Results and problems in the regularity theory of functional equations *

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In this talk we survey results and open problems concerning regularity of solutions of functional equations. Although some results concerning composite equations and implicite equations will be mentioned, our central topic is theorems proving that "week" regularity (for example measurability or Baire property) implies "strong" regularity (for example $\mathcal{C}^\infty$ or analyticity) of the solutions f of functional equation

$$f(x) = h\Big(x, y, f\big(g_1(x,y)\big), \dots, f\big(g_n(x,y)\big)\Big),$$
$$(x,y) \in D \subset \mathbb{R}^r \times \mathbb{R}^s.$$

We give a survey of results and open problems, connections with Hilbert fifth problem, and with regularity problem of partial differential equations and variational problems. We consider also algorithmic methods implemented in computer algebra systems.

References

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