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Book reviews

Jean-Paul Penot, Calculus Without Derivatives, Graduate Texts in Mathematics 266, Springer, New York - Heidelberg - Dordrecht - London, 2013, ISBN: 978-1-4614-4537-1/hbk; 978-1-4614-4538-8/ebook, xx + 524 pp.

Differential calculus offers efficient tools for the study of extrema of differentiable functions. In the case of nondifferentiable functions, defined on subsets of a Banach space X, more refined methods are needed – the differentials, which are elements of the dual space X^*, are replaced by various kinds of subdifferentials, which are subsets of X^*, and various types of cones (tangent, normal, regression, etc) enter the scene. In this new area, called nonsmooth analysis, sets and functions play interchangeable roles, leading to more flexibility in the treatment of optimization problems. As it is expected, the definitions of these new objects and the proofs of their properties use tools from set-valued analysis and differential calculus, topics that are treated in the first two chapters of the book, 1. *Metric and topological tools*, and 2. *Elements of differentiable calculus.* The first chapter contains some results from topology, set-valued analysis (continuity properties of multimaps, limits of sets), the variational principles of Ekeland, Deville-Godefroy-Zizler and Stegall, with applications to fixed point theorems, openness and regularity results (Robinson-Ursescu theorem), and well-posedness of optimization problems. In the second chapter, one studies the fundamental properties of the Fréchet, Hadamard and Gâteaux derivatives. Kantorovich's theorem on the Newton method is used to prove the Lyusternik-Graves theorem, the inverse and the implicit function theorems are proved with a special attention paid to the Lipschitz behavior of the inverse. As applications one studies the relevance of tangent and normal cones in optimization – Fermat's rule, Lagrange multipliers, Lyusternik theorem – and one gives a short introduction to the calculus of variations.

Convex functions, the subject of Chapter 3. *Elements of convex analysis*, have nice continuity and differentiability properties, allowing a good subdifferential calculus which serves as a model for more general subdifferentials studied in the subsequent chapters. This chapter contains the basic results of convex analysis – subdifferentials, the Legendre-Fenchel transform and conjugate functions. Besides the exact rules of the calculus with subdifferentials of convex functions, fuzzy rules (meaning approximate rules) are considered as well, paving the way to similar rules in the non-convex case and showing at the same time that convex analysis is a part of a more general construction.

Containing classical material, as well as some more special topics useful for nonsmooth analysis, each of these first three chapters is of independent interest and can serve as a base for a one semester course on the corresponding topic.

The rest of the book, chapters 4. *Elementary and viscosity subdifferentials*, 5. *Circa-subdifferentials, Clarke subdifferentials*, 6. *Limiting subdifferentials*, and 7. *Graded subdifferentials, Ioffe subdifferentials*, are devoted to nonsmooth analysis. As the author explains, the consideration of various kinds of subdifferentials is motivated by the optimization problem we are studying and by the choice of the space we are working in. Two, somewhat antagonistic, criteria governing the choice of a subdifferential are the accurateness of the information he supplies and the availability of calculus rules. In most cases only fuzzy calculus rules are available, a case already considered for the subdifferentials of convex functions, but this reflects the fact that, very often in real situations, only approximate values of the differentials can be computed, not the exact ones. In author's opinion, the abundance of subdifferentials is not a sign of disorder, but rather reflects the richness of the domain, a unity in diversity. He succeeds to treat in a unitary way various kinds of subdifferentials encountered in nonsmooth analysis, having as primary models convex analysis and classical differential calculus and putting in evidence connections between directional derivatives and tangent cones, on one side, and subdifferentials and normal cones, on the other side. In some cases there is not a complete duality between these two classes of objects, only "one-way routes" being available. These four notions, together with the graphical derivatives and coderivatives for multimaps, are considered by the author as "the six pillars of nonsmooth analysis".

Written by a reputed specialist in the domain, with substantial contributions to convex and nonsmooth analysis (as a significant sample we mention the Michel-Penot subdifferential), and based on a large bibliography (1003 titles from which 173 belong to the author himself, alone or in cooperation), the book presents in a unitary and systematic way a lot of results and tools used in modern optimization theory, some of them still under construction. Each subsection is followed by a set of exercises illustrating the main text by examples, or completing it, some of them with hints and for the more demanding ones a reference to a paper (or book) being given. (Some of these are rather cryptic, as, for instance Exercise

By collecting together a lot of results in nonsmooth analysis and presenting them in a coherent and accessible way, the author rendered a great service to the mathematical community. The book can be considered as an incentive for newcomers to enter this area of research, which, by the variety of tools and methods, may look discouraging at the first sight. The specialists will find also a lot of systematized information, and, as we have already told, the first three chapters can be used for independent graduate courses.

S. Cobzaş

Jean-Baptiste Hiriart-Urruty, Bases, outils et principes pour lanalyse variationnelle, Mathématiques et Applications, Springer-Verlag, Berlin - Heidelberg, 2013, ISSN 1154-483X, ISBN 978-3-642-30734-8, ISBN 978-3-642-30735-5 (eBook), DOI 10.1007/978-3-642-30735-5.

As the author explains in Preface, the book contains topics which can be taught and assimilated by the students attending the course Master 2 Recherche in the first semester (25-30 teaching ours). For this reason he made a rigorous selection, keeping only results which resisted the time and are essential for the area. The first chapter, *Prolégomènes,* contains an introduction to existence results in constrained optimization, emphasizing the roles played by topologies (norm and weak on a normed space E, and norm and weak* on its dual E^*), and by convexity. A lot of supplementary results are contained in the exercises at the end of this chapter.

The second chapter, *Conditions nécessaires d'optimalité approchée*, is concerned mainly with the variational principles of Ekeland and Borwein-Preiss (this one in Hilbert space framework). As application, a detailed study of best approximation in a Hilbert space H by elements of a nonempty closed S is done – continuity and differentiability properties of the distance function d_S and of the metric projection p_S, and dense existence results. In Annexe one discusses Fréchet, Gâteaux and Hadamard differentiability. The results of this chapter are completed and developed in the third chapter *Autour de la projection sur un convexe fermé: la decomposition de Moreau.* The term "opératoire" (operative) in the heading of the fourth chapter, *Analyse convexe opératoire*, means that the presentation is restricted to definitions, essential techniques and tools of convex analysis, destined to be used in situations where the convexity is not available.

The last chapter, Ch. 6, *Sous-différentiel généralisés de fonctions non différentiables*, is concerned with Clarke's directional generalized derivatives and subdifferentials for locally Lipschitz functions defined on an open subset of a Banach space – definitions, properties, calculus rules and applications to necessary conditions in optimization problems. Connections with tangent (contigent) cones and Clarke's normal cone are established, opening the way to nonsmooth geometry. Other types of subdifferentials – Clarke's subdifferential for arbitrary lower semi-continuous functions (not necessarily locally Lipschitz), Fréchet subdifferentials, proximal subdifferentials, viscosity subdifferentials, and back and forth rules with the corresponding tangent and normal cones – are briefly discussed at the end of this chapter.

All the notions are carefully motivated and the results are discussed and illustrated by concrete examples. Each section ends with a set of good exercises completing the main text. The bibliographic references are given at the end of each chapter.

Written in an informal and colloquial style, with witty remarks and quotations of mathematical, but also of general nature, the book is a good introduction to nonsmooth analysis. It can be used as a "très bon appéritif" to more advanced books in this domain as, for instance, the two volume book by J.-B. Hiriart-Urruty and C. Lemarechal, *Convex Analysis and Minimization Algorithms* I and II (Grundlehren des mathematischen Wissenschaften Vol. 305 and 306, Springer 1993, reprinted in 1996, or in abridged form *Fundamentals of convex analysis*, Grundlehren Text Editions

Springer-Verlag 2001, W. Schirotzek, *Nonsmooth Analysis*, Universitext, Springer 2007, and the recent one by J.-P. Penot, *Calculus Without Derivatives*, GTM, Vol. 266, Springer 2013. Or, as the author nicely says at the end of his exposition, "le lecteur-étudiant pourra se faire les dents sur des problèmes variationnels ou d'optimisation non-résolus".

S. Cobzaş

Luboš Pick, Alois Kufner, Oldřich John and Svatopluk Fučik, Function Spaces, 2nd Revised and Extended Edition, Series in Nonlinear Analysis and Applications, Vol. 14, xv+479 pp, Walter de Gruyter, Berlin - New York, 2013, ISBN: 978-3-11-025041-1, e-ISBN: 978-3-11-025042-8, ISSN: 0941-813X.

This is the second edition of the successful book by A. Kufner, O. John and S. Fučik, *Function Spaces*, Noordhoff, Leyden, and Academia, Praha, 1977. This new edition is dedicated to Professor Svatopluk Fučik who passed away not long after the first edition appeared. Over the 35 years passed since then a lot of new results appeared, several books were published, so that three of the authors (L.P., A.K. and O.J.) with the strong support of de Gruyter Publishing House, considered appropriate to write a new revised and updated edition of the book. Because this new edition is based on the 1997 version of the book, Svatopluk Fučik was included between the authors. Also, as the collected material was too long for a single volume, they decided to split in into two volumes. The first volume is dedicated to the study of function spaces, based on intrinsic properties of functions such as size, continuity, smoothness, various forms of control over the mean oscillation, and so on. The second volume will be concerned with function spaces of Sobolev type, in which the key notion is that of weak derivative of functions of several variables.

In the first chapter of the book, *Preliminaries,* for easy reference, the authors collect notions and results (without proofs) from functional analysis and measure theory used throughout the book.

The function spaces treated in the first volume are well illustrated by the headings of the chapters: Ch. 2, *Spaces of smooth functions* (completeness and compactness, separability, extension of functions, Hölder and Lipschitz spaces), Ch. 3, *Lebesgue spaces* (mollifiers, density results, separability, dual spaces, reflexivity, dual spaces, weighted Hardy inequalities, the space L^∞, weak convergence, compactness, Schauder bases), Ch. 4, *Orlicz spaces* (Young functions, Jensen inequality, the condition Δ_2, Hölder inequality, completeness and compactness, separability, isomorphisms, Schauder bases), Ch. 5, *Morrey and Campanato spaces* (these are subspaces of the Lebesgue spaces defined through a mean oscillation property). The spaces studied in Ch. 6, *Banach function spaces*, are Banach spaces of measurable functions defined through a Banach function norm. This abstract scheme covers many examples of scales of function spaces as Lebesgue, Orlicz, and Morrey spaces, as well as rearrangement invariant function spaces (studied in Chapter 7) and Lorentz spaces (studied in Chapters 8, *Lorentz spaces*, and 10, *Classical Lorentz spaces*). Ch. 9, *Generalized Lorentz-Zygmund spaces*, is concerned with a class of functions defined with the help of logarithmic functions raised to different powers near 0 and near infinity (such functions are, in a sense, "broken" in 1, a reason for which they are called also broken logarithmic functions). The last chapter of this volume, Ch. 11, *Variable-exponent Lebesgue spaces*, is concerned with a class of spaces that turned to be of great importance in the study of mathematical models of electrorheological fluids, as it was shown by M. Ružička, LNM, vol. 1748, Springer 2000.

The book contains a lot of results about various classes of function spaces, of great importance in various areas of mathematics, especially for partial differential equations, presented in a clear manner in the elegant typographical layout of de Gruyter Publishres. Undoubtedly, that, together with the second volume, this welcome new edition will become a standard reference in the domain.

Radu Precup