lh} T_{(0)}{}^h{}_{jk} - g_{jh} T_{(0)}{}^h{}_{lk} + g_{kh} T_{(0)}{}^h{}_{jl}), \\ S^*_{(\alpha)}{}^i{}_{jk} = \frac{1}{2} g^{il}(g_{lh} S_{(\alpha)}{}^h{}_{jk} - g_{jh} S_{(\alpha)}{}^h{}_{lk} + g_{kh} S_{(\alpha)}{}^h{}_{jl}),\ (\alpha = 1, 2). \end{cases} \tag{3.1}$$

Theorem 3.1. *Let $T_{(0)}{}^i{}_{jk}$, $S_{(1)}{}^i{}_{jk}$, $S_{(2)}{}^i{}_{jk}$ be given alternate d-tensor fields. Then there exists a unique conformal metrical N-linear connection $D\Gamma(N) =$ $= (L^i_{jk}, C_{(1)}{}^i_{jk}, C_{(2)}{}^i_{jk})$ with respect to $\hat{g}$, having $T_{(0)}{}^i{}_{jk}$, $S_{(1)}{}^i{}_{jk}$, $S_{(2)}{}^i{}_{jk}$ as the torsion d-tensor fields. It is given by:*

$$L^i_{jk} = \overset{0}{L}{}^i_{jk} + T^*_{(0)}{}^i{}_{jk},\ C_{(\alpha)}{}^i_{jk} = \overset{0}{C}_{(\alpha)}{}^i_{jk} + S^*_{(\alpha)}{}^i{}_{jk},\ (\alpha = 1, 2), \tag{3.2}$$

where $D\overset{0}{\Gamma}(N) = (\overset{0}{L}{}^i_{jk}, \overset{0}{C}_{(1)}{}^i_{jk}, \overset{0}{C}_{(2)}{}^i_{jk})$ is an arbitrary fixed conformal metrical N-linear connection on E.

Theorem 3.2. *There exists a unique conformal metrical N-linear connection $D\Gamma(N)$ on E, whose torsion d-tensor fields $T_{(0)}$, $S_{(\alpha)}$, $(\alpha = 1, 2)$ vanish.*

Definition 3.1. [15] *An N-linear connection* $D\Gamma(N) = (L^i_{jk}, C_{(1)}{}^i_{jk}, C_{(2)}{}^i_{jk})$ *is called semi-symmetric if the torsion d-tensor fields* $T_{(0)}{}^i{}_{jk}$, $S_{(1)}{}^i{}_{jk}$, $S_{(2)}{}^i{}_{jk}$ *have the form:*

$$\begin{cases} T_{(0)}{}^i{}_{jk} = \frac{1}{n-1}(T_{(0)j}\delta^i_k - T_{(0)k}\delta^i_j), \\ \\ S_{(\alpha)}{}^i{}_{jk} = \frac{1}{n-1}(S_{(\alpha)j}\delta^i_k - S_{(\alpha)k}\delta^i_j), \ (\alpha = 1, 2), \end{cases} \tag{3.3}$$

where $T_{(0)j} = T_{(0)}{}^i{}_{ji}$, $S_{(\alpha)j} = S_{(\alpha)}{}^i{}_{ji}$, $(\alpha = 1, 2)$.

Putting $\sigma_j = \frac{1}{n-1}T_{(0)j}$, $\tau_{(\alpha)j} = \frac{1}{n-1}S_{(\alpha)j}$, $T^*_{(0)}{}^i{}_{jk}$, $S^*_{(\alpha)}{}^i{}_{jk}$, $(\alpha = 1, 2)$ given by (3.1) become:

$$T^*_{(0)}{}^i{}_{jk} = 2\Omega^{ir}_{kj}\sigma_r, \ S^*_{(\alpha)}{}^i{}_{jk} = 2\Omega^{ir}_{kj}\tau_{(\alpha)r}, \ (\alpha = 1, 2). \tag{3.4}$$

From Theorem 3.1 we have:

Theorem 3.3. *The set of all semi-symmetric conformal metrical N-linear connections on E, which preserve the nonlinear connection N,* $D\Gamma(N) =$
$= (L^i_{jk}, C_{(1)}{}^i_{jk}, C_{(2)}{}^i_{jk})$, *is given by:*

$$\begin{cases} L^i_{jk} = \overset{0}{L}{}^i_{jk} + 2\Omega^{ir}_{kj}\sigma_r, \\ \\ C_{(\alpha)}{}^i_{jk} = \overset{0}{C}_{(\alpha)}{}^i_{jk} + 2\Omega^{ir}_{kj}\tau_{(\alpha)r}, \ (\alpha = 1, 2), \end{cases} \tag{3.5}$$

where $D\overset{0}{\Gamma}(N) = (\overset{0}{L}{}^i_{jk}, \overset{0}{C}_{(1)}{}^i_{jk}, \overset{0}{C}_{(2)}{}^i_{jk})$ *is an arbitrary fixed conformal metrical N-linear connection on E.*

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