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Professor GABRIELA KOHR – A life for research. In memoriam

Mirela Kohr and Grigore Ştefan Sălăgean

Abstract. We review the main contributions of Professor Gabriela Kohr in her studies in the Geometric function theory in several complex variables and complex Banach spaces with special emphasis on the theory of Loewner chains and the Loewner differential equation.

Mathematics Subject Classification (2010): 32A18, 32A30, 32K05, 30C45, 30C80, 30D45.

Keywords: Gabriela Kohr, geometric function theory of one and several complex variables, Loewner theory, mappings with parametric representation, extension operator, complex analysis.

1. Scientific activity of Professor Gabriela Kohr

Gabriela Kohr was born on November 20, 1967 in Teiuş (Alba), Romania.

1.1. Studies and degrees

- PhD in Mathematics, Babeş-Bolyai University, Cluj-Napoca, 1996.
  PhD thesis: *Contributions to the theory of univalent functions*, supervisor Professor Petru T. Mocanu, Member of the Romanian Academy.
- Licensed in Mathematics, Faculty of Mathematics, Cluj-Napoca, Romania (1986-1991).
- High School Aiud (Alba) (1982-1986).

1.2. Academic positions

- Teaching Assistant, Faculty of Mathematics and Computer Science, Babeş-Bolyai University, October 1991 - September 1997
- Lecturer, Faculty of Mathematics and Computer Science, Babeş-Bolyai University, October 1997 - September 2000

- Associate Professor, Faculty of Mathematics and Computer Science, Babeş-Bolyai University, October 2000-September 2006
- Professor, Faculty of Mathematics and Computer Science, Babeş-Bolyai University, October 2006-December 2020
- PhD Supervisor, Faculty of Mathematics and Computer Science, Babeş-Bolyai University, 2007-2020. PhD students: Mihai Iancu, Teodora Chirilă.

Academic awards and distinctions: The *Spiru Haret* award of the Romanian Academy in 2005 for the monograph: I. Graham, G. Kohr, Geometric Function Theory in One and Higher Dimensions, Marcel Dekker Inc., New York, Basel, 2003; Awards of Babeş-Bolyai University for excellence in science.
Research interests: Complex Analysis, Geometric Function Theory of One and Several Complex Variables.
Research visits: **Annual research visits in the period 1999-2020 at University of Toronto, Department of Mathematics, Toronto (Canada). Research collaboration with Professor Ian Graham**. Many other research visits at universities in Japan, SUA, Italy, Germany, England, Poland.
Research Projects: Coordinator of **6** national research projects (CNCSIS, UEFISCDI, Babeş-Bolyai University), member in various (international and national) research teams.
Invited/plenary speaker to many international conferences, or workshops, in: SUA, Canada, Japan, Italy, Germany, Finland, Norway, Sweden, Spain, Poland, Turkey, Romania.

1.3. Teaching activity

The lectures of Prof. Gabriela Kohr covered the following topics: Complex Analysis, Special topics in complex analysis, Complex analysis in one and higher dimensions, Geometric function theory in several complex variables, Special topics of real and complex analysis, Univalent functions and differential subordinations, Applications of complex analysis in physics, Special topics in real analysis.

The books [54] and [58] of Gabriela Kohr ([58] in collaboration) are highly appreciated for their rigorous and careful presentation and for the covered topics of real interests for all students and researchers working in Complex Analysis and related areas. Gabriela Kohr had great pedagogical skills. She captivated the students attending her courses with her clarity in teaching and passion for mathematics. The students appreciated a lot the dedication and the fairness in every single lecture, exam, or any other teaching activity of Professor Gabriela Kohr.

1.4. Research contributions

The list of publications of Gabriela Kohr contains an impressive number of scientific articles published in prestigious international journals, such as: Mathematische Annalen, Journal of Functional Analysis, Transactions of the American Mathematical Society, Journal d'Analyse Mathématique, Journal of Geometric Analysis, Annali della Scuola Normale Superiore di Pisa, Classe di Scienze, Canadian Journal of Mathematics, Israel Journal of Mathematics, Constructive Approximation, Journal of Mathematical Analysis and Applications, Annali di Matematica Pura ed Applicata, Proceedings of the American Mathematical Society, Journal of Approximation Theory, Annales Academiæ Scientiarum Fennicé, Analysis and Mathematical Physics.

The monograph of Ian Graham and Gabriela Kohr [32] is highly cited and recognized as one of the best books in Geometry function theory in one and several complex variables. Gabriela Kohr is also coauthor of the monograph [57], and her scientific work has more than 2600 citations. The complete list of publications of Gabriela Kohr can be found at [56].

The main research contributions of Professor Gabriela Kohr are devoted to

Geometric function theory in several complex variables

- Extensions of classical results in the theory of univalent functions to the case of several complex variables
- Theory of Loewner chains of several complex variables
- Approximation and control theory results for normalized biholomorphic mappings
- Extension operators and quasiconformal mappings

Geometric function theory in complex Banach spaces

- Theory of Loewner chains in complex Banach spaces
- Infinite-dimensional versions (the case of reflexive complex Banach spaces) of classical results in the theory of Loewner chains and the Loewner differential equation
- Estimation results for normalized biholomorphic mappings in complex Banach spaces

Complex analysis on bounded symmetric domains

- Bloch mappings and related results on bounded symmetric domains
- Harmonic and pluriharmonic mappings on complex Hilbert balls and bounded symmetric domains

Research collaborators

Many of the strong research contributions of Professor Gabriela Kohr have been obtained in collaboration with mathematicians from all over the world, but two of them had a very strong role in her scientific life and evolution. We refer to the outstanding research collaboration of more than 20 years, started in 1999 up to the last moment of the life of Gabi, in 2020, with Professor *Ian Graham*, University of Toronto, Canada, and Professor *Hidetaka Hamada*, Kyushu Sangyo University, Fukuoka, Japan. The remarkable collaboration and friendship of Gabi with Ian and Hidetaka highlighted her life and leaded to important and well known contributions in Geometric function theory in complex spaces. Let us note that Toronto became for Gabriela and Mirela Kohr their second home between 1999 and 2020.

Gabriela Kohr was honored to collaborate with Professor Petru T. Mocanu from Babeş-Bolyai University, Cluj-Napoca, Romania, a well known mathematician for his contributions in Geometric function theory of one complex variable. Gabi had a very important research collaboration with Professor Filippo Bracci, University

"Tor Vergata", Rome, Italy, a strong and well known mathematician for his contributions in the field. Gabi was also influenced by her collaboration with Professors Peter L. Duren, University of Michigan, Ann Arbor, USA, Ted J. Suffridge, University of Kentucky, Lexington, USA, Jerry R. Muir Jr., University of Scranton, Scranton, PA, USA, Mirela Kohr, Babeş-Bolyai University, Cluj-Napoca, Romania, Paula Curt, Babeş-Bolyai University, Mihai Iancu, Babeş-Bolyai University, Cho-Ho Chu, Queen Mary, University of London, UK, Tatsuhiro Honda, Senshu University, Japan, John A. Pfaltzgraff, University of North Carolina, USA, Piotr Liczberski, Technical University of Lodz, Poland, Martin Chuaqui, Universidad Católica de Chile, Chile, Rodrigo Hernández, Universidad Adolfo Ibánez, Vińa del Mar, Chile, and many others.

The meetings and discussions of Gabi with Professors David Shoikhet, Holon Institute of Technology and Braude Academic College, Karmiel, Israel, Oliver Roth, University of Wuerzburg, Germany, Mark Elin, Ort Braude College, Karmiel, and Grigore Sălăgean, Babeş-Bolyai University, inspired Gabi in all her research developments.

1.5. Main results

A) The class $S^0(\mathbb{B}^n)$ of normalized univalent mappings on $\mathbb{B}^n$ that have parametric representation

- The class of normalized univalent mappings in the Euclidean unit ball $\mathbb{B}^n$ of $\mathbb{C}^n$ is denoted by $S(\mathbb{B}^n)$. Therefore,

$$S(\mathbb{B}^n) := \{f : \mathbb{B}^n \to \mathbb{C}^n : f \text{ univalent}, \ f(0) = 0, \, df(0) = \mathsf{id}\}.$$

In the case of one complex variable, that is, for $n = 1$, the class $S(\mathbb{U})$ is compact, and various extremal problems have been studied by Duren, Hallenbeck and MacGregor, Pommerenke, Roth, Schaeffer and Spencer, and many others. (Recall that $\mathbb{U}$ is the open unit disc in $\mathbb{C}$.) In addition, in the case $n = 1$, every function $f \in S(\mathbb{U})$ can be embedded into a normal Loewner chain. Various results about the structure of extreme points and support points of linear problems, in particular that these points are single-slit mappings, are well known (see e.g. the first part of the book of Ian Graham and Gabriela Kohr [32]).

In higher dimensions, $n \geq 2$, the class $S(\mathbb{B}^n)$ is not compact, and there are mappings in $S(\mathbb{B}^n)$ that can not be embedded as the first element of a normalized Loewner chain (see e.g. [14] for $n = 3$), and there are no single-slit mappings.

However, in the case $n \geq 2$, the *subclass $S^0(\mathbb{B}^n)$ of $S(\mathbb{B}^n)$, consisting of mappings that have parametric representation*, is *compact*. This subclass was introduced by **I. Graham, H. Hamada** and **G. Kohr** in their remarkable paper [16]. See also the valuable book of Ian Graham and Gabriela Kohr [32] [*Geometric Function Theory in One and Higher Dimensions*, Marcel Dekker, New York, 2003]. Note that

$$S^0(\mathbb{B}^n) = \big\{f \in S(\mathbb{B}^n) : \exists\, f(z,t) \text{ Loewner chain such that } \{e^{-t} f(\cdot,t)\}_{t\geq 0} \text{ is a normal family and } f = f(\cdot,0)\big\}.$$

The class $S^0(\mathbb{B}^n)$ does not have a linear structure in the case $n = 1$, but also in higher dimensions. Therefore, it is important to analyze both linear and nonlinear extremal problems in the class $S^0(\mathbb{B}^n)$. One of the main difficulties that appears in the study of univalent mappings in higher dimensions is that the lack of an uniformization theorem does not allow to construct easily variations of a given normalized Loewner chain. **F. Bracci, I. Graham, H. Hamada** and **G. Kohr** [4] used a variational method and defined a natural class of normalized Loewner chains that they called *geräumig* Loewner chains. This method allows to construct other normalized Loewner chains with the property that from some time on, they coincide with the initial geräumig Loewner chain. The authors used their variational method in the analysis of extreme and support points of $S^0(\mathbb{B}^n)$. In particular, in the contrast to the case $n = 1$, in higher dimensions $n \geq 2$, Bracci, Graham, Hamada and Kohr [4] constructed an example of a family of mappings in $S^0(\mathbb{B}^n)$, which are bounded by a constant $M > 1$, and are not support points, nor extreme points of $S^0(\mathbb{B}^n)$. *Graham, Hamada, Kohr* and *Kohr* [26] extended these results to ***A*-normalized Loewner chains** by using their results obtained in [25] (see also [33]).

In addition, in a series of papers, [18, 26, 27, 47], Gabriela Kohr and her collaborators obtained bounded support points for various subclasses of $S(\mathbb{B}^n)$ and $S(\mathbb{U}^n)$ for $n \geq 2$, as well as for $S(\mathbb{B}_X)$, where $\mathbb{B}_X$ is a finite dimensional bounded symmetric domain with rank $r \geq 2$ in a finite dimensional complex Banach space X, by extending to such domains a shearing process due to F. Bracci [3] on the unit ball $\mathbb{B}^2$ of $\mathbb{C}^2$. They also generalized the extremal type results of W.E. Kirwan (1980) and R. Pell (1980) for Loewner chains in higher dimensions.

B) **Loewner chains and the Loewner PDE in several complex variables**

- Subordination chains in $\mathbb{C}^n$ were first studied by J.F. Pfaltzgraff in 1974, by extending to higher dimensions the results related to the subordination and the Loewner differential equation (the Loewner PDE) obtained by Ch. Pommerenke in 1965 and J. Becker in 1972 in the case $n = 1$ (see [32] for further details). Moreover, Pfaltzgraff showed the uniqueness of the solution to the Loewner PDE on the Euclidean unit ball of $\mathbb{C}^n$. The *existence result to the Loewner PDE on the unit ball of* $\mathbb{C}^n$ was proved by **I. Graham, H. Hamada** and **G. Kohr** [16]. The *uniqueness result to the Loewner PDE on the unit ball in A-normalized case* was proved by **P. Duren, I. Graham, H. Hamada** and **G. Kohr** [11] by using the *higher dimensional Carathéodory kernel convergence theorem* obtained by them.

  The existence and uniqueness theory of Loewner chains in $\mathbb{C}^n$ has been considered by several authors, and applications of this theory have been given to characterize various subclasses of biholomorphic mappings, geometric characterizations of biholomorphic mappings that have parametric representation, as well as univalence criteria. Suggestive results in this sense have been obtained by Gabriela Kohr and her collaborators in [22], [23], [24], [28], [36], [35], [40], [41], [42], [52], among many other relevant publications. The theory of Loewner chains in higher dimensions is a research area with a strong scientific contribution of Gabriela Kohr and her collaborators.

  L. Arosio, F. Bracci, H. Hamada and **G. Kohr** [2] proved the existence result for the solutions of the Loewner differential equation on complete hyperbolic complex manifolds by using a geometric construction of Loewner chains on complete hyperbolic complex manifolds based on a new interpretation of Loewner chains as the direct limit of evolution families. *This is a new and strong approach to the Loewner theory in complete hyperbolic complex manifolds, based on iteration and semigroup theory.*

C) Loewner chains and nonlinear resolvents of the Carathéodory family on the unit ball in $\mathbb{C}^n$

- Assume that f is the infinitesimal generator of a one-parameter semigroup of holomorphic self-maps of the open unit disc $\mathbb{U}$ of the complex plane $\mathbb{C}$. M. Elin, D. Shoikhet and T. Sugawa [13] studied the properties of a family of non-linear resolvent functions $J_r = (I + rf)^{-1}$ of f, $r \geq 0$, with the additional conditions $f(0) = 0$ and $f'(0) > 0$. Note that Elin, Shoikhet, and Sugawa showed that the resolvents J_r determine an inverse Loewner chain with an associated Herglotz vector field of divergence type. **I. Graham, H. Hamada** and **G. Kohr** [21] extended the result of Elin, Shoikhet, and Sugawa to the case of the Euclidean unit ball $\mathbb{B}^n$ in higher dimensions, by using their strong methods obtained during a long collaboration of 20 years, starting with [16]. Moreover, they proved that $(1 + r)J_r$ can be embedded as the first element of a normal Loewner chain, that is, $(1+r)J_r \in S^0(\mathbb{B}^n)$, and it is not an extreme point, nor a support point of $S^0(\mathbb{B}^n)$ for each $r \geq 0$, and $n \geq 1$.

D) Approximation properties by automorphisms of $\mathbb{C}^n$ and quasiconformal diffeomorphisms in $\mathbb{C}^n$

- If f is a biholomorphic mapping on the Euclidean unit ball $\mathbb{B}^n$ such that $f(\mathbb{B}^n)$ is a Runge domain, then f can be approximated locally uniformly on $\mathbb{B}^n$ by automorphisms of $\mathbb{C}^n$, whenever $n \geq 2$. This is a strong result due to E. Andersén and L. Lempert [1]. Starting from this result, it is natural to ask whether f can be approximated by automorphisms of $\mathbb{C}^n$ whose restrictions to $\mathbb{B}^n$ have the same geometric property of f. **H. Hamada, M. Iancu, G. Kohr** and **S. Schleißinger** [42] obtained a positive answer to this question whenever f is a spirallike mapping, or a convex mapping. To this end, they used the version of the Carathéodory kernel convergence theorem to the higher dimensions, a result obtained by P. Duren, I. Graham, H. Hamada and G. Kohr [11], and the property that the spirallike domains with respect to some linear operator $A \in L(\mathbb{C}^n)$ are Runge domains, a result proved by H. Hamada [36]. In particular, it follows that the first elements of A-normalized normal Loewner chains can be approximated locally uniformly on $\mathbb{B}^n$ by automorphisms of $\mathbb{C}^n$ whose restrictions to $\mathbb{B}^n$ are the first elements of A-normalized normal Loewner chains, in the case when A is nonresonant. Moreover, Hamada, Iancu and Kohr [40] proved that the first elements of A-normalized normal Loewner chains can be approximated locally uniformly on $\mathbb{B}^n$ by automorphisms of $\mathbb{C}^n$ whose restrictions to $\mathbb{B}^n$ are the first elements of A-normalized normal Loewner chains including the case when A is resonant, by using a geräumig Loewner chain similar to that used by G. Kohr and her collaborators in [4] and [26].

E) Extension operators and their mapping properties

- K. Roper and T. Suffridge (1995) introduced their extension operator which extends locally univalent functions on the unit disc $\mathbb{U}$ in $\mathbb{C}$ to locally biholomorphic mappings on the Euclidean unit ball $\mathbb{B}^n$ in $\mathbb{C}^n$. Starting with this result, various extension operators for locally univalent functions on the unit disc $\mathbb{U}$ in $\mathbb{C}$ to higher dimensional spaces have been extensively considered. Let us mention the results of Graham and Kohr [31], Graham, Kohr, and Kohr [34] and Graham, Hamada, Kohr, and Suffridge [30] in this research direction. Related to these operators, the preservation of subfamilies of starlike mappings, spirallike mappings, the first elements of Loewner chains and Bloch mappings by extension operators have been analyzed. Graham, Hamada, Kohr and Kohr [28] obtained a unified result that shows that the *first elements of g-Loewner chains are preserved by these extension operators*, where g is a convex function on $\mathbb{U}$ with $g(0) = 1$ and $\Re g(\zeta) > 0$ on $\mathbb{U}$. To show this property, they used their covering theorem for convex functions on $\mathbb{U}$. In particular, g-starlike mappings are preserved by these extension operators. This result implies that *various subfamilies of starlike mappings, such as starlike mappings of order α, strongly starlike mappings of order α and almost starlike mappings of order α are preserved by these extension operators* (see also [17], [30], [32], [55], for the preservation of starlike mappings, spirallike mappings, the first elements of Loewner chains and Bloch mappings by similar extension operators).

  By following the same idea of preservation of geometric and analytic properties of various classes of mappings, J. Pfaltzgraff and T. Suffridge (1999) introduced the extension operator which maps locally biholomorphic mappings on the Euclidean unit ball $\mathbb{B}^n$ of $\mathbb{C}^n$ to locally biholomorphic mappings on the Euclidean unit ball $\mathbb{B}^{n+1}$ of $\mathbb{C}^{n+1}$. Moreover, the extension operator of Pfaltzgraff and Suffridge preserves starlikess and the first elements of normal Loewner chains. Graham, Hamada and Kohr [20] proved that the Pfaltzgraff and Suffridge type extension operator maps the *first element of normal Loewner chains on finite dimensional bounded symmetric domains to the first element of normal Loewner chains on the unit ball in a higher dimensional space.* They used a Schwarz-Pick lemma for holomorphic self mappings of bounded symmetric domains, a result obtained by them.

F) Bloch mappings and related results on bounded symmetric domains

- The class of Bloch functions on the unit disc $\mathbb{U}$ in $\mathbb{C}$ have been analyzed by many authors, and has various applications. Bloch functions on a bounded homogeneous domain in $\mathbb{C}^n$ have been first considered by K.T. Hahn (1975) and later by R.M. Timoney (1980). Timoney showed that many of the characterizations of Bloch functions on the unit disc apply also to Bloch functions on a bounded homogeneous domain. He defined the Bloch functions by using the Bergman metric. Such an approach is not applicable in infinite dimensions. Chu, Hamada, Honda and Kohr [6] used the infinitesimal Kobayashi metric instead of the Bergman metric, and characterized Bloch functions on bounded symmetric domains, which may be infinite dimensional, by extending several known conditions for Bloch functions on the unit disc in the complex plane.

The Bloch space of a bounded symmetric domain, which may be infinite dimensional, and properties of composition operators have been studied by Chu, Hamada, Honda and Kohr in [7]. α-Bloch mappings on bounded symmetric domains in $\mathbb{C}^n$ have been studied by Hamada and Kohr [46].

- A distortion theorem for Bloch functions has been obtained by M. Bonk (1990). Distortion results for locally biholomorphic Bloch mappings on bounded symmetric domains in $\mathbb{C}^n$ have been obtained by Chu, Hamada, Honda and Kohr [5]. As an application, they obtained a lower bound for the Bloch constant for various classes of locally biholomorphic Bloch mappings. The distortion bound and a lower bound for the Bloch constant were given by using the constant $2c(\mathbb{B}_X)$, which they call "the diameter" and is defined by the Bergman metric.

G) Coefficient estimates for families of univalent mappings

- Fekete and Szegö inequality for normalized univalent functions on the unit disc $\mathbb{U}$ is an inequality involving the second and the third coefficients of univalent analytic functions. A lot of work has been done in this sense for various subclasses of $S(\mathbb{U})$. In higher dimensions, Xu and Liu (2014) obtained the Fekete and Szegö inequality for a special subclass of normalized starlike mappings on the unit ball $\mathbb{B}$ of a complex Banach space. Starting with this result, various extensions on the Fekete and Szegö inequality in higher dimensions have been obtained. However, the Fekete and Szegö inequality for the family of all starlike mappings has not been obtained. Hamada, Kohr and Kohr [50] obtained the Fekete and Szegö inequality for all starlike mappings on $\mathbb{B}$ by using a more simple proof than those provided for the previous results. As a new result (including the case on the unit disc $\mathbb{U}$), they also give the Fekete-Szegö inequality for $(1+r)J_r$, where J_r is the nonlinear resolvent mapping of f in the Carathéodory family $\mathcal{M}(\mathbb{B})$.

H) Boundary behavior of families of univalent mappings

- The Schwarz lemma at the boundary has a main role in Complex analysis due to its various applications in the geometric function theory, the theory of quasiconformal mappings, and other areas. Note that Liu, Wang and Tang (2015) obtained a variant of the Schwarz lemma for holomorphic self-mappings at the boundary of the Euclidean unit ball $\mathbb{B}^n$ in $\mathbb{C}^n$, and Wang and Ren (2017) considered holomorphic self-mappings of strongly pseudoconvex domains in $\mathbb{C}^n$. In all these results, the assumption that the mappings are holomporphic at a smooth boundary point is essential. Hamada [37] obtained a variant of the Schwarz lemma at the boundary for holomorphic self-mappings of finite dimensional irreducible bounded symmetric domains by using the Julia-Wolff-Carathéodory type condition. His proof is based on a study of the boundary behavior of the infinitesimal Kobayashi metric near smooth boundary points, by having in view an explicit expression of the infinitesimal Kobayashi metric of the unit ball of a finite dimensional JB*-triple. Extensions and related results on bounded symmetric domains have been obtained by Graham, Hamada and Kohr [19] and Hamada and Kohr [48, 49].

I) Harmonic and pluriharmonic mappings in $\mathbb{C}^n$

- Extensions to higher dimensions of corresponding results valid for planar harmonic mappings have been obtained by M. Chuaqui, H. Hamada, R. Hernández and G. Kohr [8], and H. Hamada and G. Kohr [45]. P. Duren, H. Hamada, G. Kohr [12] obtained two-point distortion theorems for univalent harmonic functions on the unit disc in $\mathbb{C}$ and pluriharmonic mappings in several complex variables.

J) One of the main research interests of Gabriela Kohr in **Geometric function theory of several complex variables** was related to the **extensions of classical results for univalent functions to the case of (finite or infinite dimensional) complex Banach spaces**. The main results in this direction are presented below.

1. Geometric function theory in several complex variables (the finite dimensional case)

(a) Univalence criteria for applications of class C^1 on the unit ball and strictly pseudo-convex domains in $\mathbb{C}^n$ for which the Bergman kernel becomes infinite on the boundary. Extensions of Jack's, Miller's and Mocanu's lemma on the unit ball and pseudo-convex domains in $\mathbb{C}^n$. Starlikeness and convexity of order α, strongly starlikeness of order α on the Euclidean unit ball in $\mathbb{C}^n$: growth and covering results, and coefficient bounds. Alpha convex mappings on the Euclidean unit ball in $\mathbb{C}^n$: necessary and sufficient conditions, geometric and analytic characterizations. Spirallike mappings of type α on the unit ball in $\mathbb{C}^n$: geometric and analytic characterizations. Covering, growth and distortion results (most of them being sharp), and coefficient bounds for certain compact subclasses of normalized biholomorphic mappings on the unit ball in $\mathbb{C}^n$. Higher dimensional versions of classical results in the theory of linear invariant families (L.I.F's) of one complex variable. Necessary and sufficient conditions of univalence for mappings in L.I.F's (two-point distortion results). Two-point distortion results for affine linear invariant familes of harmonic and pluriharmonic mappings.

(b) Geometric and analytic properties of certain subclasses of $S(B^n)$ generated by the (generalized) Roper-Suffridge and the Pfaltzgraff-Suffridge operators: Starlikeness and convexity properties associated with the Roper-Suffridge extension operator. Growth and covering results, L.I.F's generated by the (generalized) Roper-Suffridge extension operator. Bloch mappings and the Roper-Suffridge extension operator. Loewner chains associated with the Roper-Suffridge extension operator. Extreme points and support points associated with certain compact subsets of $S(B^n)$ generated by the Roper-Suffridge extension operator.

2. The theory of Loewner chains in several complex variables. We highlight the **main results of Gabriela Kohr** and her collaborators in this important research direction.

(a) *Compactness of the Carathéodory class on the unit ball in* $\mathbb{C}^n$ (*n*-dimensional version of the class of holomorphic functions on the unit disc with positive real part).

(b) *The n-dimensional version of the well known Carathéodory result, concerning the equivalence between the kernel convergence and compact convergence of univalent functions.*

(c) *Existence and uniqueness of solutions to the generalized Loewner differential equation in* $\mathbb{C}^n$.
(d) *Analytic and geometric characterizations of various subclasses of* $S(B^n)$, *by using the method of Loewner chains.*
(e) Geometric characterizations of Loewner chains on the unit ball in $\mathbb{C}^n$: *Mappings which have parametric representation on the unit ball in* $\mathbb{C}^n$. *Growth, distortion results and coefficient bounds.*
Asymptotically starlike/spirallike maps on the Euclidean unit ball in $\mathbb{C}^n$. *Asymptotically spirallike mappings and non-normalized univalent subordination chains in* $\mathbb{C}^n$. Geometric aspects.
(f) *Sufficient conditions for quasiregular holomorphic mappings, which can be imbedded in Loewner chains, to have quasiconformal extensions of* $\mathbb{R}^{2n}$ *onto itself.*
(g) *General and abstract constructions of Loewner chains on hyperbolic complex manifolds.*
(h) *Extreme points, support points and the Loewner variation in several complex variables.*
(i) *Approximation properties of biholomorphic mappings with parametric representation on the unit ball in* $\mathbb{C}^n$ *by automorphisms of* $\mathbb{C}^n$ *and smooth quasiconformal diffeomeomorphisms of* $\mathbb{C}^n$ *onto itself* $(n \geq 2)$.
(j) *Loewner chains and nonlinear resolvents of the Carathéodory family on the unit ball in* $\mathbb{C}^n$.

3. Geometric function theory in complex Banach spaces (the infinite dimensional case)

(a) The study of various subclasses of $S(B)$ (the family of normalized biholomorphic mappings on B, where B is the unit ball in a complex Banach space): spirallikeness of type $\alpha \in (-\pi/2, \pi/2)$, convexity and Φ-likeness. Geometric and analytic aspects.
(b) Sharp growth and distortion results for normalized convex (biholomorphic) mappings on the unit balls of complex Hilbert spaces.
(c) *Infinite-dimensional versions* (*the case of reflexive complex Banach spaces*) *of classical results in the theory of Loewner chains and the Loewner differential equation.*
(d) Linear invariant families on unit balls in complex Hilbert spaces.

Conclusions

Many of the above mentioned strong results of Gabriela Kohr receive a great scientific recognition, being cited in valuable publications, and open new important research directions. The complete list of Gabi's publications can be found at [56].

Gabriela Kohr was an outstanding professor of Babeş-Bolyai University, an excellent researcher with a strong research activity and valuable contributions in Complex Analysis and Geometric function theory in several complex variables and complex Banach spaces. The main contributions of Gabriela Kohr refer to the generalization of the class S to higher dimensions via the theory of Loewner chains and to the development of this theory with her collaborators, especially with Ian Graham (Canada) and Hidetaka Hamada (Japan), in order to recover important properties, but also to point out major differences. (Note that $S = S(\mathbb{U}) = S(\mathbb{B}^1)$.)

Until the last moment of her life, Gabriela Kohr was totally dedicated to the scientific research and teaching activity. Gabi attended many international conferences, enjoying them and involving herself in her presentations. Her talent for Mathematics went hand in hand with her enormously work capacity. To these two qualities, a strong sense of responsibility and high standards in her scientific activity were added.

It was a chance and privilege for all that met and worked with Professor Gabriela Kohr. Unfortunately, her life suddenly stopped when she was in full professional ascent, when she had much more to say mathematically speaking, when her disciples needed her competent guidance in their scientific research. The loss of Gabriela Kohr left a big hole in the soul of all that knew and worked with her, especially for Mirela Kohr. Gabi remains for all of us an excellent researcher, a very rigorous person, devoted to her students. She will be always alive in our thoughts trough all her great scientific contributions, and her deep ideas will continue to inspire generations of mathematicians working in Complex Analysis and Geometric Function Theory.

With the reader's permission, the first named author (M.K.) would like to add some personal recollections about her dear twin sister, Gabriela Kohr. Gabi adored and protected Mirela until the last moment of her life. They were a team working together all the time, day in, day out. An hour without math and Mirela was wasted hour in Gabi's opinion. Gabi was the essence of Mirela's life, and she will continue her mission through Mirela. This is Mirela's promise for Gabi.

The authors of this testimonial would like to express their gratitude to all collaborators, colleagues and friends of Professor Gabriela Kohr who made this special issue possible, as a tribute and recognition to her valuable research activity.

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