Almost everywhere and norm convergence of the inverse continuous wavelet transform in variable Lebesgue spaces

Kristóf Szarvas, Ferenc Weisz

Department of Numerical Analysis
Eötvös L. University

szarvaskristof@gmail.com
weisz@inf.elte.hu

The so called variable Lebesgue spaces is studied intensively in the last few years. Instead of the classical L_p-norm, the variable $L_{p(\cdot)}$-norm defined by

$$\|f\|_{p(\cdot)} := \inf \left\{ \lambda > 0 : \int_{\mathbb{R}^d} \left(\frac{f(x)}{\lambda} \right)^{p(x)} dx \leq 1 \right\}$$

and the variable $L_{p(\cdot)}$ spaces contains all f measurable functions, which $\|f\|_{p(\cdot)} < \infty$. The variable Lebesgue spaces have a lot of common property with the classical Lebesgue spaces (see in Cruz-Uribe, Fiorenza [1]).

The continuous wavelet transform of f with respect to a wavelet g is defined by

$$W_g f(x,s) := |s|^{-d/2} \int_{\mathbb{R}^d} f(t)\overline{g}(s^{-1}(t-x))\, dt = \langle f, T_x D_s g \rangle,$$

($x \in \mathbb{R}^d, s \in \mathbb{R}, s \neq 0$), when the integral does exist. The inversion formula holds for all $f \in L_2(\mathbb{R}^d)$:

$$C_{g,\gamma} \cdot f = \int_0^\infty \int_{\mathbb{R}^d} W_g f(x,s) T_x D_s \gamma \, \frac{dxds}{s^{d+1}},$$

where $C_{g,\gamma}$ is a constant depend on g and γ, but independent of f. Moreover under some conditions

$$\lim_{S \to 0} \int_S^\infty \int_{\mathbb{R}^d} W_g f(x,s) T_x D_s \gamma \, \frac{dxds}{s^{d+1}} = C_{g,\gamma} \cdot f$$

with convergence in L_p-norm, almost everywhere and each Lebesgue points for all $f \in L_p(\mathbb{R}^d)$ $(1 < p < \infty)$ see in Weisz [2].

In this paper we will investigate the convergence of the inversion formula in the $L_{p(\cdot)}$ spaces some sense, for example the norm and almost everywhere convergence or the convergence at the Lebesgue points.

References

[1] David V. Cruz-Uribe, Alberto Fiorenza. *Variable Lebesgue spaces.* Birkhäuser, Berlin, 2013.

[2] Ferenc Weisz. Inversion formulas for the continuous wavlet transform. *Acta Mathematica Hungarica*, 138:237-258, 2013.