# General linear complementarity problems: algorithms and models

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Linear complementarity problems (LCP) are usually NP-hard problems. The largest matrix class where the interior point algorithms (IPA) are polynomial is the class of $P^*(\kappa)$-matrices, for given nonnegative real parameter $\kappa$. The union for all possible $\kappa$ parameters of $P^*(\kappa)$-matrices forms the class of $P^*$-matrices. This class of matrices has been introduced by Kojima et al. in 1991.

Cottle et al. (1989) defined the class of sufficient matrices (SU). It has been proved that several variants of the criss-cross algorithm are finite for LCPs with sufficient matrices.

After all of these, it is a natural question: What is the relation between the sufficient and $P^*$-matrices? Väliaho (1996) proved that the $P^*$-matrices are exactly those which are sufficient.

Using the concept of EP-theorem of Cameron and Edmonds (1990) and the LCP duality theorem of Fukuda and Terlaky (1992), Fukuda et al. (1998) were able to generalize the criss-cross algorithm for LCP problems with arbitrary matrices. The generalized criss-cross algorithm for LCPs with rational data stops in finite number of iterations with one of the following stopping criteria: (i) primal LCP has been solved and the encoding size of the solution has a polynomial bound, (ii) dual LCP has been solved and the encoding size of the solution has a polynomial bound, (iii) the matrix of the problem is not sufficient matrix and there is a certificate whose encoding size has a polynomial bound.

Since 1998, it was an interesting open question whether the result of Fukuda et al. can be obtained using some generalization of IPAs or not? We modified some IPAs such that their stopping criteria are the same as those of the generalized criss-cross algorithm. The modified interior point algorithms running time is still polynomial, but does not give in all cases a solution for solvable LCPs [third stopping criterion, the matrix is not sufficient, but the LCP might have a solution].

Some of our interior point algorithms that solve LCPs with arbitrary matrices in the sense of the EP-theorem have been published.

Goal of this talk is to introduce algorithms that may solve general LCPs and to show their computational performance on the well-known exchange market model of Arrow and Debreu.

Selected publications of Tibor Illés related to linear complementarity problems (LCP):

1. Illés T. and Nagy A., Finiteness of the quadratic primal simplex method when s-monotone index selection rules are applied, *Operations Research Reports*, Eötvös Loránd University of Sciences, Budapest, 2014:(1) pp. 1-13, 2014.
2. Csizmadia Zs., Illés T. and Nagy A., The s-monotone index selection rule for criss-cross algorithms of linear complementarity problems, *Acta Universitatis Sapientiae Informatica* 5(1): 103-139, 2013.
3. Illés T., Nagy M. and Terlaky T., Polynomial Interior Point Algorithms for General Linear Complementarity Problems, *Algorithmic Operations Research*, 5 (1): 1-12, 2010.
4. Illés T., Nagy M. and Terlaky T., Interior point algorithms for general linear complementarity problems, *Journal of Global Optimization*,. 47:(3) 329-342, 2010.
5. Illés T., Nagy M. and Terlaky T., EP theorem for dual linear complementarity problem, *Journal of Optimization Theory and Application*, 140:233-238, 2009.
6. Illés T. and Nagy M., A Mizuno-Todd-Ye type predictor-corrector algorithm for sufficient linear complementarity problems, *European Journal of Operational Research*, 181:(3) 1097-1111, 2007
7. Csizmadia Zs. and Illés T., New criss-cross type algorithms for linear complementarity problems with sufficient matrices, *Optimization Methods and Softwares*, 21(2):247-266, 2006.
8. Akkeleş A. A., Balogh L. and Illés T., New variants of the criss-cross method for linearly constrained convex quadratic programming, *European Journal of Operational Research*, 157(1): 74-86, 2004.
9. Illés T., Peng, J. Roos, C. and Terlaky T., A strongly polynomial rounding procedure yielding a maximally complementary solution for P(*)(k) linear complementarity problems, *SIAM Journal on Optimization*, 11(2): 320-340, 2000.
10. Illés T., Roos, C. and Terlaky T., Polynomial affine-scaling algorithms for P*(k) linear complementary problems, Lecture Notes in Economics and Mathematical Systems, 452: 119-137, 1997.