A contraction principle in generalized metric spaces

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One of the generalizations of the Banach Fixed Point Theorem is due to Matkowski, who replaced contractivity by a weaker but still effective property. The aim of this note is to extend the contraction principle in this spirit for such semimetric spaces that are equipped with a natural generalization of the standard triangle inequality. The stability of fixed points is also investigated in this setting. As applications, fixed point results are presented in ultrametric spaces.

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