A quasilinear elliptic problem involving critical Sobolev exponents

Francesca Faraci, Csaba Farkas

Department of Mathematics and Informatics, University of Catania
Department of Mathematics and Informatics, Sapientia University Tg. Mureş
Department of Mathematics and Informatics, Babeş-Bolyai University Cluj-Napoca,

ffaraci@dmi.unict.it
farkas.csaba2008@gmail.com

In the present talk we consider the following quasilinear elliptic equation

$$\begin{cases} -\Delta_p u = |u|^{p^*-2}u + g(u), & \text{in } \Omega \\ u = 0, & \text{on } \partial\Omega \end{cases} \qquad (\mathscr{P})$$

where Ω is a bounded domain of $\mathbb{R}^N$ with smooth boundary $\partial\Omega$, g is a continuous function with suitable growth condition. The main obstacle in dealing with existence and multiplicity results for quasilinear problems with critical nonlinearity is represented by the lack of compactness of the embedding $W_0^{1,p}(\Omega) \hookrightarrow L^{p^*}(\Omega)$.

We will prove the existence of a weak solution for problem by combining semicontinuity argument with direct methods of calculus of variations. The existence of a local minimum for the energy functional is ensured provided a suitable algebraic inequality is fulfilled.