

Counting solutions of quadratic congruences in several variables revisited

László Tóth

Department of Mathematics, University of Pécs, Hungary

ltoth@gamma.ttk.pte.hu

Consider the quadratic congruence $a_1x_1^2 + \dots + a_kx_k^2 \equiv n \pmod{r}$, where $\mathbf{a} = (a_1, \dots, a_k) \in \mathbb{Z}^k$, $n \in \mathbb{Z}$, $r \in \mathbb{N}$. Let $N_k(n, r, \mathbf{a})$ denote the number of its incongruent solutions. In the talk I sketch short direct proofs, using the Gauss quadratic sum for certain less known compact formulas on $N_k(n, r, \mathbf{a})$, valid for r odd. These formulas are in terms of the Ramanujan sum and the Jacobi symbol, and go back to the work of Paul Bachmann [1], Eckford Cohen [2] and Hermann Minkowski [3]. I also discuss some other related identities and asymptotic formulas which seem to not appear in the literature.

References

- [1] P. Bachmann, *Zahlentheorie*, vol. 4: Arithmetik der quadratischen Formen, Leipzig, 1898.
- [2] E. Cohen, Rings of arithmetic functions. II: The number of solutions of quadratic congruences, *Duke Math. J.* **21** (1954), 9–28.
- [3] H. Minkowski, *Gesammelte Abhandlungen*, Leipzig, 1911.