k-Dyck words: generation and application

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Let $B = \{0, 1\}$ be a binary alphabet and $x_1x_2 \dots x_n \in B^n$ a word. Let $h : B \to \{-1, 1\}$ be a valuation function with $h(0) = 1$, $h(1) = -k$, where $k \geq 1$ is a given natural number, and

$$h(x_1x_2 \dots x_n) = \sum_{i=1}^{n} h(x_i)$$

A word $x_1x_2 \dots x_{(k+1)n} \in B^{(k+1)n}$ is called a *k-Dyck word* (similarly as in [2]) if satisfy the following conditions:

$$h(x_1x_2 \dots x_i) \geq 0, \qquad \text{for } 0 \leq i \leq (k+1)n - 1,$$
$$h(x_1x_2 \dots x_{(k+1)n}) = 0.$$

The number of k-Dyck words of length n is the so-called k-Catalan number [4]:

$$C_n^k = \frac{1}{kn+1}\binom{(k+1)n}{n}.$$

In this paper we deal with the generation of k-Dyck words and codification of $(k + 1)$-ary trees using k-Dyck words for $k > 1$. The case $k = 1$ was treated in [1, 3].

References

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