

Book reviews

Teodor Bulboacă, Santosh B. Joshi, and Pranay Goswami, Complex Analysis: Theory and Applications, de Gruyter, 2019, xii + 409 pages, ISBN 978-3-11-065782-1, e-ISBN (PDF) 978-3-11-065786-9, e-ISBN (EPUB) 978-3-11-065803-3

The present book provides a modern presentation of various classical topics in complex analysis. As it is mentioned in the preface, the level of difficulty of the material increases gradually from chapter to chapter.

The book is divided into seven chapters, as follows.

The first chapter is an introductory chapter, in which there are reviewed the complex numbers, the topological and metric structures of the complex plane \mathbb{C} , and the topological structure of the extended complex plane $\widehat{\mathbb{C}}$.

The second chapter is devoted to the notion of holomorphy in the case of functions of one complex variable. There are presented basic properties regarding the derivative of a complex function, including the fundamental Cauchy-Riemann theorem of characterization the complex differentiability. Also, there are included useful examples of elementary entire functions. A special attention is paid to Möbius transformations and their basic properties (the invariance of the cross ratio, and the preservation of circles in \mathbb{C}_∞ onto circles in \mathbb{C}_∞).

The third chapter is concerned with the theory of the complex integral. For this aim, first there are defined the notions of paths, homotopy of two paths, simply connected domain, etc. Then there are presented the notion of the complex integral and basic properties regarding this notion. Among them, we mention here the fundamental Cauchy integral theorem for holomorphic functions, and the Cauchy integral formulas with its important consequences and applications. The notion of the index of a path, and the analytic branches of multi-valued functions are also discussed.

In the fourth chapter, the authors are concerned with the local properties of analytic functions in terms of the power series expansions. The chapter begins with a review of locally uniformly convergence of sequences of holomorphic functions, and continues with important properties of power series, the treatment of zeros of holomorphic functions, followed by the maximum modulus theorem and the Schwarz lemma. Finally, the Laurent series, the notion of an isolated singular point, and basic properties of meromorphic functions are also presented.

The fifth chapter deals with the residue theory and various applications of the fundamental residue theorem in the computation of complex integrals as well as on

some real integrals. The residue theory is then applied to study the number of zeros and poles of meromorphic functions. There are proved the argument principle and Rouché's theorem with its main consequences and applications in the theory of holomorphic functions.

The sixth chapter is one of the main chapters of this book. In the first section, there is proved the classical result of Montel concerning the equivalence between the notions of locally uniformly convergence and normal families in the case of holomorphic functions. In the second section, there is studied the notion of univalence for holomorphic functions, and there are obtained a necessary condition of univalence (the non-vanishing of the first derivative of a univalent function) and the Hurwitz theorem concerning the locally uniformly convergence of sequences of univalent functions. In the third section, there is treated a fundamental problem in the theory of univalent mappings, namely the conformal (biholomorphic) equivalence between simply connected domains $D \subsetneq \mathbb{C}$ and the open unit disc \mathbb{U} . In the fourth section, there is proved the famous Riemann mapping theorem, and there are deduced some consequences of this significant result in complex analysis.

Each chapter contains a useful collections of exercises of different level. The solutions to these exercises are carefully presented in the seventh chapter of this book.

The present book is clearly written, in an accessible style, and the proofs of the main results are rigorous. The examples and exercises help the reader to become acquainted with the theory of functions of one complex variable. It is recommended to undergraduate and graduate students, and to all researchers that are interested in classical and advanced topics of complex analysis.

Gabriela Kohr