

Lattice-like subsets of Euclidean Jordan algebras ^{*}

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While studying some properties of linear operators in a Euclidean Jordan algebra, Gowda, Sznajder and Tao have introduced generalized lattice operations based on the projection onto the cone of squares. In two recent papers of the authors of the present paper it has been shown that these lattice-like operators and their generalizations are important tools in establishing the isotonicity of the metric projection onto some closed convex sets. These kinds of results are motivated by methods for proving the existence of solutions of variational inequalities and methods for finding these solutions in a recursive way. It turns out, that the closed convex sets admitting isotone projections are exactly the sets which are invariant with respect to these lattice-like operations, called lattice-like sets. In this paper it is shown that the Jordan subalgebras are lattice-like sets, but the converse in general is not true. In the case of simple Euclidean Jordan algebras of rank at least three the lattice-like property is rather restrictive, e.g., there are no lattice-like proper closed convex sets with interior points.

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