Rodrigues formula for the Cayley transform of the groups SO(n) and SE(n)

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The Cayley transform of the group of rotations $\mathbf{SO}(n)$ of the Euclidean space $\mathbb{R}^n$ is defined by $Cay : \mathfrak{so}(n) \to \mathbf{SO}(n), Cay(A) = (I_n + A)(I_n - A)^{-1}$, where $\mathfrak{so}(n)$ is the Lie algebra of $\mathbf{SO}(n)$. Because the inverse of the matrix $I_n - A$ can be written as $(I_n - A)^{-1} = I_n + A + A^2 + \ldots$ on a sufficiently small neighborhood of O_n, from the well-known Hamilton-Cayley Theorem, it follows that $Cay(A)$ has the polynomial form

$$Cay(A) = b_0(A)I_n + b_1(A)A + \ldots + b_{n-1}(A)A^{n-1},$$

where the coefficients $b_0, b_1, \ldots, b_{n-1}$ depend on the matrix A and are uniquely defined. By analogy with the case of the exponential map (see [1] and [2]), they are called *Rodrigues coefficients* of A with respect to the Cayley transform.

Using the main result in [3] (see also [4]), in this paper we present a method to derive the Rodrigues coefficients for $\mathbf{SO}(n)$. The case of the Euclidean group $\mathbf{SE}(n)$ is also discussed.

References

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[3] D.Andrica, R.-A.Rohan *Computing the Rodrigues coefficients of the exponential map of the Lie groups of matrices*, Balkan Journal of Geometry and its Applications, Vol.18(2013), No.2, 1-10.

[4] D.Andrica, R.-A. Rohan, *A new way to derive the Rodrigues formula for the Lorentz group*, Carpathian J.Math., 30(2014), No.1, 23-29.