Stud. Univ. Babeş-Bolyai Math. 57(2012), No. 2, 309–313

Book reviews

Daniel Liberzon, Calculus of Variations and Optimal Control. A Concise Introduction, Princeton University Press, Princeton and Oxford, 2012, xv + 235 pp, ISBN: 978-0-691-15187-8.

It is tautology to say that there are many books on calculus of variations and optimal control. Besides this D. Liberzon decided to write one more and the result is in front of us. The author explicitly motivates his decision on writing a new book in its Preface. He wanted a book satisfying all the following features:

- appropriate presentation level – the author wanted a friendly introductory text accessible to graduate students;
- logical and notational consistency among topics – the author wanted a unification of notations and presentation between calculus of variations, optimal control, and the Hamilton-Jacobi-Bellman theory;
- proof of the maximum principle – the author emphasizes that a complete proof is rather long, but is indispensable in a consistent book;
- historical perspective – the topic allows some very instructive views on the development of the calculus of variations, optimal control, and dynamic programming, based on the contributions of many experts and schools of research. At the same time at each step there are fascinating and challenging examples in technological and in economic fields;
- manageable size – the present book is designed for a one semester lecture and therefore its length is rather limited.

We have to mention clearly and undoubtedly that the author succeeded to achieve his goals.

Chapter 1, *Introduction*, presents some results related to finite-dimensional and infinite-dimensional optimization.

Chapter 2, *Calculus of variations*, introduces some examples, then basic calculus of variations problem, first-order conditions for weak extrema, Hamiltonian formalism and mechanics, variational problems with constraints, and second-order conditions.

Chapter 3, *From calculus of variations to optimal control*, deals with necessary conditions for strong extrema, calculus of variations versus optimal control, optimal control problem formulation and assumptions, and variational approach to the fixed-time, free-endpoint problem.

Chapter 4, *The maximum principle*, introduces statement of the maximum principle, proof of the maximum principle (when the Lagrangian does not depend on time), discussion of the maximum principle, time-optimal control problems, and existence of optimal controls.

Chapter 5, *The Hamilton-Jacobi-Bellman theory*, deals with dynamic programming and the HJB equation, HJB equation versus the maximum principle, and viscosity solutions of the HJB equations.

Chapter 6, *The linear quadratic regulator*, contains the topics: finite-horizon LQR problem and infinite-horizon LQR problem.

Chapter 7, *Advanced topics*, is dedicated to maximum principle on manifolds, HJB equation, canonical equations, and characteristics, Riccati equations and inequalities in robust control, and maximum principle for hybrid control systems.

Each chapter ends with a rich and useful section of notes and references. The exercises are merely problems or even theorems. The author of the book presents a large list of references and a detailed index of notions, names, and symbols. The graphical presentation of the book is pleasant.

As final remarks we have to emphasize that this book is well written, it fully deserves all its goals mentioned at the beginning of the review, and is a pleasure to read it.

Marian Mureşan

Advanced Courses of Mathematical Analysis, IV, *Proceedings of the Fourth International School – In the Memory of Professor Antonio Aizpuru Tomás*, F. Javier-Pérez-Fernández and Fernando Rambla-Barreno, (Editors), World Scientific Publishers, London - Singapore 2012, xii+247 pp, ISBN:13-978-981-4335-80-5 and 10-981-4335-80-0.

This is the fourth International Course of Mathematical Analysis in Andalusia held in Jerez (Cádiz), 8–12 September 2009. The first one took place in Cádiz (2002), the second in Granada and the third in Rábida (Huelva) (2007), all their Proceedings being published with World Scientific. This course was initially planned to be held in Alméria, but after the sad and premature death of Professor Antonio Aizpuru at the age of 53, one of the pioneers of these events, the organizers decided to change the place to Cádiz and to dedicate it to the memory of Professor Aizpuru. A biographical sketch presented at the opening ceremony is included in the present volume: F. Javier Pérez-Fernández, *In memoriam: Professor Antonio Aizpuru.* The lectures consisted of seminars, called mini-courses, taking three hours along three days, and plenary talks of one hour each. Presented by renown experts in their fields, the lectures survey some specific domains, contain the state-of-the-art and emphasize open problems deserving further attention.

The first part of the book, Part A, contains the written versions of three mini-courses, Pietro Aiena, *Weyl theorems for bounded linear operators on Banach spaces*, Joe Diestel and Angela Spalsbury, *Finitely additive measures in action*, and Thomas Schlumprecht, *Sampling and recovery of bandlimited functions and applications to signal processing.*

Part B contains some plenary lectures, two of them, F. J. García-Pacheco, *Some results on the local theory of normed spaces since 2002*, and J. B. Seoane-Sepúlveda, *Summability and lineability in the work of Antonio Aizpuru Tomás*, are concerned with areas related to the work Professor Aizpuru, surveying some of his essential contributions.

Other lectures (there are 6 besides the two mentioned above) deal with topics as isometric shifts between spaces of continuous (J. Araujo), uniform algebras of holomorphic functions on the unit ball of some complex Banach spaces (R. M. Aron and P. Galindo), linear preservers on Banach algebras and Kaplansky problem (M. Mbekhta), bounded approximation property and Banach operator ideals (E. Oja), generalizations of Banach-Stone theorem to linear and bilinear mappings on spaces of continuous or Lipschitz functions (F. Rambla-Barreno), and Hardy-minus-Identity operator on some function spaces (J. Soria).

As for the previous ones, the aim of the present course was to bring together prominent specialists in real, complex and functional analysis to expose new results and to pose open questions and techniques which could be effective in their solution. By collecting results in various domains of Complex Analysis, Functional Analysis and Measure Theory, in the focus of current research but scattered in the literature, the present volume is a valuable reference for researchers and graduate students and a valuable source inspiration for future research.

S. Cobzaş

Douglas S. Kurtz and Charles W. Swartz Theories of Integration - The integrals of Riemann, Lebesgue, Henstock-Kurzweil and Mc Shane, World Scientific, London - Singapore - Beijing, 2012, xv + 294 pages, ISBN: 13 978-981-4368-99-5 and 10 981-4368-99-7.

The book contains a clear and thorough presentation of four types of integrals – Riemann, Lebesgue, Henstock-Kurzweil and McShane – first on intervals in $\mathbb{R}$ and then on subsets of $\mathbb{R}^n$. The integrals of Denjoy and Perron are only briefly mentioned. The main idea of the authors is to show how successive generalizations of the notion of integral fix some deficiencies of the previous ones – the advantage of Lebesgue integral over the Riemann integral consists in countable additivity and better convergence theorems, while the Henstock-Kurzweil offers a very general form of the fundamental theorem of calculus.

The book starts with a short presentation of the notion of areas of plane figures, including the exhaustion method of Archimedes for the calculation of the area of a disc.

Riemann's integration theory is developed in the second chapter and includes Lebesgue's criterion of integrability, the change of variable formula, and a treatment of improper integrals.

The third chapter, *Convergence theorems and the Lebesgue integral*, starts with Lebesgue's axioms for a general integral and their consequences. Lebesgue measurable sets in $\mathbb{R}^n$ are introduced via outer measures (Carathéodori's definition) and then one proves some fundamental results for measurable functions – Egorov and Luzin theorems, the approximation by step functions. The chapter ends with the presentation of some fundamental results on the Lebesgue integral, including Mikusinski's remarkable characterization of Lebesgue integrability through series of step functions, which is then used in the proofs of the theorems of Fubini and Tonelli.

The definition and the basic properties of Henstock-Kurzweil integral are presented in the fourth chapter, *Fundamental theorem of calculus and the Henstock-Kurzweil integral*. Here it is also proved the a.e. differentiability of monotone and absolutely continuous functions and variants of the fundamental theorem of calculus (the validity of $\int_a^b f' = f(b) - f(a)$) for various types of integrals. One shows also that a function f is Lebesgue integrable iff $|f|$ is Henstock-Kurzweil integrable.

The last chapter of the book, Ch. 5, *Absolute integrability and the McShane integral*, is concerned with the properties of the McShane integral which is, in fact, equivalent to the Lebesgue integral, allowing a Riemann type treatment of it. A key notion used in Chapters 4 and 5 in the proofs of convergence results is that of uniform integrability.

The book is clearly written with cleaver proofs of some fundamental results in real analysis. All the notions and results are accompanied by comments and examples and each chapter ends with a set of exercises completing the main text (some of them ask to fill in details of some proofs).

This is the second edition of a successful book. With respect to the first one, beside corrections and new proofs to some results, some new material has been added as, for instance, the convolution product and approximate identities with applications to Weierstrass type approximation results. The chapters are relatively independent, so that each one can be used for an introduction to a specific topic. The book can be recommended as a base text for introductory courses in real analysis or for self-study.

Valer Anisiu

Martin Moskowitz and Fortios Paliogiannis Functions of several variables, World Scientific, London - Singapore - Beijing, 2011, xv + 716 pages, ISBN: 13 978-981-4299-27-5 (pbk) and 10 981-4299-27-8 (pbk).

This is a course on Mathematical Analysis dedicated to functions of several variables. The Calculus of real-valued functions of one real variable is assumed to be known by the potential reader, so that the authors pass directly in the first two chapters, 1. *Basic features of Euclidean space* $\mathbb{R}^n$, and 2. *Functions on Euclidean spaces*, to several variables by an introduction to the Euclidean space $\mathbb{R}^n$, its topology and continuous functions on subsets of $\mathbb{R}^n$. The differentiability theory is studied in the third chapter, *Differential calculus in several variables*, which includes mean value theorems, Taylor's formula, extrema and conditioned extrema, studied through the implicit function theorem. Some pleasant surprises here are a proof of Sylvester's criterium on the positivity of a quadratic form and the inclusion of Morse lemma on critical values of differentiable vector functions. Integral calculus, meaning Riemann and Darboux integrals over Jordan measurable subsets of $\mathbb{R}^n$ and including Fubini's theorem, is treated in Ch. 4, *Integral calculus in several variables.* As more advanced topics we mention Sard's theorem on singular values of differentiable mappings and Urysohn's theorem on the partition of unity. The change of variables formula, one of the most challenging theorems in Calculus is treated in the fifth chapter, containing also a fairly complete treatment of improper multiple integrals with applications to some remarkable improper integrals as the classical $\int_{-\infty}^{+\infty} \exp(-x^2)dx$ and Euler's Gamma and Beta functions. This chapter includes also a brief introduction to the Fourier transform and to the Schwartz space.

For the sake of simplicity and accessibility, the presentation in Chapter 6, *Line and surface integrals*, is restricted to $\mathbb{R}^3$. This allows to the authors to treat in details, and at the same time in an intuitive form, some deep results as Green's and Stokes' theorems, and Poincaré's lemma on closed forms. A brief discussion on differential forms in $\mathbb{R}^3$ and exterior differentiation as well as Milnor's analytic proof of the Brouwer's fixed point theorem are also included.

In the last two chapters, 7, *Elements of ordinary and partial differential equations*, and 8, *An introduction to the calculus of variations*, the so far developed machinery is applied to these two important areas of mathematics.

To make the book more attractive the authors have include a lot of examples and applications from physics, mechanics and economy.

Each chapter ends with a set of solved problems and exercises, some of these being accompanied with solutions, inviting the reader to check his/her comprehension of the theoretical matter.

Written at an intermediate level, between undergraduate and upper undergraduate, the present book could be a good companion for both undergraduate students desiring to know more on Mathematical Analysis, as well as for upper level undergraduate (or even graduate) who want to clarify some notions from their undergraduate course.

Tiberiu Trif