Reconstructing graphs from a deck of all distinct cards

Miklós Bartha and Amitesh S. Shuva

Department of Computer Science, Memorial University of Newfoundland, St. John's, Canada

bartha@mun.ca, ss4545@mun.ca

If v is a vertex of graph G, then $G - v$ is the graph obtained from G by deleting the vertex v and its incident edges. We call $G - v$ a *vertex-deleted subgraph* of G, or the *card* associated with vertex v. The *deck* of G is the multiset of the cards associated with all of its vertices. One of the most well-known unsolved problems of graph theory asks wether a graph can be reconstructed up to isomorphism from its deck. The conjecture that the answer is true for all graphs having at least three vertices was formulated by Kelly and Ulam in 1942, but very little progress has been made towards its general proof since then. See [1] and [2] for two extensive surveys on the graph reconstruction problem. In this paper we show that graph G is uniquely reconstructible from its deck, provided that the deck of G is a *set*, that is, there are no two distinct vertices in G having the same card associated with them. Since any duplication of cards indicates the presence of a kind of symmetry within graph G, our result is in accordance with [3] saying that the probability that a randomly chosen graph on n vertices is not reconstructible goes to 0 as n goes to infinity.

References

[1] Harary, F. , A survey of the reconstruction conjecture, *in:* Lecture Notes in Mathematics 406, Springer, 1974, pp. 1828.

[2] Nash-Williams, C. St. J. A., The Reconstruction Problem, *in:* Selected topics in graph theory, 1978, pp. 205236.

[3] Bollobás, B., Almost every graph has reconstruction number three, *Journal of Graph Theory*, **14**, 1990, 14.