

6-th Romanian Itinerant Seminar on
Mathematical Analysis and its Applications
RISMAA 2024

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Bogdan-Cristian Anghelina - Covers of fractal interpolation functions and surfaces

Affiliation: Faculty of Mathematics and Computer Science, Transilvania University of Braşov

Position: Ph.D. Student

In this presentation, based on the results from [On the localization of Hutchinson–Barnsley fractals, *Chaos Solitons Fractals*, 173 (2023), 113674], we generate coverings of the graph of fractal interpolation functions and fractal interpolation surfaces. In this way, we provide a method which can be used in determining the spatial extent of such structures. As a by-product we obtain estimations for the range of the interpolated graph. Additionally, we provide some improvements to the results presented in the aforementioned paper. Some concrete examples and graphical representations are given.

Sebastian Aniţa - Regional control problems for interacting populations with nonlocal reaction terms

Affiliation: "Alexandru Ioan Cuza" University of Iaşi and Octav Mayer Institute of Mathematics of the Romanian Academy

Position: Full Professor

Some regional control problems related to reaction-diffusion systems with nonlocal reaction terms are presented. The reaction-diffusion systems investigated here are describing the dynamics of some interacting populations of prey-predator type. We focus on the eradicability of a pest population acting in a small subset directly on the pest or indirectly - via the prey population. Note that these are zero-stabilizability problems with state constraints.

We find necessary and sufficient conditions for eradicability in terms of the magnitude of the principal eigenvalues to appropriate partial-integro differential operators. In case of eradicability we indicate stabilizing feedback controls with simple structure.

Vasile Berinde - Iterative algorithms for solving the fixed point problem within some classes of nonexpansive type mappings

Affiliation: Technical University of Cluj-Napoca, North University Center Baia-Mare

Position: Full Professor

The solution of many nonlinear problems, like monotone inclusion problems, convex optimization problems, variational inequality problems, equilibrium problems, convex feasibility problems, split feasibility problems, split variational inequality problems etc. can be reduced to the solution of a corresponding fixed point problem. Under its fixed point formulation, the solution of this problem can be obtained by a certain fixed point iterative scheme. Starting from this fact, there is a steadily increasing interest in designing robust and efficient iterative algorithms for approximating the solution of the above nonlinear problems, which in turn, reduce to designing efficient and stable fixed point algorithms for various classes of nonlinear mappings of nonexpansive type. In this presentation we discuss some results concerning the solution of the fixed point problem for the case of some nonexpansive type mappings in Hilbert and Banach spaces.

References:

- [1] Berinde, V. Approximating fixed points of enriched nonexpansive mappings by Krasnoselskij iteration in Hilbert spaces. *Carpathian J. Math.* **35** (2019), no. 3, 293–304.
- [2] Berinde, V.: A modified Krasnoselskii-Mann iterative algorithm for approximating fixed points of enriched nonexpansive mappings. *Symmetry*, 14(2022), No. 1, Aer. no. 123.
- [3] Berinde, V., Păcurar, M.: Krasnoselskij-type algorithms for fixed point problems and variational inequality problems in Banach spaces. *Topology Appl.* **340** (2023), Paper No. 108708, 15 pp.
- [4] Berinde, V.: Approximating fixed points of demicontractive mappings via the quasi-nonexpansive case. *Carpathian J. Math.* **39** (2023), no. 1, 73–85.
- [5] Berinde, V., Petruşel, A., Rus, I. A.: Remarks on the terminology of the mappings in fixed point iterative methods in metric spaces. *Fixed Point Theory* **24** (2023), no. 2, 525–540.

Monica Bota - Fixed point theorems for Subrahmanyam contractions in b -metric spaces

Affiliation: Babeş-Bolyai University Cluj-Napoca

Position: Associate Professor

The well-known Banach contraction principle is generalized in two main directions: on the one hand, the work space is changed, on the other, one can consider single-valued or multivalued operators satisfying different contraction conditions. We consider both directions and work in a b -metric space. For the operator, we choose to work with a multivalued Subrahmanyam contraction. This notion was introduced for the single-valued case by Chaocha and Sudprakhon in 2017, and for the multivalued operators by A. Petruşel and G. Petruşel in 2020. Fixed point results are presented in the context of a b -metric space. One can easily notice that this notion generalizes the notion of graph

contraction. We consider the case of multivalued operators. We also deduce the Ulam-Hyers stability property of the fixed point inclusion. These results expand on some recent theorems proved in classical metric spaces.

Brigitte Breckner - A homage to Professor Varga Csaba

Affiliation: Babeş-Bolyai University Cluj-Napoca

Position: Associate Professor

There is given an overview on results obtained, in collaboration with Prof. Varga, in the domain of analysis on fractals.

Adriana Buică - On the bifurcation of periodic solutions of discrete dynamical systems

Affiliation: Universitatea Babeş-Bolyai Cluj-Napoca

Position: Associate Professor

We consider nonlinear periodic difference systems with periodic solutions. Some of them can persist after perturbing the system with a nonlinear periodic term that depends on a small parameter. This is decided by the existence of zeros of the so-called bifurcation function. We give its expression. The main result is the analogue of the celebrated Malkin theorem for periodic differential systems.

Aurelian Cernea - On some proportional fractional differential inclusions of Hilfer type

Affiliation: University of Bucharest

Position: Full Professor

Existence of solutions for a class of boundary value problems associated to a Hilfer generalized proportional fractional integro-differential inclusion is provided.

Ştefan Cobzaş - The strong Ekeland principle in quasi-metric spaces

Affiliation: Babeş-Bolyai University Cluj-Napoca

Position: Full Professor

The well known Ekeland's Variational Principle (EkVP), Ivar Ekeland (1974), asserts the existence of strict minima of some perturbed version of a lower semi-continuous function defined on a complete metric space. Later Pando Georgiev (1988), and Tomonari Suzuki (2010) proved a Strong Ekeland Variational Principle, meaning the existence of strong minima for such perturbations (a strong minimum means that any minimizing sequence converges). The aim of the present paper is to extend the strong Ekeland principle to the quasi-metric case, i.e., to "metric" spaces (X, d) with asymmetric distance (i.e., it is possible that $d(x, y) \neq d(y, x)$ for some $x, y \in X$).

Nicuşor Costea - Steklov-like eigenvalues for the ϕ -Laplacian

Affiliation: National University of Science and Technology Politehnica Bucharest

Position: Associate Professor

In this talk we discuss the existence of nontrivial solutions for the problem $\operatorname{div} \left(\frac{\phi(|\nabla u|)}{|\nabla u|} \nabla u \right) = \frac{\phi(|u|)}{|u|} u$ in a bounded smooth domain $\Omega \subset \mathbb{R}^N$, with a nonlinear boundary condition given by $\frac{\phi(|\nabla u|)}{|\nabla u|} \frac{\partial u}{\partial \nu} = \lambda V(x) \frac{\psi(|u|)}{|u|} u$, on $\partial\Omega$, where $\psi, \psi : \mathbb{R} \rightarrow \mathbb{R}$ are odd strictly increasing homeomorphisms and V is a nonnegative weight function. The existence of a sequence of positive eigenvalues $\{\lambda_n\}$ such that $\lambda_n \nearrow \infty$ is established via the Ljusternik-Schnirelman Principle. A full description of the spectrum is provided when the function ψ has subcritical and either sublinear or superlinear growth w.r.t. ϕ .

Rodica Curtu - Existence of diverse spatiotemporal dynamic patterns in a model for neuronal activity

Affiliation: The University of Iowa, USA

Position: Full Professor

In this talk I will discuss a two-dimensional model for neuronal activity and show that, depending on the choice of parameters, it could give rise to several

interesting spatiotemporal patterns. In our analysis we implement a series of methods from Dangelmayr & Knobloch (1987), to prove the existence of patterns such as traveling waves, standing waves and modulated waves. The analysis is complemented by numerical simulations that confirm the theoretical results.

Vasile Drăgan - Linear differential (difference) equations with positive evolutions on an ordered Hilbert space. Applications to stochastic control problems

Affiliation: Institute of Mathematics "Simion Stoilow" of the Romanian Academy & Academy of Romanian Scientists, 3 Ilfov, 050044, Bucharest, Romania

Position: Researcher

Linear differential equations and linear difference equations on an ordered Hilbert space are considered. The order relation is induced by a self dual, solid, closed, convex cone. We say that a differential (difference) equation defines a positive evolution on the ordered Hilbert space if the cone which induce the order relation is an invariant subset for the considered equation.

Using the norm induced by the Minkovski functional defined for a suitable open convex subset, we emphasize some interesting aspects regarding the exponential stability of the linear differential (difference) equations with positive evolution under consideration. Necessary and sufficient conditions for the exponential stability of such a kind of linear differential (difference) equations are provided. Those criteria are expressed in terms of the existence of globally defined and uniform positive solutions of some suitable non-homogeneous backward linear differential (difference) equations. These criteria will be then applied in the study of the exponential stability in mean square of a large class of time-varying stochastic linear differential equation of Ito type with the coefficients perturbed by a Markov process. Also, we shall show how the properties of linear differential (difference) equations with positive evolution can be involved in the characterization of some structural properties as: stabilizability, detectability of the linear stochastic controlled systems.

Valerian Alin Fodor - The projected regions over the faces of a polyhedron

Affiliation: Babeş-Bolyai University Cluj-Napoca

Position: Ph.D. Student

We characterize analytically the closure of the regions of \mathbb{R}^n which are projected over the relative interiors of some faces of a polyhedral set, through the

metric projection of the polyhedral set itself. We show that these regions are polyhedral sets by explicitly characterizing them through systems of linear inequalities. The action of the metric projection over the polyhedral set, restricted to precisely such a region, is showed to be the same as the one over the affine hull of the considered face.

Ștefan Garoiu - A Voronovskaya type theorem associated to geometric series of Bernstein - Durrmeyer type operators

Affiliation: Transilvania University of Brasov
Position: Ph.D. Student

In this paper we give a Voronovskaya type theorem for the operators introduced by U. Abel, which are defined as the geometric series of Bernstein - Durrmeyer operators.

Andrei Gasparovici - Mixed Dirichlet-Robin problem for coupled anisotropic Darcy-Forchheimer-Brinkman equations

Affiliation: Babeș-Bolyai University Cluj-Napoca
Position: Ph.D. Student

We study mixed Dirichlet-Robin boundary value problems for the nonlinear anisotropic Darcy-Forchheimer-Brinkman system and a system of coupled Darcy-Forchheimer-Brinkman equations using a variational approach and fixed-point techniques. We also provide applications and numerical results related to viscous incompressible fluid flows in multidisperse porous media.

Cristina Gheorghe - Picard operators, retraction-displacement condition and admissible perturbation of a multi-valued operator

Affiliation: Babeș-Bolyai University Cluj-Napoca
Position: Ph.D. Student

In this talk, I will present some strict fixed-point results, together with some stability properties for multi-valued operators satisfying some contraction type conditions.

References:

- [1] C. Gheorghe, Picard operators, retraction-displacement condition and admissible perturbation, to appear.
- [2] A. Petruşel, I.A. Rus, M.A. Şerban, Basic problems of the metric fixed point theory and the relevance of a metric fixed point theorem for multivalued operators, *J. Nonlinear Convex Anal.* 15 (2014), no. 3, 493-513.
- [3] I. A. Rus, Weakly Picard mappings: retraction-displacement condition, quasicontraction notion and weakly Picard admissible perturbation, *Studia Univ. Babeş-Bolyai Math.*, 69 (2024), no. 1, 211-221.

Anca Grad - Set-valued optimality conditions

Affiliation: Babeş-Bolyai University Cluj-Napoca

Position: Lecturer

Some new duality approaches in set-valued optimization are presented. The main ideas gravitate about the concepts of solutions defined with the help of the nonempty quasi interior of a convex cone, employing a set-type criterion. Such a procedure guarantees the existence of a so-called qi-subdifferential, in analogy to the conjugate function and subdifferential from scalar optimization. Weak duality theorems and theorems containing optimality conditions are proved for general unconstrained set-valued optimization problems, in connection to a newly proposed dual problem. In the particular case when the perturbation function extends the Lagrange perturbation from scalar optimization we prove a strong duality theorem for constrained set-valued optimization problems. An application of our strong set-valued Lagrange duality theorem by means of qi-efficiency,

Eduard Ştefan Grigoriuc - The Graham-Kohr extension operator in complex Banach spaces

Affiliation: Babeş-Bolyai University & Tiberiu Popoviciu Institute of Numerical Analysis, Romanian Academy

Position: Ph.D. Student

We focus our attention on the Graham-Kohr extension operator and its properties on different domains in complex Banach spaces. The main result consists in proving that the Graham-Kohr extension operator preserves the notion of g -Loewner chain (in particular, the g -starlikeness, spirallikeness and other geometric properties are preserved from the unit disc to complex Banach spaces).

Teodor Havârneanu - A Trotter scheme for Navier-Stokes equations (joint with C. Popa)

Affiliation: "Alexandru Ioan Cuza" University of Iași

Position: Full Professor

We consider a splitting approximation scheme for the two-dimensional Navier-Stokes equations with Navier boundary conditions, proving its convergence in an appropriate space.

Veronica Ilea - Fixed point results for non-self operators on vector-valued metric spaces

Affiliation: Babeş-Bolyai University Cluj-Napoca

Position: Lecturer

The purpose of this paper is to discuss some problems of the fixed point theory for non-self operators on vector-valued metric spaces. The results complement and extend some known results given in the paper: A. Chis-Novac, R. Precup, I.A. Rus, Data dependence of fixed points for non-self generalized contractions, Fixed Point Theory, 10 (2009), No. 1, 73–87.

Daniela Ioana Inoan - Calmness of the solution of a variational relation problem

Affiliation: Technical University of Cluj-Napoca

Position: Associate Professor

We study a parametric variational relation problem, that depends on two parameters. Holder calmness of the solution mapping is investigated, obtaining a bound on the distance between unperturbed and perturbed solutions. The main abstract result is then applied to some particular cases: a variational inclusion and an equilibrium problem.

Nataliia Kolun - Stratified steady inviscid water flows with effects of surface tension and constant non-zero vorticity

Affiliation: Babeş-Bolyai University Cluj-Napoca

Position: Researcher

We consider stratified steady inviscid three-dimensional water flows of finite depth with a free surface and an interface. The interface plays the role of an internal wave that separates two layers of constant and different density. Considering effects of surface tension, we prove that if the vorticity vectors in the two layers are constant, non-vanishing and parallel to each other, then the bounded solutions to the three-dimensional equations are essentially two-dimensional, and the free surface, the interface, the pressure and the velocity field present no variations in the direction orthogonal to the direction of motion.

Szilárd Csaba László - A Nesterov type algorithm with double Tikhonov regularization: fast convergence of the function values and strong convergence to the minimal norm solution

Affiliation: Technical University of Cluj-Napoca

Position: Full Professor

We investigate the strong convergence properties of a Nesterov type algorithm with two Tikhonov regularization terms in connection to the minimization problem of a smooth convex function f . We show that the generated sequences converge strongly to the minimal norm element from $\text{argmin } f$. We also show that from a practical point of view the Tikhonov regularization does not affect Nesterov's optimal convergence rate of order $O(n^{-2})$ for the potential energies. Further, we obtain fast convergence to zero of the discrete velocity, but also some estimates concerning the value of the gradient of the objective function in the generated sequences.

Radu Miculescu - On the localization of H-B fractals

Affiliation: Transilvania University of Braşov

Position: Full Professor

We present a method to determine a finite family of closed balls whose union contains the attractor of a given finite iterated function system. Our approach is similar to the one provided by D. Canright in "Estimating the spatial extent of attractors of iterated function systems", *Comput. & Graphics*, 18 (1994), 231–238. The central idea is to replace the system that is used by Canright to determine the radii of the balls with a simpler one. More precisely he finds algorithmically a solution of his system, but we are able to determine the algebraic expression of a solution of our system. As a by-product we are able to find a cover of the attractor with a finite family of closed balls having the radii

smaller than a prescribed value, at the cost of increasing the number of family's elements. Finally a comparison, from the graphical and numerical simulation point of view, between our method and Canright's approach is presented.

Mădălina Moga - Fixed point and stability results for multi-valued nonlinear graph contractions in complete metric spaces

Affiliation: Babeş-Bolyai University Cluj-Napoca

Position: Ph.D. Student

In this work we will present a generalization of Covitz-Nadler's contraction principle, as well as of the multi-valued graph contraction principle. More precisely, we will consider the case of multi-valued nonlinear graph contractions in complete metric spaces for which fixed points and strict fixed points results are discussed. We will give existence, approximation, Ulam-Hyers stability and localization results for the fixed points of a multi-valued nonlinear graph contraction with closed graph.

Bogdan Daniel Moldovan - Conditions for the preservation of Motzkin decomposability and the Pareto bordered property under addition

Affiliation: Babeş-Bolyai University Cluj-Napoca

Position: Ph.D. Student

We provide some sufficient conditions on pairs of Motzkin decomposable sets and Pareto bordered sets in order to get the Minkowski sum of their components Motzkin decomposable and Pareto bordered respectively. Note that the none of the two classes, i.e. Motzkin decomposable and Pareto bordered, is closed with respect to the Minkowski sum. We also prove that minimal faces of a closed convex set are also extreme faces of the set and vice-versa. This result allows us to define the generalized Minkowski sets by using the extreme faces. A Minkowski type theorem is proved with extreme faces playing the role of the extreme points in the classical Minkowski Theorem. The special class of Pareto bordered sets, which is a subclass of that of generalized Minkowski sets, is also taken into account. Indeed, as mentioned above, we show that the Minkowski sum of some Pareto bordered sets with the same lineality remains Pareto bordered. Note that the class of generalized Minkowski sets is not closed with respect to the Minkowski sum. It is however worth to mention that the class of closed convex sets which are both Motzkin decomposable and generalized Minkowski (or shortly, $MdGM$ sets) is closed both with respect to Minkowski

sum and Cartesian product [J. E. Martínez-Legaz, C. Pinteá, Closed convex sets that are both Motzkin decomposable and generalized Minkowski sets, J. Nonlinear Var. Anal. 8 (2024), No. 4, pp. 571-579].

Gheorghe Moroşanu - Second-order differential inclusions with two small parameters

Affiliation: Babeş-Bolyai University Cluj-Napoca and Academy of Romanian Scientists Bucharest

Position: Researcher

Consider in a real Hilbert space H the following problem, denoted $(P_{\varepsilon\mu})$,

$$\begin{cases} -\varepsilon u''(t) + \mu u'(t) + Au(t) + Bu(t) \ni f(t), & 0 < t < T, \\ u(0) = u_0, \quad u'(T) = 0, \end{cases}$$

where $T > 0$ is a given time instant, $\varepsilon > 0$, $\mu \geq 0$ are small parameters, $A : D(A) \subset H \rightarrow H$ is a (possibly multivalued) maximal monotone operator, and $B : H \rightarrow H$ is a Lipschitz operator. Consider also the following reduced problem, denoted (P_μ) ,

$$\begin{cases} \mu u'(t) + Au(t) + Bu(t) \ni f(t), & 0 < t < T, \\ u(0) = u_0, \end{cases}$$

where $\mu > 0$, as well as the algebraic equation (inclusion)

$$Au(t) + Bu(t) \ni f(t), \quad 0 \leq t \leq T. \quad (E_{00})$$

We are concerned with the following topics:

- (a) existence, uniqueness and regularity of the solutions to the above problems and to equation (E_{00}) ;
- (b) continuity of the solution to $(P_{\varepsilon\mu})$ with respect to $\varepsilon > 0$ and $\mu \geq 0$;
- (c) convergence of the solution of problem $(P_{\varepsilon\mu})$ to the solution of problem (P_{μ_0}) , as $\varepsilon \rightarrow 0$ and $\mu \rightarrow \mu_0$, where μ_0 is a fixed positive number;
- (d) convergence of the solution of problem $(P_{\varepsilon\mu})$ to the solution of equation (E_{00}) , as $\varepsilon \rightarrow 0$ and $\mu \rightarrow 0_+$;
- (e) applications to problems involving PDEs.

Anna-Monika Muscaş - About the N-body problem with logarithmic potential

Affiliation: University of Oradea

Position: Ph.D. Student

We present some contributions on the study of the bi-dimensional gravitational systems with logarithmic potential.

Mihai Nechita - Elliptic unique continuation and optimal approximation

Affiliation: Babeş-Bolyai University Cluj-Napoca

Position: Lecturer

We consider numerical approximations of ill-posed elliptic problems with conditional stability. The notion of optimal error estimates is defined including both convergence with respect to discretization and perturbations in data. The rate of convergence is determined by the conditional stability of the underlying continuous problem and the polynomial order of the approximation space. A proof is given that no approximation can converge at a better rate than that given by the definition without increasing the sensitivity to perturbations, thus justifying the concept. A recently introduced class of primal-dual finite element methods with weakly consistent regularisation is recalled and the associated error estimates are shown to be optimal in the sense of this definition.

Adriana Nicolae - Means of finite sets in metric spaces with nonpositive curvature

Affiliation: Babeş-Bolyai University Cluj-Napoca

Position: Associate Professor

Convex hulls are hard to analyze even for three points in geodesic metric spaces. Instead, we focus on the question of computing and recognizing weighted means of a finite set of points in the setting of particular Alexandrov spaces with nonpositive curvature where geodesics can be computed efficiently. This talk is based on joint work with A. Goodwin (Cornell University), A. S. Lewis (Cornell University), and G. Lopez-Acedo (University of Seville).

Anamaria Paştiu - The univalence criteria based on the theory of Loewner chains on the unit ball of C^n

Affiliation: Babeş-Bolyai University Cluj-Napoca

Position: Ph.D. Student

We present various univalence criteria for holomorphic mappings on the unit ball B^n in C^n . We use several results related to the Loewner chains and the Loewner differential equation on the ball B^n in C^n .

Radu Păltănea - Generalized Voronovskaya theorem

Affiliation: Transilvania University of Braşov

Position: Full Professor

A generalized Voronovskaya theorem is presented.

Adrian Petruşel - Fixed point theory for multi-valued operators via admissible perturbation approach (joint work with G. Petruşel and J.-C. Yao)

Affiliation: Babeş-Boyai University Cluj-Napoca and Academy of Romanian Scientists Bucharest

Position: Full Professor

In this paper, we will consider the following problem: if a fixed point inclusion $x \in B(x)$ (where B is a "bad" multi-valued operator) is replaced by another fixed point inclusion $x \in G(x)$ (where G is now a "good/better" multi-valued operator), then which are the properties (existence, data dependence, stability) can be obtained for our initial problem in terms of some conditions on the new operator G ? We will do this by the help of the admissible perturbation technique, which will generate a "good" multi-operator G related to the "bad" multi-valued operator B by the fact that the fixed point set of B and respectively of G coincide.

Cornel Pinteă - Closed convex sets that are both Motzkin decomposable and generalized Minkowski sets (joint work with Juan Enrique Martínez Legaz)

Affiliation: Babeş-Boyai University Cluj-Napoca

Position: Associate Professor

We consider and characterize closed convex subsets of the Euclidean space which are simultaneously Motzkin decomposable and generalized Minkowski

or, shortly, $Mdgm$ sets. We also prove the existence of suitably defined fixed points for, possibly multivalued, functions defined on $Mdgm$ sets along with existence of classical fixed points for some single valued self functions of $Mdgm$ sets. The first mentioned type of existence results are based on Kakutani fixed point theorem, and the second type are based both on the Brouwer fixed point theorem and the Banach contraction principle.

Dorian Popa - Set-valued solutions of some functional equations

Affiliation: Technical University of Cluj-Napoca

Position: Full Professor

In this talk I present results on the characterization and on the representation of set-valued solutions for some functional equations of quadratic type and Jensen type.

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[1] Baias, A.R., Moşneguţu, B., Popa, D.: Set-valued solutions of a generalized quadratic functional equation. *Results Math.* 73(4), 1–8 (2018).

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[3] Baias, A.R., Popa, D. & Rassias, Michael Th.: Set-valued solutions of an equation of Jensen type, *Quaestiones Mathematicae* 2023, 46(6): 1237–1244.

[4] Iz-iddine EL-Fassi, Nikodem, K. and Popa, D.: Set-valued solutions of a two-variable functional equation with involutions, *Aequat. Math.* 96 (2022), 453–464.

Ioan Raşa - Positive linear operators and convex stochastic orders

Affiliation: Technical University of Cluj-Napoca

Position: Full Professor

Inequalities involving convex functions are instrumental in the theory of positive linear operators. They can be expressed in terms of convex stochastic orders. We will present old and new results in this direction.

Adina Luminița Sasu - Input-output criteria, ergodic theory approaches and asymptotic properties of dynamical systems

Affiliation: West University of Timișoara and Academy of Romanian Scientists Bucharest

Position: Full Professor

One of the most important classes of criteria in the asymptotic theory of dynamical systems is represented by the input-output methods. These properties facilitate the description of an asymptotic behavior by means of the solvability of an associated input-output system between well-chosen spaces. In this lecture we present a new philosophy in exploring asymptotic properties of discrete dynamical systems via the input-output conditions that take place only on a set of the parameter space of positive measure, by employing ergodic theory approaches. We discuss various admissibility conditions, examples, related applications as well as open problems in this framework. Our results apply to broad classes of dynamical systems.

The results presented in this lecture were obtained in collaboration with D. Dragičević (University of Rijeka, Croatia) and B. Sasu (West University of Timișoara, Academy of Romanian Scientists, Romania).

References:

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[3] D. Dragičević, A. L. Sasu, B. Sasu, L. Singh, Nonuniform input-output criteria for exponential expansiveness of discrete dynamical systems and applications, *Journal of Mathematical Analysis and Applications*, 515 (2022), 1-37.

[4] D. Dragičević, A. L. Sasu, B. Sasu, Input-output criteria for stability and expansiveness of dynamical systems, *Applied Mathematics and Computation* 414 (2022), 1-22.

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Marcel-Adrian Șerban - Some research directions in fibre contraction theory and its applications

Affiliation: Babeș-Bolyai University Cluj-Napoca

Position: Associate Professor

In this paper, we will discuss the fixed point theory of triangular operators in the setting of generalized metric spaces and for contraction type operators. Global asymptotic stability of the fixed point, well-posedness of the fixed point problem, Ulam-Hyers stability and Ostrowski property are investigated. Some applications of the basic fibre contraction principles are also considered.

Timea Melinda Szemely Fulop - Uniform h - dichotomy in mean for skew-evolution semiflows in Banach spaces

Affiliation: West University of Timisoara
Position: Ph.D. Student

The main aim of this paper is to give characterizations for the uniform dichotomy in mean with growth rates concept for stochastic skew-evolution semiflows in Banach spaces. We will present the connection between uniform h -dichotomy in mean in discrete time and uniform h -dichotomy in mean in continuous time with respect to both invariant projection families and strongly invariant projection families associated with the stochastic skew-evolution semiflows.

Truşcă Radu - Maia type theorems for some multi-valued generalized contractions

Affiliation: Babeş-Bolyai University Cluj-Napoca
Position: Ph.D. Student

In this work we will present a generalization of Covitz-Nadler's contraction principle, as well as of the multi-valued graph contraction principle. More precisely, we will consider the case of multi-valued nonlinear graph contractions in complete metric spaces for which fixed points and strict fixed points results are discussed. We will give existence, approximation, Ulam-Hyers stability and localization results for the fixed points of a multi-valued nonlinear graph contraction with closed graph.

Bianca Ioana Vasian - Generalized Kantorovich operators

Affiliation: Transilvania University of Braşov
Position: Ph.D. Student

In this paper we will propose a class of generalized Kantorovich type operators constructed using a general differential operator with non-constant coefficients $D^l g(x) = \sum_{i=0}^l a_i(x) g^{(i)}(x)$ and its corresponding antiderivative operator I^l with respect to the composition $D^l \circ I^l = Id$. The operators studied are of the type $K_n = D^l \circ L_n \circ I^l$ where L_n are positive linear operators. For these operators we will prove an approximation result and a Voronovskaja type theorem. Also, a simultaneous approximation result will be provided for a particular case. The operators studied in this paper are linear but not positive operators.

Cristian Vladimirescu - Stability for systems of coupled nonlinear oscillators described by second-order ODEs with coefficient functions in $W_{loc}^{1,1}(0, \infty)$

Affiliation: University of Craiova

Position: Associate Professor

In this work we investigate the stability of the null solution of different systems of differential equations describing the motion of several coupled nonlinear oscillators, in the case when its coefficient functions are in $W_{loc}^{1,1}(0, \infty)$. Under assumptions more general than those from [G. Moroşanu, C. Vladimirescu, Stability for systems of coupled nonlinear oscillators, *Nonlinear Anal. Real World Appl.* 59 (2021), 103242] we obtain new stability results relying on classical differential inequalities and the Lyapunov's method. Our theoretical results are illustrated with numerical simulations.