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ON SEMI-DECOMPOSABLE PSEUDO-SYMMETRIC WEYL SPACES

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Dedicated to Professor Pavel Enghiş at his 70th anniversary

Abstract. In this paper, we first prove that, if a semi-decomposable Weyl space W_n can be written as the product of two Weyl spaces $\bar{W}_q$ and W^*_{n-q}, then W_n has homothetic metrics. Next, after having given the definitions of symmetric and pseudo-symmetric Weyl spaces, we have shown that the symmetric Weyl space W_n can be written as the product of the symmetric subspaces $\bar{W}_q$ and W^*_{n-q}, if and only if the complementary vector field of $\bar{W}_q$ is the gradient of $\ln\sqrt{\sigma}$. Finally, we prove two theorems concerning semi-decomposable pseudo-symmetric Weyl spaces.

1. Introduction

An n-dimensional manifold W_n is said to be a Weyl space if it has a conformal metric tensor g_{ij} and a symmetric connection ∇_k satisfying the compatibility condition given by the equation

$$\nabla_k g_{ij} - 2T_k g_{ij} = 0 \ , \tag{1.1}$$

where T_k denotes a covariant vector field [1].

Under a renormalization of the fundamental tensor of the form

$$\widetilde{g_{ij}} = \lambda^2 g_{ij} \tag{1.2}$$

the complementary vector field T_k is transformed by the law

$$\widetilde{T_i} = T_i + \partial_i \ln\lambda \ , \tag{1.3}$$

where λ is a scalar function defined on W_n.

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The coefficients Γ^i_{kl} of the Weyl connection ∇_k are given by

$$\Gamma^i_{kl} = \left\{ \begin{matrix} i \\ kl \end{matrix} \right\} - g^{im} \left(g_{mk} T_l + g_{ml} T_k - g_{kl} T_m \right) . \tag{1.4}$$

A quantity A is called a satellite with weight $\{p\}$ of the tensor g_{ij}, if it admits a transformation of the form

$$\widetilde{A} = \lambda^p A$$

under the renormalization (1.2) of the metric tensor g_{ij}[2].

The prolonged covariant derivative of a satellite A of the tensor g_{ij} with weight $\{p\}$ is defined by [2]

$$\dot{\nabla}_k A = \nabla_k A - p T_k A . \tag{1.5}$$

2. SEmi-decomposable Weyl spaces

As in the Riemannian case [3], we will say that an n-dimensional Weyl space W_n $(n > 2)$ is a semi-decomposable space if its metric can be given in some coordinate system by

$$ds^2 = g_{ij}\, dx^i dx^j = \overline{g}_{ab}\, dx^a dx^b + \sigma\, g^*_{\alpha\beta}\, dx^\alpha dx^\beta \tag{2.1}$$

$$(i, j, k, ... = 1, 2, ..., n \ ; \ a, b, c, ... = 1, 2, ..., q \ ; \ \alpha, \beta, \gamma, ... = q+1, q+2, ..., n)$$

where

$$g_{ab} = \overline{g}_{ab} (x^c) \ , \ g_{\alpha\beta} = \sigma g^*_{\alpha\beta} (x^\gamma) \tag{2.1$'$}$$

and σ is a function of $x^1, x^2, ..., x^q$ with weight $\{0\}$. The two parts of (2.1) are the metrics of the two Weyl spaces $\bar{W}_q$ and W^*_{n-q} which are called the complementary spaces of W_n.

Throughout this paper, objects denoted by a bar or a star will respectively assumed to be formed by $\overline{g}_{ab}$ and $g^*_{\alpha\beta}$ while $\dot{\nabla}$, $\dot{\overline{\nabla}}$, $\dot{\nabla}^*$ indicate prolonged covariant differentiation in W_n , $\bar{W}_q$ and W^*_{n-q} respectively. If, in particular $\sigma = 1$, then W_n reduces to a decomposable space.

Suppose that $\bar{\Gamma}^a_{bc}$, $\bar{R}_{abcd}$, $\bar{T}_a$ denote, respectively the connection coefficients, the curvature tensor and the complementary vector field of $\bar{W}_q$ and let $\overset{*}{\Gamma}{}^\alpha_{\beta\gamma}$, $\overset{*}{R}_{\alpha\beta\gamma\delta}$, T^*_α

refer to the subspace W^*_{n-q} of a semi-decomposable Weyl space with non-constant function σ. We then have

$$g_{ab} = \overline{g}_{ab} \ , \ g_{\alpha\beta} = \sigma g^*_{\alpha\beta} \ , \ g^{ab} = \overline{g}^{ab} \ , \ g^{\alpha\beta} = \frac{1}{\sigma} g^{*\alpha\beta} \ , \ g_{a\alpha} = 0 \ , \ g^{a\alpha} = 0. \qquad (2.2)$$

From the compatibility condition (1.1) we get

$$T_a = \bar{T}_a \ , \ T_\alpha = T^*_\alpha \qquad (2.3)$$

and consequently the connection coefficients are related by

$$\Gamma^a_{bc} = \bar{\Gamma}^a_{bc} \ , \ \Gamma^\alpha_{\beta\gamma} = \Gamma^{*\alpha}_{\beta\gamma} \qquad (2.4)$$

$$\Gamma^a_{\beta\gamma} = -\sigma \overline{g}^{ab} \overline{u}_b g^*_{\beta\gamma} \ , \ \Gamma^\alpha_{a\beta} = \overline{u}_a \delta^\alpha_\beta \ , \ \Gamma^a_{b\alpha} = -\delta^a_b T^*_\alpha \ , \ \Gamma^\alpha_{ab} = \frac{1}{\sigma} \overline{g}_{ab} g^{*\alpha\beta} T^*_\beta \qquad (2.5)$$

where

$$\sigma_{,a} = \frac{\partial\sigma}{\partial x^a} \ , \ \overline{u}_a = \frac{1}{2\sigma}\sigma_{,a} - \overline{T}_a \ .$$

On the other hand, using the expression [4]

$$R_{ijkl} = g_{ih} R^h_{jkl} \ , \ R^h_{jkl} = \frac{\partial}{\partial x^k}\Gamma^h_{jl} - \frac{\partial}{\partial x^l}\Gamma^h_{jk} + \Gamma^h_{ki}\Gamma^i_{jl} - \Gamma^h_{li}\Gamma^i_{jk}$$

for the covariant curvature tensor R_{ijkl}, we show that the curvature tensors of W_n , $\bar{W}_q$ and W^*_{n-q} are related by

$$\begin{aligned} R_{abcd} &= \bar{R}_{abcd} + \frac{1}{\sigma} T^*_\alpha T^*_\beta g^{*\alpha\beta} \overline{A}_{abcd} \\ R_{\alpha\beta\gamma\delta} &= \sigma \overset{*}{R}_{\alpha\beta\gamma\delta} + \sigma^2 \overline{u}_a \overline{u}_b \overline{g}^{ab} \overset{*}{A}_{\alpha\beta\gamma\delta} \\ R_{a\alpha b\beta} &= -R_{a\alpha\beta b} = -R_{\alpha a b\beta} = -\sigma g^*_{\alpha\beta} \bar{A}_{ab} - \overline{g}_{ab} B^*_{\alpha\beta} \ , \end{aligned} \qquad (2.6)$$

where we have put

$$\overline{A}_{abcd} = \overline{g}_{ad}\overline{g}_{bc} - \overline{g}_{ac}\overline{g}_{bd} \ , \ \bar{A}_{ab} = \dot{\overline{\nabla}}_b \overline{u}_a + \overline{u}_a \overline{u}_b \ , \ B^*_{\alpha\beta} = -\dot{\nabla}^*_\beta T^*_\alpha + T^*_\alpha T^*_\beta \ . \qquad (2.7)$$

These relations are the Weyl versions of the relations obtained in [5] for a Riemannian semi-decomposable space. After some calculations and simplifications we find that

$$R_{a\beta\gamma\delta} = \sigma \overline{u}_a \left(g^*_{\beta\gamma} T^*_\delta - g^*_{\beta\delta} T^*_\gamma \right) , \; R_{\alpha a\gamma\delta} = \sigma \overline{u}_a \left(g^*_{\alpha\delta} T^*_\gamma - g^*_{\alpha\gamma} T^*_\delta \right)$$

$$R_{\alpha\beta a\delta} = \sigma \overline{u}_a \left(g^*_{\alpha\delta} T^*_\beta - g^*_{\beta\delta} T^*_\alpha \right) , \; R_{\alpha\beta\gamma a} = \sigma \overline{u}_a \left(g^*_{\beta\gamma} T^*_\alpha - g^*_{\alpha\gamma} T^*_\beta \right)$$

$$R_{\alpha bcd} = T^*_\alpha \left(\overline{g}_{bc} \overline{u}_d - \overline{g}_{bd} \overline{u}_c \right) , \; R_{a\alpha cd} = T^*_\alpha \left(\overline{g}_{ad} \overline{u}_c - \overline{g}_{ac} \overline{u}_d \right)$$

$$R_{ab\alpha d} = T^*_\alpha \left(\overline{g}_{ad} \overline{u}_b - \overline{g}_{bd} \overline{u}_a \right) , \; R_{abc\alpha} = T^*_\alpha \left(\overline{g}_{bc} \overline{u}_a - \overline{g}_{ac} \overline{u}_b \right)$$

$$R_{ab\alpha\beta} = \overline{g}_{ab} \left(\frac{\partial T^*_\alpha}{\partial x^\beta} - \frac{\partial T^*_\beta}{\partial x^\alpha} \right) , \; R_{\alpha\beta ab} = \sigma g^*_{\alpha\beta} \left(\frac{\partial \bar{T}_a}{\partial x^b} - \frac{\partial \bar{T}_b}{\partial x^a} \right) . \tag{2.8}$$

If the Weyl space W_n is Riemannian, then all the quantities in (2.8) become zero which explain a well-known result for a semi-decomposable Riemannian space [5].

We first prove the following theorem concerning semi-decomposable Weyl spaces.

Theorem 2.1. *A semi-decomposable Weyl space which can be written as the product of two Weyl spaces has homothetic metrics.*

Proof. For the conformal change of the metric tensors g_{ab} , $\overline{g}_{ab}$ and $g^*_{\alpha\beta}$ we have

$$\widetilde{g}_{ij} = \lambda^2 g_{ij} \; , \; \widetilde{\overline{g}}_{ab} = \overline{\lambda}^2 \overline{g}_{ab} \; , \; \widetilde{g}^*_{\alpha\beta} = \lambda^{*2} g^*_{\alpha\beta}$$

where

$$\lambda = \lambda(x^1, x^2, ..., x^n) \; , \; \overline{\lambda} = \overline{\lambda}(x^1, x^2, ..., x^q) \; , \; \lambda^* = \lambda^*(x^{q+1}, x^{q+2}, ..., x^n). \tag{2.9}$$

Then, using (2.2) and (2.9) we obtain

$$\lambda = \overline{\lambda} = \lambda^*$$

which states that λ , $\overline{\lambda}$, λ^* are equal to the same constant c. But this means that W_n has a homothetic metric.

For a Weyl space with a homothetic metric tensor, the complementary vector field T_k is invariant under the transformation (1.2). So, such a Weyl space will be Riemannian if and only if the complementary vector field T_k is identically zero. □

Remark 1. It can be easily seen that a Weyl space W_n can not be written as the product of a Weyl space and a Riemannian space, unless W_n is Riemannian.

3. Pseudo-symmetric Weyl spaces

The Weyl space W_n whose curvature tensor R_{hijk} satisfies the condition

$$\dot{\nabla}_l R_{hijk} = 2\lambda_l\, R_{hijk} + \lambda_h\, R_{lijk} + \lambda_i\, R_{hljk} + \lambda_j\, R_{hilk} + \lambda_k\, R_{hijl} \tag{3.1}$$

will be called a pseudo-symmetric space and will be denote by PSW_n, λ_i being a covariant vector field with weight $\{0\}$.

Since the weight of R_{hijk} is $\{2\}$, by (1.5) we get

$$\dot{\nabla}_l R_{hijk} = \nabla_l\, R_{hijk} - 2T_l\, R_{hijk} \tag{3.2}$$

so that (3.1) becomes

$$\nabla_l R_{hijk} = 2(T_l + \lambda_l)\, R_{hijk} + \lambda_h\, R_{lijk} + \lambda_i\, R_{hljk} + \lambda_j\, R_{hilk} + \lambda_k\, R_{hijl}\ . \tag{3.3}$$

If $T_l = 0$, W_n becomes a Riemannian space and (3.3) reduces to

$$\nabla_l R_{hijk} = 2\lambda_l\, R_{hijk} + \lambda_h\, R_{lijk} + \lambda_i\, R_{hljk} + \lambda_j\, R_{hilk} + \lambda_k\, R_{hijl} \tag{3.4}$$

which is the definition of a pseudo-symmetric Riemannian space [6].

We will say that a Weyl space is symmetric if the condition

$$\dot{\nabla}_l R_{hijk} = 0 \tag{3.5}$$

is satisfied. This definition reduces to the definition of a symmetric Riemannian space if we take $T_k = 0$ in (3.5).

It can be shown that a symmetric Weyl space with $\lambda \neq$ *const.* is Riemannian since, in this case, the complementary vector field becomes locally a gradient [7].

Theorem 3.1. *A semi-decomposable, symmetric elliptic Weyl space W_n $(n > 2)$ with $\sigma \neq$ const. can be written as the product of two symmetric Weyl spaces $\bar{W}_q$ and W^*_{n-q}, if and only if* $\overline{T}_a = \left(\frac{\partial \ln \sqrt{\sigma}}{\partial x^a}\right)$.

Proof. Remembering that

$$\nabla_l R_{hijk} = \partial_l R_{hijk} - \Gamma^m_{hl} R_{mijk} - \Gamma^m_{il} R_{hmjk} - \Gamma^m_{jl} R_{himk} - \Gamma^m_{kl} R_{hijm}$$

and using (1.5) , (2.4) , (2.5) , (2.6) , (2.7) and (2.8), after some calculations and simplifications we obtain

$$\dot{\nabla}_e R_{abcd} = \dot{\overline{\nabla}}_e \overline{R}_{abcd} - \frac{1}{\sigma} g^{*\alpha\beta} T^*_\alpha T^*_\beta [\overline{A}_{ebcd} \overline{u}_a + \overline{A}_{aecd} \overline{u}_b + \overline{A}_{abed} \overline{u}_c + \overline{A}_{abce} \overline{u}_d + 2\overline{A}_{abcd} \overline{u}_e] \tag{3.6}$$

$$\dot{\nabla}_\eta R_{\alpha\beta\gamma\delta} = \sigma \dot{\nabla}^*_\eta \overset{*}{R}_{\alpha\beta\gamma\delta} + \sigma^2 \overline{g}^{ab} \overline{u}_a \overline{u}_b [\overset{*}{A}_{\eta\beta\gamma\delta} T^*_\alpha + \overset{*}{A}_{\alpha\eta\gamma\delta} T^*_\beta + \overset{*}{A}_{\alpha\beta\eta\delta} T^*_\gamma + \overset{*}{A}_{\alpha\beta\gamma\eta} T^*_\delta + 2\overset{*}{A}_{\alpha\beta\gamma\delta} T^*_\eta] \tag{3.7}$$

First, suppose that $\overline{W}_q$ and W^*_{n-q} are symmetric. By the definition we get

$$\dot{\overline{\nabla}}_e \overline{R}_{abcd} = 0 \ , \ \dot{\nabla}^*_\eta \overset{*}{R}_{\alpha\beta\gamma\delta} = 0.$$

On the other hand, since W_n is symmetric we have

$$\dot{\nabla}_e R_{abcd} = 0 \ , \ \dot{\nabla}_\eta R_{\alpha\beta\gamma\delta} = 0.$$

Under these symmetry conditions, (3.6) and (3.7) reduce, respectively to

$$g^{*\alpha\beta} T^*_\alpha T^*_\beta [\overline{A}_{ebcd} \overline{u}_a + \overline{A}_{aecd} \overline{u}_b + \overline{A}_{abed} \overline{u}_c + \overline{A}_{abce} \overline{u}_d + 2\overline{A}_{abcd} \overline{u}_e] = 0 \tag{3.8}$$

$$\overline{g}^{ab} \overline{u}_a \overline{u}_b [\overset{*}{A}_{\eta\beta\gamma\delta} T^*_\alpha + \overset{*}{A}_{\alpha\eta\gamma\delta} T^*_\beta + \overset{*}{A}_{\alpha\beta\eta\delta} T^*_\gamma + \overset{*}{A}_{\alpha\beta\gamma\eta} T^*_\delta + 2\overset{*}{A}_{\alpha\beta\gamma\delta} T^*_\eta] = 0 \ . \tag{3.9}$$

Since the space W_n is assumed to be elliptic, i.e. the metric is positive definite and W^*_{n-q} is not Riemannian the factor $g^{*\alpha\beta} T^*_\alpha T^*_\beta$ in (3.8) can not be zero. On the other hand, if in (3.9) $\overline{g}^{ab} \overline{u}_a \overline{u}_b = 0$, it follows that $\overline{u}_a = 0$, i.e. $\overline{T}_a = \left(\frac{\partial \ln \sqrt{\sigma}}{\partial x^a}\right)$ and consequently (3.8) and (3.9) are automatically satisfied.

Suppose now that

$$g^{*\alpha\beta} T^*_\alpha T^*_\beta \neq 0 \ , \ \overline{g}^{ab} \overline{u}_a \overline{u}_b \neq 0 \ .$$

In this case (3.8) and (3.9) are reduced to

$$[\overline{A}_{ebcd} \overline{u}_a + \overline{A}_{aecd} \overline{u}_b + \overline{A}_{abed} \overline{u}_c + \overline{A}_{abce} \overline{u}_d + 2\overline{A}_{abcd} \overline{u}_e] = 0 \tag{3.8$'$}$$

$$[\overset{*}{A}_{\eta\beta\gamma\delta} T^*_\alpha + \overset{*}{A}_{\alpha\eta\gamma\delta} T^*_\beta + \overset{*}{A}_{\alpha\beta\eta\delta} T^*_\gamma + \overset{*}{A}_{\alpha\beta\gamma\eta} T^*_\delta + 2\overset{*}{A}_{\alpha\beta\gamma\delta} T^*_\eta] = 0 \ . \tag{3.9$'$}$$

Now, transvecting $(3.8)'$ by $\overline{g}^{ad}$ and $\overline{g}^{bc}$ and $(3.9)'$ by $g^{*\beta\gamma}$ and $g^{*\alpha\delta}$, we get respectively

$$(q-1)\,\overline{u}_e\,(q+2)=0\ . \tag{3.10}$$

$$(n-q+2)(n-q-1)\,T^*_\eta=0 \tag{3.11}$$

from which it follows that, since $n>2$ and $T^*_\eta\neq 0$, the latter case can not happen. This proves the necessity of the condition.

Conversely, suppose that $\overline{T}_e=\dfrac{\partial(\ln\sqrt{\sigma})}{\partial x^e}$, i.e. $\overline{u}_e=0$. From (3.6) and (3.7) we conclude that

$$\dot{\overline{\nabla}}_e\overline{R}_{abcd}=0\ ,\ \dot{\nabla}^*_\eta\,\overset{*}{R}_{\alpha\beta\gamma\delta}=0\ .$$

showing that the two subspaces $\bar{W}_q$ and W^*_{n-q} are symmetric . □

Theorem 3.2. *For a semi-decomposable PSW_n $(n>2)$, we have*

$$\lambda_a=-\overline{u}_a\ ,\ \lambda_\alpha=T^*_\alpha$$

unless $\overline{T}_a$ and T^*_α are gradients.

Proof. For a PSW_n we have from (3.1) that

$$\dot{\nabla}_a R_{\alpha\beta\gamma\delta}+\dot{\nabla}_a R_{\beta\alpha\gamma\delta}=2\lambda_a\left(R_{\alpha\beta\gamma\delta}+R_{\beta\alpha\gamma\delta}\right) \tag{3.12}$$

$$\dot{\nabla}_\alpha R_{abcd}+\dot{\nabla}_\alpha R_{bacd}=2\lambda_\alpha\left(R_{abcd}+R_{bacd}\right). \tag{3.13}$$

By using the relations (1.5) , (2.5) , (2.6) , (2.8), the left hand sides of (3.12) and (3.13) may be put into the form

$$\dot{\nabla}_a R_{\alpha\beta\gamma\delta} + \dot{\nabla}_a R_{\beta\alpha\gamma\delta} = -2\sigma\,\overline{u}_a\,(\overset{\bullet}{R}_{\alpha\beta\gamma\delta} + \overset{\bullet}{R}_{\beta\alpha\gamma\delta}) \tag{3.14}$$

$$\dot{\nabla}_\alpha R_{abcd} + \dot{\nabla}_\alpha R_{bacd} = 2T^*_\alpha\,(\overline{R}_{abcd} + \overline{R}_{bacd})\ . \tag{3.15}$$

If the relation [7]

$$R_{ijkl} + R_{jikl} = 2g_{ij}\,(T_{k,l} - T_{l,k}) \tag{3.16}$$

is taken into account, from (3.12),(3.13),(3.14),(3.15), we finally get

$$g^*_{\alpha\beta}\,(T^*_{\gamma,\delta} - T^*_{\delta,\gamma})\,(\overline{u}_a + \lambda_a) = 0 \tag{3.17}$$

$$\overline{g}_{ab}\,(\overline{T}_{c,d} - \overline{T}_{d,c})(T^*_\alpha - \lambda_\alpha) = 0. \tag{3.18}$$

Since the complementary vector fields $\overline{T}_a$ and T^*_α are not gradients, from (3.17) and (3.18) it follows that

$$\overline{u}_a + \lambda_a = 0\ ,\ T^*_\alpha - \lambda_\alpha = 0 \tag{3.19}$$

which completes the proof. □

Theorem 3.3. *For a semi-decomposable PSW_n the subspaces $\bar{W}_q$ and W^*_{n-q} are also pseudo-symmetric unless $\overline{T}_a$ and T^*_α are gradients.*

Proof. Using (2.6) , (3.1) , (3.6) , (3.7) and (3.19), after some calculations we obtain

$$\dot{\overline{\nabla}}_e \overline{R}_{abcd} = -2\overline{u}_e\,\overline{R}_{abcd} - \overline{u}_a\,\overline{R}_{ebcd} - \overline{u}_b\,\overline{R}_{aecd} - \overline{u}_c\,\overline{R}_{abed} - \overline{u}_d\,\overline{R}_{abce} \tag{3.20}$$

$$\dot{\overset{\bullet}{\nabla}}_\eta \overset{\bullet}{R}_{\alpha\beta\gamma\delta} = 2T^*_\eta\,\overset{\bullet}{R}_{\alpha\beta\gamma\delta} + T^*_\alpha\,\overset{\bullet}{R}_{\eta\beta\gamma\delta} + T^*_\beta\,\overset{\bullet}{R}_{\alpha\eta\gamma\delta} + T^*_\gamma\,\overset{\bullet}{R}_{\alpha\beta\eta\delta} + T^*_\delta\,\overset{\bullet}{R}_{\alpha\beta\gamma\eta} \tag{3.21}$$

stating that $\bar{W}_q$ and W^*_{n-q} are pseudo-symmetric. □

Corollary 3.4. *For a semi-decomposable PSW_n with $\sigma \neq$ const., the condition $\overline{T}_a = \frac{\partial}{\partial x^a}(\ln\sqrt{\sigma})$ implies that $\bar{W}_q$ is symmetric and that W^*_{n-q} is pseudo-symmetric provided that T^*_α is not a gradient.*

Proof. The truth of this assertion is clear from (2.6) , (3.7) , (3.17) and (3.20) if we take $\overline{u}_a = 0$. □

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