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# ON THE PERIOD OF QUASI-CIRCULAR MOTION IN A SPHERICAL POST-NEWTONIAN GRAVITATIONAL FIELD

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**REZUMAT. - Asupra perioadei mişcării cvasicirculare într-un câmp gravitaţional post-newtonian sferic.** Utilizându-se teoria clasică a perturbaţiilor, se studiază evoluţia perioadei nodale în raport cu perioada kepleriană corespunzătoare în mişcarea cvasicirculară a unei particule de probă într-un câmp gravitaţional post-newtonian sferic (caracterizat de parametri $\alpha$, $\beta$, $\gamma$). Se deduc analitic (cu o precizie de ordinul întâi în excentricitate) perturbaţiile relativiste de ordinele întâi şi al doilea ale perioadei nodale. Considerându-se cazul câmpului post-newtonian sferic al lui Einstein ($\beta = \gamma = 1$), se discută evoluţia perioadei nodale pentru trei valori ale parametrului $\alpha$, atât în cazul general, cât şi în două cazuri particulare. Se discută, de asemenea, influenţa aceluiaşi câmp Einstein asupra mişcării circulare, în trei sisteme de coordonate diferite.

**Introduction.** One of the oldest methods intended to study the motion in a post-Newtonian (not necessarily relativistic) field used the classic theory of perturbations. According to this method, the force acting on a test particle in such a field is written as a sum of two terms: the Newtonian attraction and a post-Newtonian perturbing force, while the deviations of the orbit from a Keplerian orbit are regarded as perturbations (e.g. [2]).

Such a method was used by different authors (e.g. [3-5]) to determine first

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order relativistic changes of some Keplerian orbital parameters over one anomalistic period. First and second order perturbations in orbital elements over one nodal period were determined in [1, 9, 10] for different relativistic and nonrelativistic post-Newtonian fields.

Few authors dealt with the nodal period behaviour in such a field. An approximate formula for the nodal period as functio. of the orbital elements was given in [5], for the Schwarzschild field, but without expressing the variation of this period. The first and second order changes of the nodal period were obtained in [10, 11] for the Mücket-Treder field, in [1, 7] for the Schwarzschild - de Sitter field, and in [9] for Fock's field.

In this paper we shall treat perturbatively the quasi-circular motion of a test particle in a spherical post-Newtonian gravitational field. We shall determine the first and second order relativistic perturbations of the nodal period.

Notice that the orbits are in fact unperturbed in the considered field, but we shall hereafter use, by abuse of language, a perturbation theory terminology.

**2. Starting equations**. Let a central body of mass $M$ be the source which generates a spherical post-Newtonian gravitational field, and let $\mu = GM$ be its gravitational parameter ($G$ = gravitational constant). Consider a test particle orbiting $M$ under the action of this field. The relative motion of the test particle can be described in coordinates ($t$, $x$) in the form [12]

$$dV/dt = -\mu x/r^3 + a_{PN}. \tag{1}$$

The left-hand side of the above equation is the total acceleration of the test particle. The first term in the right-hand side is nothing but the Newtonian attraction per unit mass ($r$ = radial coordinate), while $a_{PN}$ is the virtual perturbing post-Newtonian acceleration, which has the expression (e.g. [12]; see also [13])

$$\begin{aligned} a_{PN} = (\mu/c^2)(2(\beta + \gamma - 2\alpha)\mu x/r^4 - (\gamma + \alpha)(V^2/r^3)x + \\ + 3\alpha(x\cdot V)^2 x/r^5 + 2(\gamma + 1 - \alpha)(x\cdot V)V/r^3), \end{aligned} \tag{2}$$

where $c$ = speed of light; $\alpha$ = gauge parameter [3]; $\beta$, $\gamma$ are the Eddington-Robertson parameters [14]: $\beta$ = post-Newtonian parameter describing the amout of nonlinearity of the gravitational field, $\gamma$ = post-Newtonian parameter describing the space curvature.

Choose a reference frame originated in the mass centre of the body $M$; and feature the motion of the test particle with respect to this frame throung the Keplerian orbital parameters $\{y \in Y;\ u\}$, all time-dependent, where

$$Y = \{p,\ q = e\cos\omega,\ k = e\sin\omega,\ \Omega,\ i\} \tag{3}$$

and $p$ = semilatus rectum, $e$ = eccentricity, $\omega$ = argument of pericentre, $\Omega$ = longitude of ascending note, $i$ = inclination, $u$ = argument of latitude.

For our purposes we shall use the definition relation of the nodal period

$$T_\Omega = \int_0^{2\pi} (dt/du)\,du \tag{4}$$

and Newton-Euler equations written with respect to in the form(e.g. [1, 9, 10])

$$\begin{aligned}
dp/du &= 2(Z/\mu)\,r^3 T,\\
dq/du &= (Z/\mu)\,(r^3 kBCW/(pD) + r^2 T(r(q + A)/p + A) + r^2 BS),\\
dk/du &= (Z/\mu)\,(-r^3 qBCW/(pD) + r^2 T(r(k + B)/p + B) - r^2 AS),\\
d\Omega/du &= (Z/\mu)\,r^3 BW/(pD),\\
di/du &= (Z/\mu)\,r^3 AW/p,\\
dt/du &= Zr^2(\mu p)^{-1/2},
\end{aligned} \tag{5}$$

where $Z = (1 - r^2 C\dot{\Omega}/(\mu p)^{1/2})^{-1}$, $A = \cos u$, $B = \sin u$, $C = \cos i$, $D = \sin i$, $S$, $T$, $W$ = radial, transverse, and binormal components of the perturbing acceleration, respectively.

The change of $y \in Y$ between the initial ($u_0$) and current ($u$) positions, which will be used below, is

$$\Delta y = \int_{u_0}^{u} (dy/du)\,du,\ y \in Y, \tag{6}$$

with the integrands given by (5). The integrals are estimated by successive approximations, with $Z \approx 1$.

**3. Perturbing acceleration and corresponding equations of motion.** The components of the perturbing acceleration $\boldsymbol{a}_{PN}$ have the following expressions [12]

$$S = (\mu/c^2)(\mu/(a^3(1-e^2)^3))(1+e\cos v)^2((2\beta+\gamma-3\alpha) +$$

$$+ (\gamma+2)e^2 + 2(\beta-2\alpha)e\cos v - (2\gamma+2-\alpha)e^2\cos^2 v), \tag{7}$$

$$T = 2(\mu/c^2)(\mu/(a^3(1-e^2)^3))(1+e\cos v)^3(\gamma+1-\alpha)e\sin v,$$

$$W = 0,$$

with $a$ = semimajor axis, $v$ = true anomaly.

Replacing in (7) the well-known formulae

$$p = a(1-e^2), \tag{8}$$

$$u = \omega + v, \tag{9}$$

the definition expression of $q$ and $k$, and the orbit equation in polar coordinates

$$r = p/(1+e\cos v), \tag{10}$$

then retaining only terms to first order in $q$ and $k$ (because we deal with quasi-circular orbits), the components of the perturbing acceleration reduce to

$$S = (\mu^2/(c^2 p r^2))(L_1 + L_2 A q + L_2 B k),$$

$$T = (\mu^2/(c^2 r^3)) L_3 (B q - A k), \tag{11}$$

$$W = 0,$$

where we abbreviated

$$L_1 = 2\beta + \gamma - 3\alpha,$$

$$L_2 = 2(\beta - 2\alpha), \tag{12}$$

$$L_3 = 2(\gamma + 1 - \alpha).$$

Focus now our attention to equations (5). It is easy to observe, by the fourth and the sixth equations (5) and by the expression of $Z$, that $Z = 1$ (because $W = 0$). Substituting (11) in (5), using (1) in the equivalent form

$$r = p/(1 + Aq + Bk), \tag{13}$$

and performing all necessary calculations, the equations of motion become

$$dp/du = 2(\mu/c^2) L_3 (Bq - Ak),$$

$$dq/du = \mu/(c^2 p))(L_1 B + (L_2 + 2L_3) ABq + (L_2 B^2 - 2L_3 A^2) k),$$

$$dk/du = -(\mu/(c^2 p))(L_1 A + (L_2 A^2 - 2L_3 B^2) q + (L_2 + 2L_3) ABk), \tag{14}$$

$$d\Omega/du = 0,$$

$$di/du = 0,$$

$$dt/du = p(p/\mu)^{1/2}(1 + Aq + Bk)^{-2}.$$

**4. Variations of orbital elements.** Let us now perform the integrals (6) with the integrands provided by the first five equations (14). We use the successive approximations method, limiting the process to the first order approximation. Accordingly, we consider $y = y_0 = y(u_0)$, $y \in Y$, in the right-hand side of equations (14), and integrate these ones separately. Performing the integrations, and denoting

$$x = \mu/(c^2 p_0) \tag{15}$$

and

$$\begin{aligned} b_1 &= L_3 = 2(\gamma + 1 - \alpha), \\ b_2 &= L_1 = 2\beta + \gamma - 3\alpha, \\ b_3 &= (L_2 + 2L_3)/2 = \beta + 2\gamma + 2 - 4\alpha, \\ b_4 &= (L_2 - 2L_3)/2 = \beta - 2\gamma - 2, \end{aligned} \tag{16}$$

we get the first order (in $x$) relativistic changes

$$\begin{aligned} \Delta p &= 2xp_0 b_1(-Aq_0 - Bk_0 + A_0 q_0 + B_0 k_0), \\ \Delta q &= x(-b_2 A + b_3 B^2 q_0 - (b_3 AB - b_4 u)k_0 + \\ &\quad + b_2 A_0 - b_3 B_0^2 q_0 + (b_3 A_0 B_0 - b_4 u_0)k_0), \\ \Delta k &= x(-b_2 B - (b_3 AB + b_4 u)q_0 - b_3 B^2 k_0 + \\ &\quad + b_2 B_0 + (b_3 A_0 B_0 + b_4 u_0)q_0 + b_3 B_0^2 k_0), \end{aligned} \tag{17}$$

$$\Delta\Omega = 0,$$

$$\Delta_i = 0,$$

where, obviously, $A_0 = A(u_0)$, $B_0 = B(u_0)$.

Observe that, due to the post-Newtonian conservation of the angular momentum, the motion is restricted to a fixed plane (see the last two expressions (17)).

Although this is not the goal of our paper, let us examine briefly what changes ubdergoes the orbit over one nodal period (that is, letting $u$ vary between 0 and $2\pi$). Putting $u_0 = 0$, $u = 2\pi$ in (17), the first three expressions become

$$\begin{aligned} \Delta p &= 0, \\ \Delta q &= 2\pi x b_4 k_0, \\ \Delta k &= -2\pi x b_4 q_0. \end{aligned} \tag{18}$$

Observing that for quasi-circular orbits $p \approx a$, using the definitions of $q$ and $k$, and taking into account the last notation (16), relations (18) lead easily to

$$\begin{aligned} \Delta a &= 0, \\ \Delta e &= 0, \\ \Delta\omega &= -2\pi x(\beta - 2\gamma - 2). \end{aligned} \tag{19}$$

This means that the only first order relativistic effect in the considered field consists of a rotation of the orbit in its plane (apsidat motion). If we particularize the field to the spherical Einstein post-Newtonian gravitational field ($\beta = \gamma = 1$), then, taking into account (15), the last formula (19) reads

$$\Delta\omega = 6\pi\mu/(c^2 p_0), \tag{20}$$

that is, the well-known expression for the relativistic shift of pericentre.

**5. Nodal period.** Now, let us come back to the main purpose of our paper. As shown in Section 1, we shall determine the first and second order (in $x$) relativistic perturbations of the nodal period. To do that, we shall resort to the method proposed in [15], extended in [6], and generalized in [8] for some special situations.

According to this method, to second order in a small parameter characterizing the perturbing factor, the nodal period is given by

$$T_\Omega = T_0 + \Delta_1 T + \Delta_2 T, \tag{21}$$

where $T_0$ (the Keplerian period corresponding to $u_0$) is determined from (see (4) and the last equation (14))

$$T_0 = p_0 (p_0/\mu)^{1/2} \int_0^{2\pi} g^{-2} du, \tag{22}$$

with the abbreviation $g = g(u) = 1 + Aq_0 + Bk_0$.

The general expression of $\Delta_1 T$ and $\Delta_2 T$ were given and explicited in [6, 7] and will not be repeated here. We shall directly particularize them to our perturbing factor (taking into account, for the beginning, the fact that $W = 0$ and the small parameter is just $x$).

The first order (in $x$) perturbation of the noda' )eriod has (in our case) the form

$$\Delta_1 T = p_0 (p_0/\mu)^{1/2} (-2I_q - 2I_k + (3/2)\, I_p/p_0), \tag{23}$$

with [6]

$$\begin{aligned} I_p &= \int_0^{2\pi} g^{-2} \Delta p \, du, \\ I_q &= \int_0^{2\pi} g^{-3} A \Delta q \, du, \\ I_k &= \int_0^{2\pi} g^{-3} B \Delta k \, du. \end{aligned} \tag{24}$$

The second order (in $x$) perturbation of the nodal period has (in our case) the form

$$\begin{aligned} \Delta_2 T = 3p_0 (p_0/\mu)^{1/2} (I_{qq} + I_{kk} + 2I_{qk} - \\ (I_{pq} + I_{pk})/p_0 + I_{pp}/(8p_0^2)), \end{aligned} \tag{25}$$

with [6]

$$I_{pp} = \int_0^{2\pi} g^{-2}(\Delta p)^2 du,$$

$$I_{qq} = \int_0^{2\pi} g^{-4}A^2(\Delta q)^2 du,$$

$$I_{kk} = \int_0^{2\pi} g^{-4}B^2(\Delta k)^2 du, \tag{26}$$

$$I_{pq} = \int_0^{2\pi} g^{-3}A\Delta p\Delta q\, du,$$

$$I_{pk} = \int_0^{2\pi} g^{-3}B\Delta p\Delta k\, du,$$

$$I_{qk} = \int_0^{2\pi} g^{-4}AB\Delta q\Delta k\, du.$$

**6. Results.** Replacing (17) in (24) and (26), expanding $g^{-n}$, $n = \overline{2,4}$, to first order in $q_0$, $k_0$, and performing the integrations, formulae (24) and (26) become respectively

$$I_p = 4\pi x p_0 b_1(A_0 q_0 + B_0 k_0),$$

$$I_q = -\pi x b_2(1 + 3A_0 q_0), \tag{27}$$

$$I_k = -\pi x(b_2 - 2b_4 q_0 + 3b_2 B_0 k_0),$$

and

$$I_{pp} = 0,$$

$$I_{qq} = \pi x^2 b_2(b_2(3/4 + A_0^2) + (6b_2 + b_3(1/2 - 2B_0^2))A_0 q_0 +$$

$$+ 2(b_3 A_0 B_0 + b_4(\pi - u_0))A_0 k_0),$$

$$I_{kk} = \pi x^2 b_2(b_2(3/4 + B_0^2) + 2((b_3 A_0 B_0 - b_4(\pi - u_0))B_0 -$$

$$- 4b_4/3)q_0 + (6b_2 - b_3(3/2 + 2B_0^2))B_0 k_0), \tag{28}$$

$$I_{pq} = -2\pi x^2 p_0 b_1 b_2(2A_0 q_0 + B_0 k_0),$$

$$I_{pk} = -2\pi x^2 p_0 b_1 b_2(A_0 q_0 + 2B_0 k_0),$$

$$I_{qk} = \pi x^2 b_2(b_2/4 + ((b_2 - b_3/4 + b_4/2)A_0 - 2b_4/3)q_0 +$$

$$+ (b_2 - b_3/4 - b_4/2)B_0 k_0).$$

With these expressions, (23) acquires the form

$$\Delta_1 T = 2\pi p_0 (p_0/\mu)^{1/2} x(2b_2 - (2b_4 - 3b_1 + b_2)A_0)q_0 +$$

$$+ 3(b_1 + b_2)B_0 k_0), \tag{29}$$

while (25) becomes

$$\Delta_2 T = 3\pi p_0 (p_0/\mu)^{1/2} x^2 b_2(3b_2 - (4b_4 - (6b_1 + 8b_2 + b_4)A_0 +$$

$$+ 2b_4(\pi - u_0)B_0)q_0 + ((6b_1 + 8b_2 - b_4)B_0 +$$

$$+ 2b_4(\pi - u_0)A_0)k_0). \tag{30}$$

Finally, replacing (16) in (29) and (30), denoting

$$f_1 = 2(2\beta + \gamma - 3\alpha) + (2(-\beta + 2\gamma + 2) + 3(2\beta + 3\gamma + 2 - 5\alpha)A_0)q_0 +$$

$$+ 3(2\beta + 3\gamma + 2 - 5\alpha)B_0 k_0), \tag{31}$$

$$f_2 = (3/2)(2\beta + \gamma - 3\alpha)(3(2\beta + \gamma - 3\alpha) + (4(-\beta + 2\gamma + 2) +$$
$$+ (17\beta + 18\gamma + 10 - 36\alpha)A_0 - 2(\beta - 2\gamma - 2)(\pi - u_0)B_0)q_0 +$$
$$+ ((15\beta + 22\gamma + 14 - 36\alpha)B_0 + 2(\beta - 2\gamma - 2)(\pi - u_0)A_0)k_0),$$

substituting the resulting expressions in (21), and writing (22) to first order in $q_0$, $k_0$ as

$$T_0 = 2\pi p_0(p_0/\mu)^{1/2}, \tag{32}$$

the nodal period (to second order in $x$) reads

$$T_\Omega = T_0(1 + xf_1 + x^2 f_2). \tag{33}$$

This is the basic formula we searched for and which will be used in the next sections (with $f_1$, $f_2$ provided by (31), and with $x$ given by (15)).

**7. Two particular cases.** We shall consider two particular cases: initial orbital elements corresponding to ascending node, and initially circular orbit. In the first situation we $u_0$, hence $A_0 = 1$, $B_0 = 0$. So, (31) become

$$f_1 = 2(2\beta + \gamma - 3\alpha) + (4\beta + 13\gamma + 10 - 15\alpha)q_0,$$
$$f_2 = (3/2)(2\beta + \gamma - 3\alpha)(3(2\beta + \gamma - 3\alpha) + (13\beta + 26\gamma + \tag{34}$$
$$+ 18 - 36\alpha)q_0 + 2\pi(\beta - 2\gamma - 2)k_0).$$

If the initial orbit is circular (of radius $r_0$), then we have $q_0 = 0$, $k_0 = 0$, hence (31) acquire the form

$$f_1 = 2(2\beta + \gamma - 3\alpha),$$

$$f_2 = (9/2)(2\beta + \gamma - 3\alpha)^2. \tag{35}$$

It is easy to see that in this last particular case the perturbation of the nodal period does not depend on the initial position of the test particle.

**8. Spherical Einstein post-Newtonian field.** Consider that the field in which the test particle moves is the spherical Einstein post-Newtonian gravitational field. In this case $\beta = \gamma = 1$, and formulae (31) read

$$f_1 = 3(2(1 - \alpha) + (2 + (7 - 5\alpha)A_0)q_0 + (7 - 5\alpha)B_0k_0),$$

$$f_2 = (27/2)(1 - \alpha)(3(1 - \alpha) + (4 + 3(5 - 4\alpha)A_0 + 2(\pi - u_0)B_0)q_0 + \tag{36}$$

$$+ ((17 - 12\alpha)B_0 - 2(\pi - u_0)A_0)k_0).$$

The two particular cases (34) and (35) become respectively

$$f_1 = 3(2(1 - \alpha) + (9 - 5\alpha)q_0),$$

$$f_2 = (27/2)(1 - \alpha)(3(1 - \alpha) + (19 - 12\alpha)q_0 - 2\pi k_0), \tag{37}$$

and

$$f_1 = 6(1 - \alpha), \quad f_2 = (81/2)(1 - \alpha)^2. \tag{38}$$

Let us now assign to the gauge parameter $\alpha$ some particular values, which mean some systems of coordinates. The case $\alpha = 0$ means the use of standard post-Newtonian coordinates (spatially isotropic). Expressions (36) become in this case

$$f_1 = 3(2 + (2 + 7A_0)q_0 + 7B_0k_0),$$
$$f_2 = (27/2)(3 + (4 + 15A_0 + 2(\pi - u_0)B_0)q_0 + \qquad (39)$$
$$+ (17B_0 - 2(\pi - u_0)A_0)k_0),$$

while the particular cases (37) and (38) become respectively

$$f_1 = 3(2 + 9q_0), \; f_2 = (27/2)(3 + 19q_0 - 2\pi k_0), \qquad (40)$$

and

$$f_1 = 6, \; f_2 = 81/2. \qquad (41)$$

If we consider $\alpha = 1$, namely the spatial standard coordinate system is used, formulae (36) read

$$f_1 = 6((1 + A_0)q_0 + B_0k_0),$$
$$f_2 = 0, \qquad (42)$$

while (37) and (38) acquire respectively the form

$$f_1 = 12q_0, \; f_2 = 0, \qquad (43)$$

and

$$f_1 = 0, \; f_2 = 0. \qquad (44)$$

Lastly, put $\alpha = 2$. This value of the gauge parameter leads to

$$f_1 = -3(2 - (2 - 3A_0)q_0 + 3B_0k_0),$$

$$f_2 = (27/2)(3 - (4 - 9A_0 + 2(\pi - u_0)B_0)q_0 + \tag{45}$$

$$+ (7B_0 + 2(\pi - u_0)A_0)k_0)$$

for the expressions (36), and to

$$f_1 = -3(2 + q_0), \quad f_2 = (27/2)(3 + 5q_0 + 2\pi k_0), \tag{46}$$

and

$$f_2 = -6, \quad f_2 = 81/2 \tag{47}$$

for the particular cases (37) and (38), respectively.

**9. Period behaviour for circular orbits.** To end, let us compare the nodal period with the corresponding Keplerian period for circular orbits in the spherical Einstein post-Newtonian gravitational field. Taking into account (38), formula (33) can be written in this situation

$$T_\Omega = T_0(1 + (3/2)(1 - \alpha)x(4 + 27(1 - \alpha)x)). \tag{48}$$

Consider the standard post-Newtonian coordinates ($\alpha = 0$); formula (48) becomes in this case

$$T_\Omega = T_0(1 + (3/2)x(4 + 27x)). \tag{49}$$

Since $x$ is a positive quantity, we have $T_\Omega > T_0$. In other words, for $\alpha = 0$ the post-Newtonian perturbing force acts to decelerate the motion.

For $\alpha = 1$, formula (48) leads immediately to $T_\Omega = T_0$, that is, if we use the spatial standard coordinate system, the motion keeps its Keplerian period.

Lastly, for $\alpha = 2$, expression (48) reads

$$T_\Omega = T_0(1 - (3/2)x(4 - 27x)). \tag{50}$$

This means that there exists a critical value of $x$, $x_c$ say, such that for $x = x_c = 4/27$ the nodal period and the corresponding Keplerian period coincide. Having in view the expression (15) of $x$ (with $p_0$ = orbit radius), and recalling the expression of the Schwarzschild radius $R_{Sch} = 2\mu/c^2$, the above coincidence criterion can be formulated as

$$p_0 = (27/8)R_{Sch}. \tag{51}$$

In other words, for an initial radius smaller than $(27/8)R_{Sch}$ the post-Newtonian perturbing force acts to decelerate the motion, and conversely. We may therefore conclude that for concrete astronomical situations the case $\alpha = 2$ entails generally an acceleration of the circular motion as against the Keplerian motion.

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