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MAXWELL EQUATIONS FOR A GENERALIZED LAGRANGE SPACE OF ORDER 2 IN INVARIANT FRAMES

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Dedicated to Professor Pavel Enghiş at his 70th anniversary

Abstract. The study of higher order Lagrange spaces founded on the notion of bundle of velocities of order k has been given by Radu Miron and Gheorghe Atanasiu in [2]. The bundle of accelerations correspond in this study to k=2. The notion of invariant geometry of order 2 was introduced by the author in [4]. In this paper we shall give the Maxwell equations of a generalized Lagrange space of order 2 in invariant frames.

1. General Invariant Frames

Let us consider the bundle $E = Osc^2 M$, a nonlinear connection N with the coefficients $\left(\underset{(1)}{N^i{}_j} \ , \ \underset{(2)}{N^i{}_j} \right)$ and the duals $\left(\underset{(1)}{M^i{}_j} \ , \ \underset{(2)}{M^i{}_j} \right)$.

The invariant frames adapted to the direct decomposition

$$T_u(\ Osc^2 M\ ) \ = \ N_0(u) \oplus N_1(u) \oplus V_2(u) \quad \forall u \in E \tag{1}$$

will be $\Re \ = \ (\ e^{(0)i}_{\alpha}, e^{(1)i}_{\alpha}, e^{(2)i}_{\alpha}\ )$ and the dual $\Re^* \ = \ (\ f^{(0)\alpha}_{i}, f^{(1)\alpha}_{i}, f^{(2)\alpha}_{i}\ )$.

The duality conditions are

$$< \ e^{(A)i}_{\alpha}, f^{(B)\alpha}_{j} \ > = \ \delta^i_j \delta^A_B \quad (\,A, B = 0, 1, 2\,) \tag{2}$$

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In this frame the adapted basis has the representation

$$\frac{\delta}{\delta x^i} = f_i^{(0)\alpha}\frac{\delta}{\delta s^{(0)\alpha}} \quad \frac{\delta}{\delta y^{(1)i}} = f_i^{(1)\alpha}\frac{\delta}{\delta s^{(1)\alpha}} \quad \frac{\delta}{\delta y^{(2)i}} = f_i^{(2)\alpha}\frac{\delta}{\delta s^{(2)\alpha}} \tag{3}$$

and the cobasis

$$\delta x^i = e_\alpha^{(0)i}\delta s^{(0)\alpha} \quad ;\delta y^{(1)i} = e_\alpha^{(1)i}\delta s^{(1)\alpha} \quad ;\delta y^{(2)i} = e_\alpha^{(2)i}\delta s^{(2)\alpha} \tag{4}$$

and we have the relations

$$\left\langle \frac{\delta}{\delta s^{(A)\alpha}}, \delta s^{(B)\beta}\right\rangle = \delta_\alpha^\beta\,\delta_A^B \qquad (A,B = 0,1,2) \tag{5}$$

This representation lead us to an invariant frames transformation group with the analitycal expressions

$$\bar{e}_\alpha^{(A)i} = \overset{A}{C}{}_\alpha^\beta\ (x, y^{(1)}, y^{(2)}), e_\beta^{(A)i} \quad ; \quad f_j^{(B)\alpha} = \overset{B}{\bar{C}}{}_\beta^\alpha\,\bar{f}_j^{(B)\beta} \tag{6}$$

isomorphic with the multiplicative nonsingular matrix group

$$\begin{pmatrix} \overset{0}{C}{}_\beta^\alpha & 0 & 0 \\ 0 & \overset{1}{C}{}_\beta^\alpha & 0 \\ 0 & 0 & \overset{2}{C}{}_\beta^\alpha \end{pmatrix}$$

A N-linear connection D has in the frame $\Re$ the coefficients

$$\overset{0A}{L}{}_{\beta\alpha}^\gamma = f_m^{(A)\gamma}\left(\frac{\delta e_\beta^{(A)m}}{\delta s^{(0)\alpha}} + e_\alpha^{(0)i}e_\beta^{(A)j}L_{ij}^m\right) \quad (A = 0,1,2) \tag{7}$$

$$\overset{BA}{C}{}_{\beta\alpha}^\gamma = f_m^{(A)\gamma}\left(\frac{\delta e_\beta^{(A)m}}{\delta s^{(B)\alpha}} + e_\alpha^{(B)i}e_\beta^{(A)j}\ \underset{(B)}{C_{ij}^m}\right) \quad (A = 0,1,2\,;\,B = 1,2) \tag{8}$$

Definition 1.1. *If the vector field $X \in \chi(E)$ has the invariant components $X^{(A)\alpha}$ (A=0,1,2) and we denote by '$)$', $\overset{B}{)}$ the h− and v_B, B=1,2 the covariant invariant derivative operators then*

$$X^{(A)\alpha}{}_{)\beta} = \frac{\delta X^{(A)\alpha}}{\delta s^{(0)\beta}} + \overset{0A}{L^{\alpha}_{\varphi\beta}} X^{(A)\varphi} \tag{9}$$
$$X^{(A)\alpha} \overset{(B)}{)}_{\beta} = \frac{\delta X^{(A)\alpha}}{\delta s^{(B)\beta}} + \overset{BA}{C^{\alpha}_{\varphi\beta}} X^{(A)\varphi}$$

The definition of the Lie bracket conduces us to the introduction of the non-holonomy coefficients of Vranceanu

$$\left[\frac{\delta}{\delta s^{(A)\alpha}}, \frac{\delta}{\delta s^{((B)\beta}}\right] = \underset{(AB)}{\overset{0}{W^{\gamma}_{\alpha\beta}}} \frac{\delta}{\delta s^{(0)\gamma}} + \underset{(AB)}{\overset{1}{W^{\gamma}_{\alpha\beta}}} \frac{\delta}{\delta s^{(1)\gamma}} + \underset{(AB)}{\overset{2}{W^{\gamma}_{\alpha\beta}}} \frac{\delta}{\delta s^{(2)\gamma}} \tag{10}$$

$(A, B = 0, 1, 2; \quad A \leq B)$.

2. Torsion and Curvature d-tensor Fields

The torsion tensor of the N-linear connection D on E

$$\mathcal{T}(X,Y) = D_X Y - D_Y X - [X,Y] \qquad \forall\, X,Y \in \chi(E) \tag{11}$$

in the invariant frame $\Re$, has a number of horizontal and vertical components corresponding to D^h , D^{v_1} , D^{v_2}

Theorem 2.1. *The torsion tensor of a N-linear connection D in the invariant frame $\Re$ is characterized by the d-tensor fields with local components*

$$\begin{cases} \underset{(0)}{T^{\gamma}{}_{\beta\alpha}} = \overset{(00)}{L^{\gamma}{}_{\beta\alpha}} - \overset{(00)}{L^{\gamma}{}_{\alpha\beta}} - \underset{(00)}{\overset{(0)}{W^{\gamma}{}_{\beta\alpha}}} \\[2ex] \underset{(0A)}{R^{\gamma}{}_{\beta\alpha}} = \underset{(00)}{\overset{(A)}{W^{\gamma}{}_{\beta\alpha}}} \end{cases} \tag{12}$$

$$\begin{cases} \underset{(1)}{K}{}^{\gamma}{}_{\beta\alpha} = -\underset{(1)}{\overset{(10)}{C}}{}^{\gamma}{}_{\beta\alpha} - \underset{(01)}{\overset{(0)}{W}}{}^{\gamma}{}_{\beta\alpha} \\ \underset{(11)}{P}{}^{\gamma}{}_{\beta\alpha} = \overset{(01)}{L}{}^{\gamma}{}_{\beta\alpha} + \underset{(01)}{\overset{(1)}{W}}{}^{\gamma}{}_{\beta\alpha} \\ \underset{(12)}{P}{}^{\gamma}{}_{\beta\alpha} = \underset{(01)}{\overset{(2)}{W}}{}^{\gamma}{}_{\beta\alpha} \end{cases} \tag{13}$$

$$\begin{cases} \underset{(2)}{K}{}^{\gamma}{}_{\beta\alpha} = -\underset{(2)}{\overset{(20)}{C}}{}^{\gamma}{}_{\beta\alpha} - \underset{(02)}{\overset{(0)}{W}}{}^{\gamma}{}_{\beta\alpha} \\ \underset{(21)}{P}{}^{\gamma}{}_{\beta\alpha} = \underset{(02)}{\overset{(1)}{W}}{}^{\gamma}{}_{\alpha\beta} - \underset{(01)}{\overset{(1)}{W}}{}^{\gamma}{}_{\beta\alpha} \\ \underset{(22)}{P}{}^{\gamma}{}_{\beta\alpha} = \overset{(02)}{L}{}^{\gamma}{}_{\beta\alpha} - \underset{(02)}{\overset{(2)}{W}}{}^{\gamma}{}_{\beta\alpha} \end{cases} \tag{14}$$

$$\begin{cases} \underset{(11)}{Q}{}^{\gamma}{}_{\beta\alpha} = \underset{(1)}{\overset{(11)}{C}}{}^{\gamma}{}_{\beta\alpha} - \underset{(1)}{\overset{(11)}{C}}{}^{\gamma}{}_{\alpha\beta} - \underset{(11)}{\overset{(1)}{W}}{}^{\gamma}{}_{\beta\alpha} \\ \underset{(21)}{\overset{(2)}{Q}}{}^{\gamma}{}_{\beta\alpha} = \underset{(11)}{\overset{(2)}{W}}{}^{\gamma}{}_{\beta\alpha} \end{cases} \tag{15}$$

$$\begin{cases} \underset{(12)}{Q^{\gamma}{}_{\beta\alpha}} = \underset{(2)}{\overset{(21)}{C^{\gamma}{}_{\beta\alpha}}} - \underset{(12)}{\overset{(1)}{W^{\gamma}{}_{\beta\alpha}}} \\ \underset{(22)}{Q^{\gamma}{}_{\beta\alpha}} = -\underset{(12)}{\overset{(1)}{C^{\gamma}{}_{\alpha\beta}}} - \underset{(12)}{\overset{(2)}{W^{\gamma}{}_{\beta\alpha}}} \end{cases} \tag{16}$$

$$\begin{cases} \underset{(2)}{S^{\gamma}{}_{\beta\alpha}} = \underset{(2)}{\overset{(22)}{C^{\gamma}{}_{\beta\alpha}}} - \underset{(2)}{\overset{(22)}{C^{\gamma}{}_{\alpha\beta}}} - \underset{(22)}{\overset{(2)}{W^{\gamma}{}_{\beta\alpha}}} \end{cases} \tag{17}$$

Theorem 2.2. *The components given by Theorem 2.1 are the invariant components of the d-tensor fields of torsion of the N-linear connection D*

The curvature tensor field $\mathcal{R}$ of the N-linear connection D on $Osc^2(M)$ has the expression

$$\mathcal{R}(X,Y) = [\, D_X \, , \, D_Y \,]\, Z - D_{[X,Y]}Z \tag{18}$$

Theorem 2.3. *The curvature tensor field $\mathcal{R}$ of a N-linear connection D in the invariant frame $\mathfrak{R}$ is characterized by the following d-tensor fields on $Osc^2(M)$:*

$$\begin{aligned} R_{\gamma}{}^{\varphi}{}_{\beta\alpha} = {} & \frac{\delta \overset{(00)}{L^{\varphi}_{\gamma\beta}}}{\delta s^{(0)\alpha}} - \frac{\delta \overset{(00)}{L^{\varphi}_{\gamma\alpha}}}{\delta s^{(0)\beta}} + \overset{(00)}{L^{\eta}_{\gamma\beta}} \overset{(00)}{L^{\varphi}_{\eta\alpha}} - \overset{(00)}{L^{\eta}_{\gamma\alpha}} \overset{(00)}{L^{\varphi}_{\eta\beta}} - \\ & - \underset{(00)}{\overset{(0)}{W^{\psi}_{\beta\alpha}}} \overset{(00)}{L^{\varphi}_{\gamma\psi}} + \underset{(00)}{\overset{(1)}{W^{\psi}_{\beta\alpha}}} \underset{(1)}{\overset{(10)}{C^{\varphi}_{\gamma\psi}}} + \underset{(00)}{\overset{(2)}{W^{\psi}_{\beta\alpha}}} \underset{(2)}{\overset{(20)}{C^{\varphi}_{\gamma\psi}}} \end{aligned} \tag{19}$$

$$\underset{(1)}{P}{}_{\gamma\ \beta\alpha}^{\ \varphi} = \frac{\delta \overset{(10)}{\underset{(1)}{C}}{}^{\varphi}_{\gamma\beta}}{\delta s^{(0)\alpha}} - \frac{\delta \overset{(00)}{L}{}^{\varphi}_{\gamma\alpha}}{\delta s^{(1)\beta}} + \overset{(10)}{\underset{(1)}{C}}{}^{\eta}_{\gamma\beta} \overset{(00)}{L}{}^{\varphi}_{\eta\alpha} - \overset{(00)}{L}{}^{\eta}_{\gamma\alpha} \overset{(10)}{\underset{(1)}{C}}{}^{\varphi}_{\eta\beta} - \overset{(0)}{\underset{(01)}{W}}{}^{\psi}_{\beta\alpha} \overset{(00)}{L}{}^{\varphi}_{\gamma\psi} + \overset{(1)}{\underset{(01)}{W}}{}^{\psi}_{\beta\alpha} \overset{(10)}{\underset{(1)}{C}}{}^{\varphi}_{\gamma\psi} + \overset{(2)}{\underset{(01)}{W}}{}^{\psi}_{\beta\alpha} \overset{(20)}{\underset{(2)}{C}}{}^{\varphi}_{\gamma\psi} \tag{20}$$

$$\underset{(2)}{P}{}_{\gamma\ \beta\alpha}^{\ \varphi} = \frac{\delta \overset{(20)}{\underset{(2)}{C}}{}^{\varphi}_{\gamma\beta}}{\delta s^{(0)\alpha}} - \frac{\delta \overset{(00)}{L}{}^{\varphi}_{\gamma\alpha}}{\delta s^{(2)\beta}} + \overset{(20)}{\underset{(2)}{C}}{}^{\eta}_{\gamma\beta} \overset{(00)}{L}{}^{\varphi}_{\eta\alpha} - \overset{(00)}{L}{}^{\eta}_{\gamma\alpha} \overset{(20)}{\underset{(2)}{C}}{}^{\varphi}_{\eta\beta} - \overset{(0)}{\underset{(02)}{W}}{}^{\psi}_{\beta\alpha} \overset{(00)}{L}{}^{\varphi}_{\gamma\psi} + \overset{(1)}{\underset{(02)}{W}}{}^{\psi}_{\beta\alpha} \overset{(10)}{\underset{(1)}{C}}{}^{\varphi}_{\gamma\psi} + \overset{(2)}{\underset{(02)}{W}}{}^{\psi}_{\beta\alpha} \overset{(20)}{\underset{(2)}{C}}{}^{\varphi}_{\gamma\psi} \tag{21}$$

$$\underset{(11)}{S}{}_{\gamma\ \beta\alpha}^{\ \varphi} = \frac{\delta \overset{(10)}{\underset{(1)}{C}}{}^{\varphi}_{\gamma\beta}}{\delta s^{(1)\alpha}} - \frac{\delta \overset{(01)}{\underset{(1)}{C}}{}^{\varphi}_{\gamma\alpha}}{\delta s^{(1)\beta}} + \overset{(10)}{\underset{(1)}{C}}{}^{\eta}_{\gamma\beta} \overset{(10)}{\underset{(1)}{C}}{}^{\varphi}_{\eta\alpha} - \overset{(10)}{\underset{(1)}{C}}{}^{\eta}_{\gamma\alpha} \overset{(10)}{\underset{(1)}{C}}{}^{\varphi}_{\eta\beta} - \overset{(1)}{\underset{(11)}{W}}{}^{\psi}_{\beta\alpha} \overset{(10)}{\underset{(1)}{C}}{}^{\varphi}_{\gamma\psi} - \overset{(2)}{\underset{(11)}{W}}{}^{\psi}_{\beta\alpha} \overset{(20)}{\underset{(2)}{C}}{}^{\varphi}_{\gamma\psi} \tag{22}$$

$$\underset{(21)}{S}{}_{\gamma}{}^{\varphi}{}_{\beta\alpha} = \frac{\delta \underset{(2)}{\overset{(20)}{C}}{}^{\varphi}_{\gamma\beta}}{\delta s^{(1)\alpha}} - \frac{\delta \underset{(1)}{\overset{(10)}{C}}{}^{\varphi}_{\gamma\alpha}}{\delta s^{(2)\beta}} + \underset{(2)}{\overset{(20)}{C}}{}^{\eta}_{\gamma\beta} \underset{(1)}{\overset{(10)}{C}}{}^{\varphi}_{\eta\alpha} - \underset{(1)}{\overset{(10)}{C}}{}^{\eta}_{\gamma\alpha} \underset{(2)}{\overset{(20)}{C}}{}^{\varphi}_{\eta\beta} - \underset{(12)}{\overset{(1)}{W}}{}^{\psi}_{\beta\alpha} \underset{(1)}{\overset{(10)}{C}}{}^{\varphi}_{\gamma\psi} - \underset{(12)}{\overset{(2)}{W}}{}^{\psi}_{\beta\alpha} \underset{(2)}{\overset{(20)}{C}}{}^{\varphi}_{\gamma\psi} \tag{23}$$

$$\underset{(22)}{S}{}_{\gamma}{}^{\varphi}{}_{\beta\alpha} = \frac{\delta \underset{(2)}{\overset{(20)}{C}}{}^{\varphi}_{\gamma\beta}}{\delta s^{(2)\alpha}} - \frac{\delta \underset{(2)}{\overset{(20)}{C}}{}^{\varphi}_{\gamma\alpha}}{\delta s^{(2)\beta}} + \underset{(2)}{\overset{(20)}{C}}{}^{\eta}_{\gamma\beta} \underset{(2)}{\overset{(20)}{C}}{}^{\varphi}_{\eta\alpha} - \underset{(2)}{\overset{(20)}{C}}{}^{\eta}_{\gamma\alpha} \underset{(2)}{\overset{(20)}{C}}{}^{\varphi}_{\eta\beta} - \underset{(22)}{\overset{(2)}{W}}{}^{\psi}_{\beta\alpha} \underset{(2)}{\overset{(20)}{C}}{}^{\varphi}_{\gamma\psi} \tag{24}$$

Theorem 2.4. *The components given by Theorem 2.3 are the invariant components of the d-tensor fields of curvature of the N-linear connection D*

Theorem 2.5. *In the frame $\mathfrak{R}$ the essential components of the curvature tensor field $\mathcal{R}$ are those given by Theorem 2.3.*

3. Fundamental Identities, Maxwell Equations

Begining from Jacoby identities we obtain

Theorem 3.1. *The non-holonomy coefficients W given by satisfy the following fundamental identities called Vranceanu identities:*

$$\sum_{\substack{\text{cicl}\\(\alpha\,\beta\,\gamma)}} \left\{ \underset{(00)}{\overset{(I)}{W^{\sigma}{}_{\beta\gamma}}}\ \underset{(0I)}{\overset{(J)}{W^{\eta}{}_{\alpha\sigma}}} + \frac{\underset{(00)}{\overset{(J)}{W^{\eta}{}_{\beta\gamma}}}}{\delta s^{(I)\alpha}} \right\} = 0 \tag{25}$$

(I,J=0,1,2; summation also by I)

$$\underset{(\alpha,\beta)}{\succ\prec} \left\{ \underset{(0K)}{\overset{(I)}{W^{\sigma}{}_{\beta\gamma}}}\ \underset{(0I)}{\overset{(J)}{W^{\eta}{}_{\alpha\sigma}}} + \frac{\underset{(0K)}{\overset{(J)}{W^{\eta}{}_{\beta\gamma}}}}{\delta s^{(0)\alpha}} + \frac{1}{2}\ \underset{(00)}{\overset{(I)}{W^{\sigma}{}_{\alpha\beta}}}\ \underset{(IK)}{\overset{(J)}{W^{\eta}{}_{\sigma\gamma}}} + \frac{1}{2}\frac{\underset{(00)}{\overset{(J)}{W^{\eta}{}_{\alpha\beta}}}}{\delta s^{(K)\gamma}} \right\} = 0 \tag{26}$$

$$\underset{(\beta,\gamma)}{\succ\prec} \left\{ \underset{(0K)}{\overset{(I)}{W^{\sigma}{}_{\gamma\alpha}}}\ \underset{(0I)}{\overset{(J)}{W^{\eta}{}_{\beta\alpha}}} + \frac{\underset{(0K)}{\overset{(J)}{W^{\eta}{}_{\gamma\alpha}}}}{\delta s^{(1)\beta}} + \frac{1}{2}\ \underset{(KK)}{\overset{(I)}{W^{\sigma}{}_{\beta\gamma}}}\ \underset{(KI)}{\overset{(J)}{W^{\eta}{}_{\alpha\sigma}}} + \frac{1}{2}\frac{\underset{(KK)}{\overset{(J)}{W^{\eta}{}_{\beta\gamma}}}}{\delta s^{(0)\alpha}} \right\} = 0 \tag{27}$$

(I,J=0,1,2; K=1,2; $\succ\prec$ meaning permutation of indexes and subtraction of results)

$$\underset{(12)}{\overset{(1)}{W^{\sigma}{}_{\beta\gamma}}}\ \underset{(01)}{\overset{(0)}{W^{\eta}{}_{\alpha\sigma}}} + \underset{(12)}{\overset{(2)}{W^{\sigma}{}_{\beta\gamma}}}\ \underset{(02)}{\overset{(0)}{W^{\eta}{}_{\alpha\sigma}}} + \underset{(02)}{\overset{(0)}{W^{\sigma}{}_{\gamma\alpha}}}\ \underset{(01)}{\overset{(0)}{W^{\eta}{}_{\sigma\beta}}} -$$

$$- \underset{(01)}{\overset{(0)}{W^{\sigma}{}_{\alpha\beta}}} \quad \underset{(02)}{\overset{(0)}{W^{\eta}{}_{\sigma\gamma}}} + \Sigma \frac{\underset{(12)}{\overset{(0)}{W^{\eta}{}_{\beta\gamma}}}}{\delta s^{(0)\alpha}} = 0 \tag{28}$$

$$\underset{(12)}{\overset{(I)}{W^{\sigma}{}_{\beta\gamma}}} \quad \underset{(0I)}{\overset{(1)}{W^{\eta}{}_{\alpha\sigma}}} + \underset{(02)}{\overset{(I)}{W^{\sigma}{}_{\gamma\alpha}}} \quad \underset{(I1)}{\overset{(1)}{W^{\eta}{}_{\sigma\beta}}} - \underset{(01)}{\overset{(I)}{W^{\sigma}{}_{\alpha\beta}}} \quad \underset{(I2)}{\overset{(1)}{W^{\eta}{}_{\sigma\gamma}}} + \Sigma \frac{\underset{(12)}{\overset{(1)}{W^{\eta}{}_{\beta\gamma}}}}{\delta s^{(0)\alpha}} = 0 \tag{29}$$

$$\underset{(12)}{\overset{(I)}{W^{\sigma}{}_{\beta\gamma}}} \quad \underset{(0I)}{\overset{(2)}{W^{\eta}{}_{\alpha\sigma}}} + \underset{(02)}{\overset{(I)}{W^{\sigma}{}_{\gamma\alpha}}} \quad \underset{(I1)}{\overset{(2)}{W^{\eta}{}_{\sigma\beta}}} - \underset{(01)}{\overset{(I)}{W^{\sigma}{}_{\alpha\beta}}} \quad \underset{(I2)}{\overset{(2)}{W^{\eta}{}_{\sigma\gamma}}} + \Sigma \frac{\underset{(12)}{\overset{(2)}{W^{\eta}{}_{\beta\gamma}}}}{\delta s^{(0)\alpha}} = 0 \tag{30}$$

(I=0,1,2; summation by I; Σ *meaning sumation on simultaneous cycle on pairs* $(0,\alpha);(1,\beta);(2,\gamma)$

Denoting by

$$q^{((1)\alpha} = s^{(1)\alpha}\ ; \quad q^{(2)\alpha} = s^{(2)\alpha} + \frac{1}{2} \underset{(1)}{M^{\alpha}{}_{\beta}}\ s^{(1)\beta} \tag{31}$$

then in the considerated invariant frame the Liouville vector fields are:

$$\overset{1}{\Gamma} = q^{(1)\alpha} e^{(1)i}_{\alpha} f^{(2)\beta}_{i} \frac{\delta}{\delta s^{(2)\beta}}$$

$$\overset{2}{\Gamma} = q^{(1)\alpha} \frac{\delta}{\delta s^{(1)\alpha}} + 2\, q^{(2)\alpha} \frac{\delta}{\delta s^{(2)\alpha}} \tag{32}$$

Let us consider the generalized Lagrange space $GL^{(2n)} = (M, g_{ij}(x, y^{(1)}, y^{(2)}))$ with g_{ij} symmetric and nondegenerated, the canonical metrical linear N-connection $L\Gamma(N)$ and the case when the three frames adapted to the three distributions are the same. Then

$$e^{(0)i}_{\alpha} = e^{(1)i}_{\alpha} = e^{(2)i}_{\alpha} = e^{i}_{\alpha} \tag{33}$$

and simillar for the duals.

In this frames the canonical metrical linear N-connection has the coefficients:

$$\overset{(00)}{L^{\alpha}_{\beta\gamma}} = \frac{1}{2}\overset{(0)}{\underset{(00)}{W^{\alpha}_{\beta\gamma}}} + f^{\alpha}_{i} e^{j}_{\beta} e^{k}_{\gamma} L^{i}_{jk}$$

$$\overset{(11)}{\underset{(1)}{C^{\alpha}_{\beta\gamma}}} = \frac{1}{2}\overset{(1)}{\underset{(11)}{W^{\alpha}_{\beta\gamma}}} + f^{\alpha}_{i} e^{j}_{\beta} e^{k}_{\gamma} \underset{(1)}{C^{\alpha}_{\beta\gamma}}$$

$$\overset{(22)}{\underset{(2)}{C^{\alpha}_{\beta\gamma}}} = \frac{1}{2}\overset{(2)}{\underset{(22)}{W^{\alpha}_{\beta\gamma}}} + f^{\alpha}_{i} e^{j}_{\beta} e^{k}_{\gamma} \underset{(2)}{C^{\alpha}_{\beta\gamma}} \tag{34}$$

We shall consider now the tensor fields

$$D^{(A)\alpha}_{\beta} = q^{(A)\alpha}_{\ \ )\beta} \qquad d^{(AB)\alpha}_{\beta} = q^{(A)\alpha} \overset{(B)}{)} \tag{35}$$

(A,B=1,2)

Theorem 3.2.. *The tensor fields defined above represent the invariant conponents of the deflection tensor of the canonical metrical N-linear connection.*

We define the invariant electromagnetic tensor field by:

$$F^{(A)}_{\alpha\beta} = \frac{1}{2}\left\{\frac{\delta q^{(A)}_{\alpha}}{\delta s^{(0)\beta}} - \frac{\delta q^{(A)}_{\beta}}{\delta s^{(0)\alpha}}\right\}$$

$$f^{(AB)}_{\alpha\beta} = \frac{1}{2}\left\{\frac{\delta q^{(A)}_{\alpha}}{\delta s^{(B)\beta}} - \frac{\delta q^{(A)}_{\beta}}{\delta s^{(B)\alpha}}\right\} \tag{36}$$

(A,B=1,2)

Theorem 3.3.. *The electromagnetic tensor fields have the expressions*

$$F^{(A)}_{\alpha\beta} = \frac{1}{2}\left(D^{(A)}_{\alpha\beta} - D^{(A)}_{\beta\alpha}\right)$$

$$f^{(AB)}_{\alpha\beta} = \frac{1}{2}\left(d^{(AB)}_{\alpha\beta} - d^{(AB)}_{\beta\alpha}\right) \tag{37}$$

and represent the invariant components of the electromagnetic tensor fields of the cononical metrical N-linear connection.

Using Ricci identities with respect to $C\Gamma(N)$ we prove

Theorem 3.4. *The electromagnetic tensor fields $F^{(A)}_{\alpha\beta}$ and $f^{(AB)}_{\alpha\beta}$ of the generalized Lagrange space $GL^{(2n)}$ satisfy the following Maxwell generalized equations:*

$$\Sigma F^{(A)}_{\alpha\beta|\gamma} = \Sigma \left\{ q^{(A)\eta} R_{\eta\beta\alpha\gamma} - \sum_{B=1}^{2} d^{(AB)}_{\beta\eta} \underset{(0B)}{R^{\eta}{}_{\alpha\gamma}} \right\}$$

$$\Sigma F^{(A)}_{\alpha\beta} \overset{(B)}{|}_{\gamma} + \Sigma f^{(AB)}_{\alpha\beta|\gamma} = \Sigma \left\{ q^{(A)\eta} \left(\underset{(B)}{P_{\eta\beta\alpha\gamma}} - \underset{(B)}{P_{\eta\beta\gamma\alpha}} \right) - \sum_{B=1}^{2} d^{(AB)}_{\beta\eta} \left(\underset{(B)}{P_{\eta\beta\alpha\gamma}} - \underset{(B)}{P_{\eta\beta\gamma\alpha}} \right) \right\}$$

$$\Sigma f^{(AB)}_{\alpha\beta} \overset{(B)}{|}_{\gamma} = \Sigma \left\{ q^{(A)\eta} \underset{(A)}{S_{\eta\beta\alpha\gamma}} - \sum_{C=1}^{2} d^{(AB)}_{\beta\eta} \left(\underset{(BC)}{R^{\eta}{}_{\alpha\gamma}} \right. \right\}$$

$$\Sigma f^{(AB)}_{\alpha\beta} \overset{(C)}{|}_{\gamma} = \Sigma \left\{ q^{(A)\eta} \underset{(BC)}{P_{\eta\beta\alpha\gamma}} - \sum_{D=1}^{2} d^{(AB)}_{\beta\eta} \left(\underset{(BC)}{\overset{(D)}{P^{\eta}{}_{\alpha\gamma}}} - \right. \right.$$
$$\left. -d^{(AB)}_{\beta\eta} \left(\underset{(B)}{C^{\eta}{}_{\alpha\gamma}} - \underset{(B)}{C^{\eta}{}_{\gamma\alpha}} \right) \right\} \qquad B \neq C \tag{38}$$

Theorem 3.5. *If the canonical metrical N-linear connection is torsionless then* $F^{(A)}_{\alpha\beta}$ *and* $f^{(AB)}_{\alpha\beta}$ *satisfy the folowing generalized Maxwell equations:*

$$\begin{aligned} \Sigma F^{(A)}_{\alpha\beta|\gamma} &= 0 \\ \Sigma F^{(A)}_{\alpha\beta} \overset{(B)}{|}_{\gamma} + \Sigma f^{(AB)}_{\alpha\beta|\gamma} &= 0 \\ \Sigma f^{(AB)}_{\alpha\beta} \overset{(B)}{|}_{\gamma} &= 0 \\ \Sigma f^{(AB)}_{\alpha\beta} \overset{(C)}{|}_{\gamma} &= 0 \end{aligned} \qquad (39)$$

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