Stud. Univ. Babeş-Bolyai Math. 59(2014), No. 2, 263–266

Book reviews

Łukasz Piasecki, Classification of Lipschitz Mappings, CRC Press, Taylor & Francis Group, Boca Raton 2014, x + 224 pp, ISBN: 13: 978-1-4665-9521-7.

The book is concerned with the study of Lipschitz mappings on metric spaces in connection to fixed point theory. One denotes by $L(k)$ the class of Lipschitz mappings with constant $k > 0$ on a metric space (M, ρ), that is mappings $T\colon M \to M$ satisfying the condition $\rho(Tx, Ty) \le k\rho(x, y)$ for all $x, y \in M$. The smallest Lipschitz constant of the mapping T is denoted by $k(T)$ (or by $k_\rho(T)$, if necessary). The mapping T is called uniformly Lipschitz if there exists $k > 0$ such that $\rho(T^n x, T^n y) \le k\rho(x, y)$ for all $x, y \in M$ and all $n \in \mathbb{N}$. This class is characterized by the condition $k_\infty(T) := \limsup_{n\to\infty} \sqrt[n]{k(T^n)} < \infty$.

It follows that $k(T^{m+n}) \le k(T^m)k(T^n)$ so that one can define the characteristic $k_0(T) = \lim_{n\to\infty} \sqrt[n]{k(T^n)} = \inf\{ \sqrt[n]{k(T^n)} : n \in \mathbb{N}\}$ – the analog of the spectral radius of a continuous linear operator on a Banach space. It turns up that $k_0(T) = \inf_d k_d(T)$, where the infimum is taken over all metrics d on M that are Lipschitz equivalent to ρ. An important class of Lipschitz mappings is formed by the nonexpansive ones, i.e. Lipschitz mappings with $k = 1$. The fixed point theory for this class of mappings acting on a Banach space X is tightly connected with the geometric properties of the underlying Banach space X (uniform rotundity, superreflexivity, uniform nonsquareness) as well as with those of the convex set $C \subset X$ on which they act (having normal structure, for instance). Some basic results along with some recent ones in this domain are presented in the seventh chapter of the book.

The main class studied by the author is that of mean Lipschitz functions. A multi-index is an n-tuple $\alpha = (\alpha_1, \ldots, \alpha_n)$ with $\alpha_1, \alpha_n > 0, \alpha_i \ge 0$ and $\sum_{i=1}^n \alpha_i = 1$. A mapping $T : M \to M$ is called α-Lipschitzian with constant k if $\sum_{i=1}^n \alpha_i \rho({}^i x, T^i y) \le k\rho(x, y)$ for all $x, y \in M$. The class of these mappings is denoted by $L(\alpha, k)$. Any α-Lipschitz mapping is Lipschitz and $k(T) \le k(\alpha, T)/\alpha_1$. Uniformly k-Lipchitzian mappings are (α, k)-Lipschitz for any multi-index α. Another class is that of the mappings satisfying, for $p \ge 1$ and some $k \ge 0$, the condition

$$\left(\sum_{i=1}^n \alpha_i \rho({}^i x, T^i y)^p\right)^{1/p} \le k\rho(x, y), \quad \forall x, y \in M\,,$$

called (α, p)-Lipschitz mappings with constant k.

The bulk of the book is formed by the chapters 4. *On Lipschitz constants for iterates of mean lipschitzian mappings*, 5. *Subclasses determined by p-averages, 6. Mean* *contractions*, 8. *Mean nonexpansive mappings*, and 9. *Mean lipschitzian mappings with* $k > 1$. These chapters are concerned with the behavior of the quantities $k_0(T)$, $k(T^n)$ and $k_\infty(T)$, including some numerical experiments, for mappings T in these classes of mean-Lipschitz mappings, and are essentially based on results obtained by the author alone or in cooperation with Víctor Pérez García.

The first two chapters 1. *The Lipschitz condition* and 2. *Basic facts on Banach spaces*, contain some preliminary notions and results.

The book is well written and contains new interesting results along with some classical ones in metric fixed point theory. The prerequisites are modest – some basic results in topology and functional analysis – so it can be used by advanced undergraduate and graduate students for an introduction to this domain and by researchers as a reference text. Experts in other areas, as differential equations, dynamical systems, will find it useful as well.

S. Cobzaş

Ioannis M. Roussos, Improper Riemann Integrals, CRC Press, Taylor & Francis Group, Boca Raton 2014, xiv + 675 pp, ISBN: 978-1-4665-8807-3.

The book contains a detailed presentation of the main improper Riemann integrals (with or without parameter) at the master level for students in mathematics, statistics, applied sciences and engineering. As it is well known, the improper Riemann integrals are important tools in various areas of mathematics (differential equations, probability theory) as well as in its applications to physics, mechanics, engineering. New classes of functions (e.g. Euler' Beta and Gamma functions) are introduced as improper Riemann integrals depending on a parameter as well as the integral transforms of Fourier and Laplace.

The presentation is restricted to Riemann integral (including double Riemann integral) and in order to make the book self-contained the principal theorems used in the calculations are included, some with proofs other without. In some cases these results are presented under some restricted conditions, accessible to the undergraduate but sufficient for applications.

The book is divided into two main parts 2. *Real analysis techniques*, and 3. *Complex analysis techniques*. An introductory chapter contains the definition of an improper integral, convergence criteria and some motivating examples.

Chapter 2 contains a detailed study of the properties of improper Riemann integrals depending on a parameter – continuity, differentiability, integrability. The treatment is based on a version of Lebesgue dominated convergence theorem for the Riemann integral. Applications are given to Frullani integrals, the functions Beta and Gamma, and to the Laplace transform.

For reader's convenience Chapter 3 contains a quick introduction to complex analysis with emphasis on the elementary holomorphic functions - the exponential, the trigonometric functions, and the multivalued holomorphic functions - the complex logarithm $\log z$, the power function $z^\alpha = e^{\alpha \log z}$. Here the powerful and relatively simple method of residues is applied to the calculation of some improper Riemann integrals, including a relatively complete treatment of the Fourier transform – definition, Riemann-Lebesgue Lemma, calculus rules, the inversion formula – and a reconsideration of the Laplace transform in the complex case.

The last chapter of the book is 4. *List of non-elementary integrals and sums in text*, contains a record of the most important integrals and sum calculated in the text, with exact reference to the places where they appear.

By collecting a lot of important improper integrals and sums used in various domains, and presenting their calculation in an accessible but rigorous way, the book will be of great use to students in mathematics and related areas and for applied scientists (statisticians, engineers, physicists) as well. A small personal objection – the presentation of some examples is too detailed, and so the abundance of these details hide to some extent the ideas behind.

T. Trif

Miroslav Pavlović, Function Classes on the Unit Disc, Studies in Mathematics, Vol. 52, xiii + 449 pp, Walter de Gruyter, Berlin - New York, 2014, ISBN: 978-3-11-028123-1, e-ISBN: 978-3-11-028190-3, ISSN: 0179-0986.

The book is concerned with spaces of harmonic and of analytic functions in the unit disc $\mathbb{D}$ – Hardy, Bergman, Besov, Lipschitz, Bloch, Hardy-Sobolev, BMO, etc. The approach proposed by the author differs from those contained in the classical books of Zygmund, Duren, Koosis, Garnett, allowing him to present new results and to give simpler and clearer proofs to some known facts (e.g. Fefferman-Stein theorem on subharmonic functions, theorems on conjugate harmonic functions, etc).

The first three chapters, 1. *The Poisson integral and Hardy spaces*, 2. *Subharmonic functions and Hardy spaces*, and 3. *Subharmonic behavior and mixed norm spaces*, are devoted to the spaces $h(\mathbb{D})$ and $H(\mathbb{D})$ of harmonic, respectively analytic, functions in the unit disc $\mathbb{D}$.

In Chapter 4. *Taylor coefficients with applications,* the approach to the mixed-norm Bergman spaces is based on a class of functions, called quasi-nearly subharmonic, introduced by the author. Besov spaces are studied in Chapter 5, while the sixth chapter, *The dual of H^1 and some related spaces*, is concerned with the duality between H^1 and BMO spaces. Chapter 7. *Littlewood-Paley theory*, contains some deep characterizations of H^p, $p > 0$, spaces as well as of hyperbolic Hardy classes.

The Lipschitz classes Λ^p_ω of analytic functions are studied in chapters 8. *Lipschitz spaces of first order*, and 9. *Lipschitz spaces of higher order*, defined by ordinary moduli of continuity, respectively by higher order moduli of smoothness. Chapter 10. *One-to-one mappings,* is devoted to the problem of membership of univalent and quasiconformal harmonic mappings in some classical spaces, while Chapter 11. *Coefficients multipliers,* some multiplier results are presented following some ideas of Kalton and of the author, including compact multipliers and multipliers on spaces with non-normal weight.

Chapter 12. *Toward a theory of vector-valued spaces,* presents some results on spaces of harmonic and analytic functions with values in a Banach or a quasi-Banach space X.

Author's booklet, *Introduction to the Function Spaces on the Disk*, Matematički Institut SANU, Special Publications, vol. 20, Belgrade 2004, contains some material included in the present book, although, as the author mentions in the Preface, this new book can not be considered as an expanded version of the former one – they have only nonempty intersection.

The reading of the book assumes familiarity with real, complex and functional analysis (at the level of Rudin's *Real and Complex Analysis*). For reader's convenience, two appendices, A. *Quasi-Banach spaces*, and B. *Interpolation and maximal functions*, are added to the main text. Sixteen research problems are included, and each chapter ends with a section of historical notes and references to further results.

The book is well written and contains a lot of deep and interesting results, including personal contributions of the author. It can be recommended to specialists as a reference text and to post-graduate students for study.

Gabriela Kohr