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RADIAL MOTION IN MANEFF'S FIELD

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Dedicated to Professor V. Ureche on his 60^{th} anniversary

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REZUMAT. - Mişcarea radială în câmpul Maneff. Se studiază mişcarea radială în cadrul problemei celor două corpuri în câmpul gravitaţional post-newtonian nerelativist propus de G.Maneff (caracterizat de un potenţial cvasiomogen). Pe baza integralei prime a energiei, se stabilesc traiectorii de coliziune sau evadare pentru toate valorile şi pentru cele două orientări posibile ale vitezei iniţiale.

Proposed in 1924, Maneff's post-Newtonian nonrelativistic gravitational law [5-8] proved itself able to describe accurately the secular motions of both perihelia of inner planets and Moon's perigee. As showed in [4], Maneff's law

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provides the same good theoretical approximation for these phenomena as the relativity. Reconsidered recently (starting with F.N.Diacu's researches), Maneff's potential appeared much less commonplace than at first sight, showing interesting and surprising properties (see [1-3,9]). This field has implications not only in physics and (celestial) mechanics, but also in astrodynamics, cosmogony, astrophysics [10], even in atomic physics (see [1]).

In this note we shall consider the radial motion in Maneff's field, more precisely the rectilinear motion in the framework of the two-body problem with the potential function (e.g. [1,3,9])

$$U = \frac{Gm_1m_2}{r}\left(1 + \frac{3G(m_1+m_2)}{2c^2r}\right), \tag{1}$$

where m_1,m_2 = the masser, r = distance between m_1 and m_2, G = Newtonian gravitational constant, c = speed of light.

It is easy to see that, with the potential function (1), the relative motion of m_2, say, with respect to m_1 will be described by the equation

$$\ddot{\mathbf{r}} = -\frac{\mu\mathbf{r}}{r^3} - 3\left(\frac{\mu}{c}\right)^2\frac{\mathbf{r}}{r^4} \tag{2}$$

with $\mu = G(m_1+m_2)$. In polar coordinates (r,u), (2) transforms as (see [9])

$$\ddot{r} - r\dot{u}^2 + \frac{\mu}{r^2} + 3\frac{(\mu/c)^2}{r^3} = 0, \tag{3}$$

$$r\ddot{u} + 2\dot{r}\dot{u} = 0, \tag{4}$$

system to which we attach the initial conditions

$$(r,u,\dot{r},\dot{u})(t_0) = (r_0,u_0,V_0\cos\alpha,V_0\sin\alpha/r_0), \tag{5}$$

where V = velocity, α = angle between initial radius vector and initial velocity (remind that we study the motion of m_2 in a frame originated in m_1).

The force field is central, so the angular momentum is conserved and (4) provides the first integral

$$r^2\dot{u} = C, \tag{6}$$

where $C = r_0 V_0 \sin\alpha$ is the constant angular momentum. The first integral of energy can also be easily obtained by the usual technique

$$V^2 = \dot{r}^2 + (r\dot{u})^2 = \frac{2\mu}{r} + 3\frac{(\mu/c)^2}{r^2} + h, \tag{7}$$

where the constant of energy h results to have the expression

$$h = V_0^2 - 2\frac{\mu}{r_0} - 3\frac{(\mu/c)^2}{r_0^2}. \tag{8}$$

In the following we shall consider only the rectilinear motion (α = 0 or $\alpha = \pi$, so C = 0). In this case (7) leads to $V^2 = \dot{r}^2$, but the integral of energy explicited by (7) and (8)

$$V^2 = V_0^2 + 2\mu\left(\frac{1}{r} - \frac{1}{r_0}\right) + 3\left(\frac{\mu}{c}\right)^2\left(\frac{1}{r^2} - \frac{1}{r_0^2}\right), \tag{9}$$

keeps the same expression as in the general case.

We shall study the motion for all values of V_0. The domains in which the motion is possible, featured by the condition $V^2 \geq 0$, will be pointed out, and the characteristics of the motion as well.

Let us first introduce the following abridging notation

$$V_1 = \frac{c}{\sqrt{3}} + \frac{\sqrt{3}\mu}{cr_0}, \quad V_2 = \sqrt{2\frac{\mu}{r_0} + 3\frac{(\mu/c)^2}{r_0^2}} . \tag{10}$$

Suppose that $V_0 > V_1$. In this case $h > c^2/3$. If $\alpha = 0$ (radial motion outwards), m_2 will follow an escape trajectory on which V decreases continuously, tending to $\sqrt{h}$ when r tends to infinity. If $\alpha = \pi$ (radial motion inwards), we have a collision trajectory with continuously increasing velocity such that $V \to \infty$ for $r \to 0$.

For $V_0 = V_1$, we have $h = c^2/3$. The possible scenarios are the same: escape path with decreasing velocity ($V \to \sqrt{h} = c/\sqrt{3}$ when $r \to \infty$) for $\alpha = 0$, and collision path with increasing velocity ($V \to \infty$ when $r \to 0$) for $\alpha = \pi$.

Let now consider $V_2 < V_0 < V_1$, which means $0 < h < c^2/3$. All is like previously: the motion directed outwards is decelerated but leads however to escape, while the motion directed inwards is accelerated and leads to collision. At limits V tends to the same values $\sqrt{h}$ and ∞, respectively.

For $V_0 = V_2$ we have $h = 0$. The scenario is identical: $\alpha = 0$ means escape trajectory with $V \to 0$ for $r \to \infty$, while $\alpha = \pi$ leads to collision (with $V \to \infty$

when $r \to 0$).

Lastly, consider $V_0 < V_2$, meaning $h < 0$. If $\alpha = 0$, then m_2 moves outwards with decreasing velocity, such that for

$$r = \frac{3\mu/c^2}{-1 + \sqrt{[1 + 3\mu/(c^2 r_0)]^2 - 3(V_0/c)^2}} \tag{11}$$

m_2 stops, then it starts inwards and collides with m_1 ($V \to \infty$ for $r \to 0$). If $\alpha = \pi$, we have a collision path with continuously increasing velocity, tending to infinity when $r \to 0$.

Notice that V_1 has no physical, but only mathematical importance (this value of V_0 annuls the discriminant of the second degree polynomial function $V = V(1/r)$ given by (9)), while V_2 has a precise physical significance (this value of V_0 annuls the constant of energy).

Concluding, in Maneff's field the radial motion has no other end but escape or collision, just like in the Newtonian field. By analogy with this last one (and by abuse of language), we shall call V_2 (for which $h = 0$) "parabolic velocity". So, the "hyperbolic/parabolic-type" ($V_0 \geq V_2$) rectilinear motion directed outwards in Maneff's field leads to escape with decreasing velocity (which tends to the corresponding value $\sqrt{h} \geq 0$ when $r \to \infty$). The "elliptictype" ($V_0 < V_2$) rectilinear motion directed outwards cannot lead to escape; m_2 stops at a finite distance (11), then reverses the sense of motion and directs itself with increasing velocity to collision. As to the rectilinear motion directed inwards from the beginning, it ends in collision for any value of the initial velocity.

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