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Book reviews

Smaïl Djebali, Lech Górniewicz and Abdelghani Ouahab, Solution sets for differential equations and inclusions, Series in Nonlinear Analysis and Applications, Vol. 18, xix + 453 pp, Walter de Gruyter, Berlin - New York, 2013, ISBN: 978-3-11-029344-9, e-ISBN: 978-3-11-029356-2, ISSN: 0941-813X.

The book is concerned with the topological structure of the solution sets of differential equations and inclusions. with the aim to offer a comprehensive exposition of classical and recent results in this area. This direction of research was initiated by G. Peano in 1890 who proved that the set S of solutions to the Cauchy problem (1) $x'(t) = f(t, x(t))$, $t \in [t_0, a]$, $x(t_0) = x_0$, where $f : [0, a] \times \mathbb{R} \to \mathbb{R}$ is a continuous function, has the property that the sections $S(t) = \{x(t) : x \in S\}$ are nonempty, compact and connected for every t in a neighborhood of t_0. The result was successively extended, first by Kneser in 1923 to n dimensions, and then to various settings by other mathematicians – Hukuhara (1928), Aronszajn (1942). A turning point in this study was represented by a paper from 1969 by F. Browder and C. Gupta, a result that appears recurrently, in various hypostases, throughout the book.

The topological properties of the solution sets considered by the authors are acyclicity, the AR (absolute retract) property, being an R_δ-set (i.e., the intersection of a decreasing sequence of compact absolute retracts). The study is done in the first chapter, *Topological structure of fixed point sets*, in the more general context of fixed point sets for various kinds of mappings or of operator equations. This chapter contains also the proofs of some fundamental fixed point theorems for single-valued mappings (Banach, Brouwer, Schauder), as well as for set-valued mappings. The second chapter, *Existence theory for differential equations and inclusions,* contains the fundamental theorems of Picard-Lindelöf, Peano and Carathéodori, as well as Nagumo's results on the existence of solutions of differential equations on non-compact intervals and of differential inclusions.

The core of the book is formed by the chapters 3, *Solution sets for differential equations and inclusions,* and 4, *Impulsive differential inclusions and solution sets,* where the authors systematically examine the topological behavior of the solutions sets in various situations.

In order to make the book self-contained, the authors have included some auxiliary material from algebraic topology in Chapter 5, *Preliminary notions of topology*, and set-valued analysis in Chapter 6, *Background in multi-valued analysis,* completed in Appendix with other results on compactness and weak compactness in function spaces, the Bochner integral, C_0-semigroups.

The book ends with a rich bibliography, counting 506 items and including lots by the authors.

Written by experts and including many of their results and of their co-workers, and presenting in a unitary way a lot of results, both classical and recent, scattered to various publications, this research monograph will become an indispensable tool for the researchers in nonlinear ordinary and partial differential equations and inclusions, applied topology and topological fixed point theory. The self-contained style of exposition adopted by the authors, with a careful presentation of the needed background material, makes the book of great help for those desiring to start research in this field, too.

Radu Precup

Carlos Boss and Charles Schwartz, Functional Calculi, World Scientific, London - Singapore, 2013, x+215 pp, ISBN 978-981-4415-97-2.

The book is devoted to an exposition of functional calculi for various classes of linear operators, including the background material needed for the presentation. By a functional calculus one understands a construction which associates to an operator, or to a family of commuting operators, a homomorphism from a function space into a subspace of continuous linear operators. The simplest case is that of a polynomial $p(z) = \sum_{k=0}^{n} a_k z^k$ and an operator A on a Banach space X, to which one associates the operator $p(A) = \sum_{k=0}^{n} a_k A^k$. The so defined correspondence, $p \mapsto p(A)$, is a homomorphism from the algebra $\mathcal{P}$ of polynomials into the algebra $L(X)$ of continuous linear operators on X, and any functional calculus should be an extension of this homomorphism. The most familiar example is that of a continuous self adjoint linear operator A on a complex Hilbert space H, in which case there exists a projection valued measure E defined on the σ-algebra of Borel subsets of the spectrum $\sigma(A)$ of A, such that $A = \int_{\sigma(A)} z dE(z)$. This case is treated in Chapter 2, *Functions of a self adjoint operator*, with the extension to several commuting self adjoint operators given in Chapter 3. The fourth chapter is concerned with the spectral theorem for normal operators. The background material is developed in Chapter 1, *Vector and operator valued measures,* which presents the integration of scalar functions with respect to vector measures, with a stretch on operator valued measures and, in particular, on projection valued measures. The integration of vector functions with respect to scalar measures is treated in Chapter 5. The sixth chapter, *An abstract operational calculus*, is concerned with an axiomatic approach to operational calculus. The Riesz functional calculus, based on the theory of analytic vector functions, in particular on Cauchy's formula for such functions, is developed in the seventh chapter. The preamble of this chapter contains a brief but thorough presentation of basic results on vector analytic functions, including a proof, due to S. Grabiner (1976), of Runge's approximation theorem.

The last chapter of the book, Chapter 8, *Weyl's functional calculus*, is concerned with a functional calculus devised by H. Weyl and having its origins in quantum mechanics. The key tool is the so called Weyl transform (the authors use a slight modification of the original one), based on the Fourier transform of vector tempered distributions.

The basic text is completed by five appendices: A. *The Orlicz-Pettis theorem* (on unconditionally convergent series in Banach spaces), B. *The spectrum of an operator*, C. *Self adjoint, normal and unitary operators*, D. *Sesquilinear functionals,* and E. *Tempered distributions and the Fourier transform.* The most consistent of these is the last one (30 pages) which contains a quick presentation (with proofs) of the basic results in this area – the spaces $\mathcal{S}(\mathbb{R}^n)$ and $\mathcal{S}'(\mathbb{R}^n)$, the Fourier transform of functions and of tempered distributions, the Paley-Wiener theorem.

The bibliography contains 47 items, mainly textbooks, but some fundamental papers in spectral theory, or containing simpler proofs of known results, are included as well.

The book presents, in an accessible, self-contained way and in a relatively small number of pages, some basic results in the spectral theory of linear operators on Banach or on Hilbert space. Of great help is the auxiliary material which prevent the reader to lose time by looking through various treatises on functional analysis or measure theory. The book can be used for courses on functional analysis and operator theory, or for self-study, as an introduction to the subject.

Tiberiu Trif