On consecutive numbers divisible by powers of their largest prime factors

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In [1], the following problem is considered: do there exist k consecutive integers such that they are all divisible by the rth power of their (respective) largest prime factors? The authors of the paper prove that there are infinitely many examples for $k = 2$, $r = 2$, provide some examples for $k = 3$, $r = 2$ and $k = 2$, $r = 3$. They also give heuristic arguments in favor of the existence of such numbers for every pair of positive integers k and r. We describe a computational search method for the $k = 3$, $r = 3$ case that narrows the search space and provides the first known examples. Our method also gives a heuristic lower bound on the number of such configurations in the general case, assuming some plausible number theoretical arguments.

We also consider possible generalizations to algebraic number fields and polynomials with examples and propose some new open questions.

References

[1] Jean-Marie De Koninck, Nicolas Doyon and Florian Luca: *Consecutive Integers Divisible by the Square of Their Largest Prime Factors,* to appear in the Journal of Combinatorics and Number Theory, **5** (2), 2013