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Book reviews

Friedrich Haslinger; Complex analysis. A functional analytic approach,
De Gruyter Graduate, De Gruyter, Berlin 2018, ix + 338 p.,
ISBN: 978-3-11-041723-4/pbk; 978-3-11-041724-1/ebook.

The book is an introduction to complex analysis in one variable and some topics in several complex variables, oriented to applications to Cauchy-Riemann equations studied via the method of Hilbert space operators. The key tool in this study is the $\bar{\partial}$-Neumann operator viewed as an operator acting on various Hilbert spaces of analytic functions. This part is largely based on author's papers, being also treated with more details and supplementary material in the related book by F. Haslinger, *The $\bar{\partial}$-Neumann problem and Schrödinger operators*, De Gruyter Expositions in Mathematics 59, De Gruyter, Berlin, 2014. It is worth to mention that the needed results on Hilbert spaces (e.g. Riesz' representation theorem) and spectral theory of linear operators (bounded or unbounded) defined on such spaces are included with full proofs in the book, making the book fairly selfcontained.

Complex analysis in one variable is treated in the first four chapters of the book: 1. *Complex numbers and functions*; 2. *Cauchy's Theorem and Cauchy's formula*; 3. *Analytic continuation*; 4. *Construction and approximation of holomorphic functions.* The author presents here, in a clear and rigorous manner, the standard results of complex analysis, some more special topics – Runge's approximation theorem, normal families with applications to Riemann's mapping theorem – being included in the fourth chapter.

Chapter 5, *Harmonic functions*, is concerned with real-valued harmonic functions defined on open subsets (called domains) of $\mathbb{R}^2$, the Dirichlet problem and subharmonic functions. The study of holomorphic functions of several variables starts in Chapter 6, *Several complex variables*, with emphasis on the differences between one and several variables. As the authors says in the introduction to this chapter:

> To think that the analysis of several complex variables is more or less the one variable theory with some more indices turns out to be incorrect.

This is proved by some facts as: in the several variable case the Cauchy-Riemann equations constitute an overdetermined system of partial differential equations, Hartogs phenomenon, the Identity Theorem, and others. A presentation of the famous example of Hans Lewy of a partial differential operator without a solution is also included.

The Bergman spaces, i.e. Hilbert spaces $A^2(\Omega)$ formed of all holomorphic functions contained $L^2(\Omega)$, where Ω is a domain in $\mathbb{C}^n$, is studied in Chapter 7, with emphasis on the Bergman kernel and the reproducing property of the Bergman spaces.

The rest of the chapters are devoted to the study of various realizations of the $\bar{\partial}$-Neumann operator: 8. *The canonical solution operator to* $\bar{\partial}$; 9. *Nuclear Fréchet spaces of holomorphic functions*; 10. *The* $\bar{\partial}$*-complex*; 11. *The twisted* $\bar{\partial}$*-complex and Schrödinger operators.*

Each chapter ends with a set of exercises completing the main text and a section of Notes referring to literature and more advanced results. The prerequisites are real analysis, basic measure theory and some topology.

The book is clearly written, in a pleasant style and an elegant layout. The first part can be used for graduate courses in complex analysis, while the more advanced second part is adequate for postgraduate courses, as an introductive text on applications of operator theory on Hilbert spaces of holomorphic functions to partial differential equations in complex variables.

S. Cobzaş

Robert C. Gunning; An introduction to analysis,
Princeton University Press, x+370 p., 2018,
ISBN: 978-0-691-17879-0/hbk; 978-1-4008-8941-9/ebook.

This is an accelerated version of a three semester honors analysis course (I - calculus in one variable, II - linear algebra, and III - calculus in several variables) taught by the author at Princeton University. As, in recent years, the undergraduate students attending this course came with a more advanced background in abstract mathematics, the author decided to offer a two semester options to these students.

The book starts with some results on set theory, mappings and algebraic structures (groups, rings, fields, vector spaces). The algebraic part is completed in the fourth chapter with some more specialized topics in linear algebra (Cayley-Hamilton theorem, Jordan form, spectral theory, etc) along with an introduction to inner product spaces. The second chapter is concerned with topology - metric spaces, normed spaces and topological spaces (including compactness and Baire category). Continuous mappings are treated in the fourth chapter, where differentiable mappings are introduced directly for vector function $f : U \to \mathbb{R}^n$, where U is an open subset of $\mathbb{R}^m$. Here one discusses also series and real analytic mappings with applications to the basic functions of analysis. Chapter 5, *Geometry of mappings*, is concerned with the basic results of differential calculus – inverse and implicit function theorems, rank theorem – as well as an introduction to topological and differentiable manifolds.

The sixth chapter is devoted to basic results on the Riemann integration theory of functions of m variables (including improper integrals). The last chapter, 6. *Differential forms*, contains a presentation of differential forms and Stokes' theorem, with the classical theorems of Green and Gauss-Ostrogradski as examples.

Each section ends with a set of problems stratified at two levels: I. problems that every student can (and must) solve without any difficulty, and II. more difficult problems, covering more theoretical aspects and tougher calculations, but still accessible.

As the author mention in the Preface, he initially intended to include a third level of very challenging problems, but finally renounced being “frustrated by a resounding lack of interest on the part of the students”.

The book is clearly written, with elegant proofs, and a cleaver selection of the topics, allowing to present in a one volume in a coherent and self-contained manner, several topics, starting with background results in abstract algebra and topology and culminating with Stokes’ theorem.

The author is a well known specialist in complex analysis of several variables. He authored several seminal books, the most known being that written jointly with Hugo Rossi, *Analytic functions of several variables*, Prentice Hall 1965 (a reprint was published by AMS in 2009), as well as several published with Princeton University Press. Some notes spread throughout the book reflects his erudition and deep understanding of mathematics.

The book will be of great help for teachers on advanced courses in analysis, or for self study by students, giving them a quick but rigorous acquittance with some fundamental results of mathematical analysis.

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