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CONNECTING ALGORITHMICAL PROBLEMS IN SEMIGROUPS WITH THE THEORIES OF LANGUAGES AND AUTOMATA

K.D. TARKALANOV

Abstract. This paper is a survey with new generalizations and their short proofs of some of our results. Its purpose is connecting as pointed out in the title.

A beginning (and end) of a word over an alphabet is called correct [4] if its length is not smaller than the half of the word. A finite determined semigroup is of the class $K_{\frac{1}{2}}$ [4] under the following conditions: if two determining words have a common section which is a correct beginning (a correct end) of one of them, then this section is a beginning (an end) of the other as well. (We give a generalization in [10]). The word $E = R_1R_2\dots R_k$ over the alphabet of a semigroup of $K_{\frac{1}{2}}$ is normal [4] if each multiplier is s subword of a determining word, the first is a correct beginning, the last being a correct end and if each R_i is a correct end of R_{i+1} is a correct beginning.

The word problem is solvable in a semigroup of the class $K_{\frac{1}{4}}$ [4].

The inequality problem which is formulated by us in [9] is in fact the problem of the deduction in a semi-true system. The one-direction substitutions represents a transformation to smaller words. By a process of a symmetrization a semigroup is obtained and the one-direction inequalities introduce an order in it. This problem is more general than the word problem. It is solvable [10] in a class of partially ordered semigroups which are given by determining inequalities. This class of ours contains the class $K_{\frac{1}{2}}$. In the basis of our generalization way suggested in [9], we are not interested in the common sections of the right determining words in the one-direction transformations from left to right.

A vocabulary (a set of words over an alphabet) is called strongly regular [7] if none of its words enters into another and no real beginning of any word is an end of any of them. The invariant deconding automata without overtaking [7] decode (the coding is an overletter one) the coding word only after its full absorption at the entrance. Until then they iet out the empty word at the exit. This sort of automata exists if and only if [7] the coding words constitute a strongly regular vocabulary.

We prove that in a (partially ordered) semigroup with a strongly regular vocabulary of the determining words the inequality/word problem is solvable [11].

In an analogical way to the relation from theorem 2.1.5 [8] (as well as theorem 1 [5]) we define [12] a congruence $\approx$ in a semigroup Π in the following way: we shall say that the element $[x]$ is in correlation to the element $[y]$ in Π if and only if for any arbitrary elements $[w]$ and $[z]$ of Π the elements $[w][x][z]$ and $[w][y][z]$ simultaneously belong or do not belong to a given subset of elements of Π. We prove [12] the theorem: *If the full prototype $\varphi^{-1}(M)$ of the subset M of the semigroup Π with a finite number of generators Σ at the natural homomorfism φ of the free semigroup Σ^* (generated by them) is a regular language in the latter, then the congruence $\approx$ for M has a finite index in Π and M consists of full classes of elements which are equivalent to it. Then the natural homomorphic image $\Pi/\approx$ is a finite semigroup. (The natural homomorphism depicts each letter from Σ in that element from Π which contains it.)*

This theorem affords a common way of obtaining finite homomorphic images of semigroups. This can be fulfilled for the semigroups of the class $K_{\frac{1}{2}}$ and with a strongly regular vocabulary of the determining words: thus, we shall say that an element of a semigroup Π_1 of the class $K_{\frac{1}{2}}$ is normal if it consists of (a finite number - according to the theorems for the solvability of the word/inequality problem) normal words. For the subsemigroup N_1 of the normal elements $\varphi^{-1}(N_1)$ is a regular language in Σ^* [12]. The proof is effected by constructing a concrete right linear grammar [8]. We shall say that an element of a semigroup Π_2 from the other class is vocabular if all of its words (a finite number [11]) are products of determining words. For the subsemigroup N_2 from the vocabular elements $\varphi^{-1}(N_2)$ is a *regular language.*

Actually, for the strongly regular vocabulary $V = \{v_1, v_2, \ldots, v_m\}$ of the determining words of Π_2 we construct the right linear grammar

$$\Gamma_2 = \{\Sigma \cup \{\mu_i |\ i = \overline{1,m}\} \cup \{\sigma\}, \Sigma, Q_w, \sigma\}$$

(the designations are clear [8]). Here Q_2 are the following rules for deduction:

$$\sigma : v_i\mu_i; \quad i = \overline{1,m};$$

$$\mu_i \to v_j\mu_j; \quad j = \overline{1,m};$$

$$\mu_i \to \Lambda.$$

It has been proved that the language generated by it coincide with the subsemigroup $\varphi^{-1}(N_2)$ of all vocabulary words in Σ^*.

The already indicated theorem from [12] gives us the opportunity by a unified method to prove

Theorem 1. *Each semigroup from the two indicated classes possesses a finite homographic image with a nontrivial subsemigroup.*

The part of this theorem for the class with strongly regular vocabularies of the determining words is not published so far.

A word from a semigroup Π_1 is called symmetric if it begins and ends in a normal word. The symmetric elements form a subsemigroups H_1 in Π_1 [12]. The word $\gamma = \gamma_1 v_{j_1} \gamma_2 v_{j_2} \ldots \gamma_t v_{j_t} \gamma_{t+1}$ in a semigroup v_{j_s}, $s = \overline{1,t}$) is called right special if γ_{t+1} does not end in a nonempty proper beginning of a determining word. The right special elements form a subsemigroup H_2 in Π_2 [11]. The method applied for separating subsemigroups in the semigroups from the two classes aims each one of them to possess the following property: if a given element of the separated subsemigroup is a product of several of its elements then each of its words is a product of a word from the first multiplier multiplied by a word from the second one and so on - until the last one. This property is not fulfilled in the general case.

A definer regular algebra over an arbitrary semigroup has been introduced in [6] analogically to Kleene's algebra of regular events over a free semigroup. The general method of separating subsemigroup with the indicated property is suggested by the author and his two results from [12] and [11] can be unified in one (not published so far).

Theorem 2. *If $\mathcal{R}$ and $\mathcal{S}$ are regular expressions in a separated subsemigroup H of a semigroup of one of the two classes with a system of generators Σ then $\mathcal{R} = \mathcal{S} \Leftrightarrow \varphi^{-1}(\mathcal{R}) = \varphi^{-1}(\mathcal{S})$.*

In this way the problem of the equality $\mathcal{R} = \mathcal{S}$ is reduced to the solvable [8], [1] identity problem in the algebra of the regular events over Σ^*.

After everything said about the strongly regular vocabularies it is justified and in co-ordination with it to continue the research of the decoding automata without overtaking. We prove [13] that a homomorphic image [2] of an invariant decondin automaton without overtaking is the same automaton as the above. The existence of a nontrivial image is proved with the help of our theorem [14] for constructing factor-automata. Much later and in the particular case of automata without exits factor-automata are been studied by another author in [3]. With the help of the correspondences from the definition for an automaton homomorphism [2] we define an image of coding:

Let (H_1, H_2, H_3) be a homomorphism of the automaton $U = (S, B, A, \lambda, \beta)$ in the automaton $U' = (S', B', A', \lambda', \beta')$ and the correspondence $H_3 : A = \{a_1, a_2, \dots, a_n\} \to A' = \{a'_1, a'_2, \dots, a'_m\}$ of the exit alphabet A in the exit alphabet A' be reversible. Let U' be an invariant decondin automaton without overtaking for the conding $K_{V'}^{A'}$. Then $n \leq m$ and let us admit that $H_3(a_i) = a'_i$, $i = \overline{1, n}$ (after permutations in A or in A' which does not reduce the generality). We chose only one prototype v_i at the correspondence H_2 for each coding owrd v'_i $(i = \overline{1, n})$ of the coding $K_{V'}^{A'}$, where the words v_i from the semigroup B^* have lengths, which are respectively equal to these of the words v'_i. In this way we obtain many codings K_V^A of the alphabet A with vocabularies $V = \{v_1, v_2, \dots, v_n\}$. Analogically we prove an unpublished theorem which answers the interesting reverse question:

Theorem 3. *If U' is an invariant deconding automaton without overtaking for the coding $K_{V'}^{A'}$ then the automaton U decodes invariantly without overtaking each one of the possible codings K_V^A.*

We prove also in [13] that the consecutive connection of two invariant decoding automata without overtaking is the same automaton as the above. As a consequence we point out that after a consecutive coding with the help of strongly regular vocabularies a coding is obtained again with the help of such a vocabulary.

A considerable range of the problems considered is well-grounded from: methodological and philosophical point of view in [15].

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KRASSIMIR D. TARKALANOV, PLOVDIV UNIVERSITY, DEPT. OF MATHEMATICS AND INFORMATICS, 24 TZAR ASSON STR., PLOVDIV 4000, BULGARIA