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A note on strong differential superordinations using a generalized Sălăgean operator and Ruscheweyh operator

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Abstract. In the present paper we establish several strong differential superordinations regardind the new operator DR_λ^m defined by convolution product of the extended Sălăgean operator and Ruscheweyh derivative, $DR_\lambda^m : \mathcal{A}_{n\zeta}^* \to \mathcal{A}_{n\zeta}^*$, $DR_\lambda^m f(z,\zeta) = (D_\lambda^m * R^m) f(z,\zeta)$, $z \in U$, $\zeta \in \overline{U}$, where $R^m f(z,\zeta)$ denote the extended Ruscheweyh derivative, $D_\lambda^m f(z,\zeta)$ is the extended generalized Sălăgean operator and $\mathcal{A}_{n\zeta}^* = \{f \in \mathcal{H}(U \times \overline{U}),\ f(z,\zeta) = z + a_{n+1}(\zeta) z^{n+1} + \ldots,\ z \in U,\ \zeta \in \overline{U}\}$ is the class of normalized analytic functions.

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1. Introduction

Denote by U the unit disc of the complex plane $U = \{z \in \mathbb{C} : \ |z| < 1\}$, $\overline{U} = \{z \in \mathbb{C} : \ |z| \le 1\}$ the closed unit disc of the complex plane and $\mathcal{H}(U \times \overline{U})$ the class of analytic functions in $U \times \overline{U}$.

Let

$$\mathcal{A}_{n\zeta}^* = \{f \in \mathcal{H}(U \times \overline{U}),\ f(z,\zeta) = z + a_{n+1}(\zeta) z^{n+1} + \ldots,\ z \in U,\ \zeta \in \overline{U}\},$$

where $a_k(\zeta)$ are holomorphic functions in $\overline{U}$ for $k \ge 2$, and

$\mathcal{H}^*[a,n,\zeta] = \{f \in \mathcal{H}(U \times \overline{U}),\ f(z,\zeta) = a + a_n(\zeta) z^n + a_{n+1}(\zeta) z^{n+1} + \ldots,\ z \in U,\ \zeta \in \overline{U}\}$,

for $a \in \mathbb{C}$, $n \in \mathbb{N}$, $a_k(\zeta)$ are holomorphic functions in $\overline{U}$ for $k \ge n$.

Denote by

$$K_{n\zeta} = \left\{ f \in \mathcal{H}(U \times \overline{U}) : \mathrm{Re} \frac{z f_z''(z,\zeta)}{f_z'(z,\zeta)} + 1 > 0 \right\}$$

the class of convex function in $U \times \overline{U}$.

We also extend the differential operators presented above to the new class of analytic functions $\mathcal{A}^*_{n\zeta}$ introduced in [10].

Definition 1.1. [5] *For* $f \in \mathcal{A}^*_{n\zeta}$, $\lambda \geq 0$ *and* $n, m \in \mathbb{N}$, *the operator* D^m_λ *is defined by* $D^m_\lambda : \mathcal{A}^*_{n\zeta} \to \mathcal{A}^*_{n\zeta}$,

$$\begin{aligned} D^0_\lambda f(z,\zeta) &= f(z,\zeta) \\ D^1_\lambda f(z,\zeta) &= (1-\lambda) f(z,\zeta) + \lambda z f'_z(z,\zeta) = D_\lambda f(z,\zeta), ..., \\ D^{m+1}_\lambda f(z,\zeta) &= (1-\lambda) D^m_\lambda f(z,\zeta) + \lambda z \left(D^m_\lambda f(z,\zeta)\right)'_z \\ &= D_\lambda \left(D^m_\lambda f(z,\zeta)\right), \quad z \in U, \zeta \in \overline{U}. \end{aligned}$$

Remark 1.2. [5] If $f \in \mathcal{A}^*_{n\zeta}$ and $f(z) = z + \sum_{j=n+1}^{\infty} a_j(\zeta) z^j$, then

$$D^m_\lambda f(z,\zeta) = z + \sum_{j=n+1}^{\infty} \left[1 + (j-1)\lambda\right]^m a_j(\zeta) z^j, \text{ for } z \in U, \ \zeta \in \overline{U}.$$

Definition 1.3. [4] *For* $f \in \mathcal{A}^*_{n\zeta}$, $n, m \in \mathbb{N}$, *the operator* R^m *is defined by*

$$\begin{aligned} R^m &: \mathcal{A}^*_{n\zeta} \to \mathcal{A}^*_{n\zeta}, \\ R^0 f(z,\zeta) &= f(z,\zeta), \\ R^1 f(z,\zeta) &= z f'_z(z,\zeta), ..., \\ (m+1) R^{m+1} f(z,\zeta) &= z \left(R^m f(z,\zeta)\right)'_z + m R^m f(z,\zeta), \quad z \in U, \ \zeta \in \overline{U}. \end{aligned}$$

Remark 1.4. [4] If $f \in \mathcal{A}^*_{n\zeta}$, $f(z,\zeta) = z + \sum_{j=n+1}^{\infty} a_j(\zeta) z^j$, then

$$R^m f(z,\zeta) = z + \sum_{j=n+1}^{\infty} C^m_{m+j-1} a_j(\zeta) z^j, \ z \in U, \ \zeta \in \overline{U}.$$

As a dual notion of strong differential subordination G.I. Oros has introduced and developed the notion of strong differential superordinations in [9].

Definition 1.5. [9] *Let* $f(z,\zeta)$, $H(z,\zeta)$ *analytic in* $U \times \overline{U}$. *The function* $f(z,\zeta)$ *is said to be strongly superordinate to* $H(z,\zeta)$ *if there exists a function* w *analytic in* U, *with* $w(0) = 0$ *and* $|w(z)| < 1$, *such that* $H(z,\zeta) = f(w(z),\zeta)$, *for all* $\zeta \in \overline{U}$. *In such a case we write* $H(z,\zeta) \prec\prec f(z,\zeta)$, $z \in U$, $\zeta \in \overline{U}$.

Remark 1.6. [9] (i) Since $f(z,\zeta)$ is analytic in $U \times \overline{U}$, for all $\zeta \in \overline{U}$, and univalent in U, for all $\zeta \in \overline{U}$, Definition 1.5 is equivalent to $H(0,\zeta) = f(0,\zeta)$, for all $\zeta \in \overline{U}$, and $H\left(U \times \overline{U}\right) \subset f\left(U \times \overline{U}\right)$.

(ii) If $H(z,\zeta) \equiv H(z)$ and $f(z,\zeta) \equiv f(z)$, the strong superordination becomes the usual notion of superordination.

Definition 1.7. [9] *We denote by* Q^* *the set of functions that are analytic and injective on* $\overline{U} \times \overline{U} \backslash E(f,\zeta)$, *where* $E(f,\zeta) = \{y \in \partial U : \lim\limits_{z \to y} f(z,\zeta) = \infty\}$, *and are such that* $f_z'(y,\zeta) \neq 0$ *for* $y \in \partial U \times \overline{U} \backslash E(f,\zeta)$. *The subclass of* Q^* *for which* $f(0,\zeta) = a$ *is denoted by* $Q^*(a)$.

We have need the following lemmas to study the strong differential superordinations.

Lemma 1.8. [9] *Let* $h(z,\zeta)$ *be a convex function with* $h(0,\zeta) = a$ *and let* $\gamma \in \mathbb{C}^*$ *be a complex number with* $\mathrm{Re}\gamma \geq 0$. *If* $p \in \mathcal{H}^*[a,n,\zeta] \cap Q^*$, $p(z,\zeta) + \frac{1}{\gamma} z p_z'(z,\zeta)$ *is univalent in* $U \times \overline{U}$ *and*

$$h(z,\zeta) \prec\prec p(z,\zeta) + \frac{1}{\gamma} z p_z'(z,\zeta), \qquad z \in U,\ \zeta \in \overline{U},$$

then

$$q(z,\zeta) \prec\prec p(z,\zeta), \qquad z \in U,\ \zeta \in \overline{U},$$

where $q(z,\zeta) = \frac{\gamma}{n z^{\frac{\gamma}{n}}} \int_0^z h(t,\zeta)\, t^{\frac{\gamma}{n}-1} dt$, $z \in U$, $\zeta \in \overline{U}$. *The function* q *is convex and is the best subordinant.*

Lemma 1.9. [9] *Let* $q(z,\zeta)$ *be a convex function in* $U \times \overline{U}$ *and let*

$$h(z,\zeta) = q(z,\zeta) + \frac{1}{\gamma} z q_z'(z,\zeta),\ z \in U,\ \zeta \in \overline{U},$$

where $\mathrm{Re}\gamma \geq 0$.

If $p \in \mathcal{H}^*[a,n,\zeta] \cap Q^*$, $p(z,\zeta) + \frac{1}{\gamma} z p_z'(z,\zeta)$ *is univalent in* $U \times \overline{U}$ *and*

$$q(z,\zeta) + \frac{1}{\gamma} z q_z'(z,\zeta) \prec\prec p(z,\zeta) + \frac{1}{\gamma} z p_z'(z,\zeta), \qquad z \in U,\ \zeta \in \overline{U},$$

then

$$q(z,\zeta) \prec\prec p(z,\zeta), \qquad z \in U,\ \zeta \in \overline{U},$$

where $q(z,\zeta) = \frac{\gamma}{n z^{\frac{\gamma}{n}}} \int_0^z h(t,\zeta)\, t^{\frac{\gamma}{n}-1} dt$, $z \in U$, $\zeta \in \overline{U}$. *The function* q *is the best subordinant.*

2. Main results

Definition 2.1. [2] *Let* $\lambda \geq 0$ *and* $m \in \mathbb{N} \cup \{0\}$. *Denote by* DR_λ^m *the operator given by the Hadamard product (the convolution product) of the extended generalized Sălăgean operator* D_λ^m *and the extended Ruscheweyh operator* R^m, $DR_\lambda^m : \mathcal{A}_{n\zeta}^* \to \mathcal{A}_{n\zeta}^*$,

$$DR_\lambda^m f(z,\zeta) = (D_\lambda^m * R^m) f(z,\zeta).$$

Remark 2.2. [2] If $f \in \mathcal{A}_{n\zeta}^*$, $f(z,\zeta) = z + \sum\limits_{j=n+1}^{\infty} a_j(\zeta) z^j$, then

$$DR_\lambda^m f(z,\zeta) = z + \sum_{j=n+1}^{\infty} C_{m+j-1}^m [1 + (j-1)\lambda]^m a_j^2(\zeta) z^j,\ z \in U,\ \zeta \in \overline{U}.$$

Remark 2.3. For $\lambda = 1$ we obtain the Hadamard product SR^m ([1], [3], [7], [8]) of the extended Sălăgean operator S^m and the extended Ruscheweyh operator R^m.

Theorem 2.4. *Let $h(z,\zeta)$ be a convex function in $U \times \overline{U}$ with $h(0,\zeta) = 1$. Let $m \in \mathbb{N}$, $\lambda \geq 0$, $f(z,\zeta) \in \mathcal{A}^*_{n\zeta}$, $F(z,\zeta) = I_c(f)(z,\zeta) = \frac{c+2}{z^{c+1}} \int_0^z t^c f(t,\zeta)\, dt$, $z \in U$, $\zeta \in \overline{U}$, $\mathrm{Re} c > -2$, and suppose that $(DR^m_\lambda f(z,\zeta))'_z$ is univalent in $U \times \overline{U}$, $(DR^m_\lambda F(z,\zeta))'_z \in \mathcal{H}^*[1,n,\zeta] \cap Q^*$ and*

$$h(z,\zeta) \prec\prec (DR^m_\lambda f(z,\zeta))'_z, \quad z \in U, \ \zeta \in \overline{U}, \tag{2.1}$$

then

$$q(z,\zeta) \prec\prec (DR^m_\lambda F(z,\zeta))'_z, \quad z \in U, \ \zeta \in \overline{U},$$

where $q(z,\zeta) = \frac{c+2}{nz^{\frac{c+2}{n}}} \int_0^z h(t,\zeta) t^{\frac{c+2}{n}-1} dt$. The function q is convex and it is the best subordinant.

Proof. We have

$$z^{c+1} F(z,\zeta) = (c+2) \int_0^z t^c f(t,\zeta)\, dt$$

and differentiating it, with respect to z, we obtain

$$(c+1) F(z,\zeta) + zF'_z(z,\zeta) = (c+2) f(z,\zeta)$$

and

$$(c+1) DR^m_\lambda F(z,\zeta) + z (DR^m_\lambda F(z,\zeta))'_z = (c+2) DR^m_\lambda f(z,\zeta), \quad z \in U, \ \zeta \in \overline{U}.$$

Differentiating the last relation with respect to z we have

$$(DR^m_\lambda F(z,\zeta))'_z + \frac{1}{c+2} z (DR^m_\lambda F(z,\zeta))''_{z^2} = (DR^m_\lambda f(z,\zeta))'_z, \quad z \in U, \ \zeta \in \overline{U}. \tag{2.2}$$

Using (2.2), the strong differential superordination (2.1) becomes

$$h(z,\zeta) \prec\prec (DR^m_\lambda F(z,\zeta))'_z + \frac{1}{c+2} z (DR^m_\lambda F(z,\zeta))''_{z^2}. \tag{2.3}$$

Denote

$$p(z,\zeta) = (DR^m_\lambda F(z,\zeta))'_z, \quad z \in U, \ \zeta \in \overline{U}. \tag{2.4}$$

Replacing (2.4) in (2.3) we obtain

$$h(z,\zeta) \prec\prec p(z,\zeta) + \frac{1}{c+2} z p'_z(z,\zeta), \quad z \in U, \ \zeta \in \overline{U}.$$

Using Lemma 1.8 for $\gamma = c+2$, we have

$$q(z,\zeta) \prec\prec p(z,\zeta), \ z \in U, \ \zeta \in \overline{U}, \ \text{i.e. } q(z,\zeta) \prec\prec (DR^m_\lambda F(z,\zeta))'_z, \ z \in U, \ \zeta \in \overline{U},$$

where $q(z,\zeta) = \frac{c+2}{nz^{\frac{c+2}{n}}} \int_0^z h(t,\zeta) t^{\frac{c+2}{n}-1} dt$. The function q is convex and it is the best subordinant. □

Corollary 2.5. *Let* $h(z,\zeta)=\frac{1+(2\beta-\zeta)z}{1+z}$, *where* $\beta\in[0,1)$. *Let* $m\in\mathbb{N}$, $\lambda\geq 0$, $f(z,\zeta)\in\mathcal{A}^*_{n\zeta}$, $F(z,\zeta)=I_c(f)(z,\zeta)=\frac{c+2}{z^{c+1}}\int_0^z t^c f(t,\zeta)\,dt$, $z\in U$, $\zeta\in\overline{U}$, $\operatorname{Re}c>-2$, *and suppose that* $(DR^m_\lambda f(z,\zeta))'_z$ *is univalent in* $U\times\overline{U}$, $(DR^m_\lambda F(z,\zeta))'_z\in\mathcal{H}^*[1,n,\zeta]\cap Q^*$ *and*

$$h(z,\zeta)\prec\prec(DR^m_\lambda f(z,\zeta))'_z,\quad z\in U,\ \zeta\in\overline{U},\tag{2.5}$$

then

$$q(z,\zeta)\prec\prec(DR^m_\lambda F(z,\zeta))'_z,\qquad z\in U,\ \zeta\in\overline{U},$$

where q *is given by* $q(z,\zeta)=2\beta-\zeta+\frac{(c+2)(1+\zeta-2\beta)}{nz^{\frac{c+2}{n}}}\int_0^z\frac{t^{\frac{c+2}{n}-1}}{t+1}dt$, $z\in U$, $\zeta\in\overline{U}$. *The function* q *is convex and it is the best subordinant.*

Proof. Following the same steps as in the proof of Theorem 2.4 and considering $p(z,\zeta)=(DR^m_\lambda F(z,\zeta))'_z$, the strong differential superordination (2.5) becomes

$$h(z,\zeta)=\frac{1+(2\beta-\zeta)z}{1+z}\prec\prec p(z,\zeta)+\frac{1}{c+2}zp'_z(z,\zeta),\qquad z\in U,\ \zeta\in\overline{U}.$$

By using Lemma 1.8 for $\gamma=c+2$, we have $q(z,\zeta)\prec\prec p(z,\zeta)$, i.e.

$$q(z,\zeta)=\frac{c+2}{nz^{\frac{c+2}{n}}}\int_0^z h(t,\zeta)t^{\frac{c+2}{n}-1}dt=\frac{c+2}{nz^{\frac{c+2}{n}}}\int_0^z\frac{1+(2\beta-\zeta)t}{1+t}t^{\frac{c+2}{n}-1}dt$$

$$=2\beta-\zeta+\frac{(c+2)(1+\zeta-2\beta)}{nz^{\frac{c+2}{n}}}\int_0^z\frac{t^{\frac{c+2}{n}-1}}{t+1}dt\prec\prec(DR^m_\lambda F(z,\zeta))'_z,\qquad z\in U,\ \zeta\in\overline{U}.$$

The function q is convex and it is the best subordinant. □

Theorem 2.6. *Let* $q(z,\zeta)$ *be a convex function in* $U\times\overline{U}$ *and let*

$$h(z,\zeta)=q(z,\zeta)+\frac{1}{c+2}zq'_z(z,\zeta),$$

where $z\in U$, $\zeta\in\overline{U}$, $\operatorname{Re}c>-2$.

Let $m\in\mathbb{N}$, $\lambda\geq 0$, $f(z,\zeta)\in\mathcal{A}^*_{n\zeta}$, $F(z,\zeta)=I_c(f)(z,\zeta)=\frac{c+2}{z^{c+1}}\int_0^z t^c f(t,\zeta)\,dt$, $z\in U$, $\zeta\in\overline{U}$, *and suppose that* $(DR^m_\lambda f(z,\zeta))'_z$ *is univalent in* $U\times\overline{U}$, $(DR^m_\lambda F(z,\zeta))'_z\in\mathcal{H}^*[1,n,\zeta]\cap Q^*$ *and*

$$h(z,\zeta)\prec\prec(DR^m_\lambda f(z,\zeta))'_z,\quad z\in U,\ \zeta\in\overline{U},\tag{2.6}$$

then

$$q(z,\zeta)\prec\prec(DR^m_\lambda F(z,\zeta))'_z,\qquad z\in U,\ \zeta\in\overline{U},$$

where $q(z,\zeta)=\frac{c+2}{nz^{\frac{c+2}{n}}}\int_0^z h(t,\zeta)t^{\frac{c+2}{n}-1}dt$. *The function* q *is the best subordinant.*

Proof. Following the same steps as in the proof of Theorem 2.4 and considering $p(z,\zeta)=(DR^m_\lambda F(z,\zeta))'_z$, $z\in U$, $\zeta\in\overline{U}$, the strong differential superordination (2.6) becomes

$$h(z,\zeta)=q(z,\zeta)+\frac{1}{c+2}zq'_z(z,\zeta)\prec\prec p(z,\zeta)+\frac{1}{c+2}zp'_z(z,\zeta),\qquad z\in U,\ \zeta\in\overline{U}.$$

Using Lemma 1.9 for $\gamma = c+2$, we have

$$q(z,\zeta) \prec\prec p(z,\zeta), \ z \in U, \ \zeta \in \overline{U}, \ \text{i.e. } q(z,\zeta) \prec\prec \left(DR_\lambda^m F(z,\zeta)\right)'_z, \ z \in U, \ \zeta \in \overline{U},$$

where $q(z,\zeta) = \frac{c+2}{nz^{\frac{c+2}{n}}} \int_0^z h(t,\zeta) t^{\frac{c+2}{n}-1} dt$. The function q is the best subordinant. □

Theorem 2.7. *Let $h(z,\zeta)$ be a convex function, $h(0,\zeta) = 1$. Let $\lambda \geq 0$, $m, n \in \mathbb{N}$, $f(z,\zeta) \in \mathcal{A}^*_{n\zeta}$ and suppose that $\left(DR_\lambda^m f(z,\zeta)\right)'_z$ is univalent and $\frac{DR_\lambda^m f(z,\zeta)}{z} \in \mathcal{H}^*[1,n,\zeta] \cap Q^*$. If*

$$h(z,\zeta) \prec\prec \left(DR_\lambda^m f(z,\zeta)\right)'_z, \quad z \in U, \ \zeta \in \overline{U}, \tag{2.7}$$

then

$$q(z,\zeta) \prec\prec \frac{DR_\lambda^m f(z,\zeta)}{z}, \quad z \in U, \ \zeta \in \overline{U},$$

where $q(z,\zeta) = \frac{1}{nz^{\frac{1}{n}}} \int_0^z h(t,\zeta) t^{\frac{1}{n}-1} dt$. The function q is convex and it is the best subordinant.

Proof. Consider

$$p(z,\zeta) = \frac{DR_\lambda^m f(z,\zeta)}{z} = \frac{z + \sum_{j=n+1}^{\infty} C^m_{m+j-1} [1+(j-1)\lambda]^m a_j^2(\zeta) z^j}{z}$$
$$= 1 + \sum_{j=n+1}^{\infty} C^m_{m+j-1} [1+(j-1)\lambda]^m a_j^2(\zeta) z^{j-1}.$$

Evidently $p \in \mathcal{H}^*[1,n,\zeta]$.

We have $p(z,\zeta) + zp'_z(z,\zeta) = \left(DR_\lambda^m f(z,\zeta)\right)'_z$, $z \in U$, $\zeta \in \overline{U}$.

Then (2.7) becomes

$$h(z,\zeta) \prec\prec p(z,\zeta) + zp'_z(z,\zeta), \quad z \in U, \ \zeta \in \overline{U}.$$

By using Lemma 1.8 for $\gamma = 1$, we have

$$q(z,\zeta) \prec\prec p(z,\zeta), \quad z \in U, \ \zeta \in \overline{U}, \quad \text{i.e.} \quad q(z,\zeta) \prec\prec \frac{DR_\lambda^m f(z,\zeta)}{z}, \quad z \in U, \ \zeta \in \overline{U},$$

where $q(z,\zeta) = \frac{1}{nz^{\frac{1}{n}}} \int_0^z h(t,\zeta) t^{\frac{1}{n}-1} dt$. The function q is convex and it is the best subordinant. □

Corollary 2.8. *Let $h(z,\zeta) = \frac{1+(2\beta-\zeta)z}{1+z}$ be a convex function in $U \times \overline{U}$, where $0 \leq \beta < 1$. Let $\lambda \geq 0$, $m, n \in \mathbb{N}$, $f(z,\zeta) \in \mathcal{A}^*_{n\zeta}$ and suppose that $\left(DR_\lambda^m f(z,\zeta)\right)'_z$ is univalent and $\frac{DR_\lambda^m f(z,\zeta)}{z} \in \mathcal{H}^*[1,n,\zeta] \cap Q^*$. If*

$$h(z,\zeta) \prec\prec \left(DR_\lambda^m f(z,\zeta)\right)'_z, \quad z \in U, \ \zeta \in \overline{U}, \tag{2.8}$$

then

$$q(z,\zeta) \prec\prec \frac{DR_\lambda^m f(z,\zeta)}{z}, \quad z \in U, \ \zeta \in \overline{U},$$

where q is given by $q(z,\zeta)=2\beta-\zeta+\frac{1+\zeta-2\beta}{nz^{\frac{1}{n}}}\int_0^z\frac{t^{\frac{1}{n}-1}}{1+t}dt$, $z\in U$, $\zeta\in\overline{U}$. *The function q is convex and it is the best subordinant.*

Proof. Following the same steps as in the proof of Theorem 2.7 and considering

$$p(z,\zeta)=\frac{DR_\lambda^m f(z,\zeta)}{z},$$

the strong differential superordination (2.8) becomes

$$h(z,\zeta)=\frac{1+(2\beta-\zeta)z}{1+z}\prec\prec p(z,\zeta)+zp'_z(z,\zeta),\quad z\in U,\ \zeta\in\overline{U}.$$

By using Lemma 1.8 for $\gamma=1$, we have $q(z,\zeta)\prec\prec p(z,\zeta)$, i.e.

$$q(z,\zeta)=\frac{1}{nz^{\frac{1}{n}}}\int_0^z h(t,\zeta)\,t^{\frac{1}{n}-1}dt=\frac{1}{nz^{\frac{1}{n}}}\int_0^z t^{\frac{1}{n}-1}\frac{1+(2\beta-\zeta)\,t}{1+t}dt$$

$$=2\beta-\zeta+\frac{1+\zeta-2\beta}{nz^{\frac{1}{n}}}\int_0^z\frac{t^{\frac{1}{n}-1}}{1+t}dt\prec\prec\frac{DR_\lambda^m f(z,\zeta)}{z},\quad z\in U,\ \zeta\in\overline{U}.$$

The function q is convex and it is the best subordinant. □

Theorem 2.9. *Let $q(z,\zeta)$ be convex in $U\times\overline{U}$ and let h be defined by*

$$h(z,\zeta)=q(z,\zeta)+zq'_z(z,\zeta).$$

If $\lambda\geq 0$, $m,n\in\mathbb{N}$, $f(z,\zeta)\in\mathcal{A}^*_{n\zeta}$, suppose that $\left(DR_\lambda^m f(z,\zeta)\right)'_z$ is univalent, $\frac{DR_\lambda^m f(z,\zeta)}{z}\in\mathcal{H}^*[1,n,\zeta]\cap Q^*$ and satisfies the strong differential superordination

$$h(z,\zeta)=q(z,\zeta)+zq'_z(z,\zeta)\prec\prec\left(DR_\lambda^m f(z,\zeta)\right)'_z,\quad z\in U,\ \zeta\in\overline{U},\tag{2.9}$$

then

$$q(z,\zeta)\prec\prec\frac{DR_\lambda^m f(z,\zeta)}{z},\quad z\in U,\ \zeta\in\overline{U},$$

where $q(z,\zeta)=\frac{1}{nz^{\frac{1}{n}}}\int_0^z h(t,\zeta)t^{\frac{1}{n}-1}dt$. *The function q is the best subordinant.*

Proof. Let

$$p(z,\zeta)=\frac{DR_\lambda^m f(z,\zeta)}{z}=\frac{z+\sum_{j=n+1}^{\infty}C_{m+j-1}^m\left[1+(j-1)\lambda\right]^m a_j^2(\zeta)\,z^j}{z}$$

$$=1+\sum_{j=n+1}^{\infty}C_{m+j-1}^m\left[1+(j-1)\lambda\right]^m a_j^2(\zeta)\,z^{j-1}.$$

Evidently $p\in\mathcal{H}^*[1,n,\zeta]$.

Differentiating, we obtain $p(z,\zeta)+zp'_z(z,\zeta)=\left(DR_\lambda^m f(z,\zeta)\right)'_z$, $z\in U$, $\zeta\in\overline{U}$, and (2.9) becomes

$$q(z,\zeta)+zq'_z(z,\zeta)\prec\prec p(z,\zeta)+zp'_z(z,\zeta),\quad z\in U,\ \zeta\in\overline{U}.$$

Using Lemma 1.9 for $\gamma = 1$, we have

$$q(z,\zeta) \prec\prec p(z,\zeta), \quad z \in U,\ \zeta \in \overline{U}, \text{ i.e.}$$

$$q(z,\zeta) = \frac{1}{nz^{\frac{1}{n}}} \int_0^z h(t,\zeta) t^{\frac{1}{n}-1} dt \prec\prec \frac{DR_\lambda^m f(z,\zeta)}{z}, \quad z \in U, \zeta \in \overline{U},$$

and q is the best subordinant. □

Theorem 2.10. *Let $h(z,\zeta)$ be a convex function, $h(0,\zeta) = 1$. Let $\lambda \geq 0$, $m, n \in \mathbb{N}$, $f(z,\zeta) \in \mathcal{A}^*_{n\zeta}$ and suppose that $\left(\frac{zDR_\lambda^{m+1} f(z,\zeta)}{DR_\lambda^m f(z,\zeta)}\right)'_z$ is univalent and $\frac{DR_\lambda^{m+1} f(z,\zeta)}{DR_\lambda^m f(z,\zeta)} \in \mathcal{H}^*[1,n,\zeta] \cap Q^*$. If*

$$h(z,\zeta) \prec\prec \left(\frac{zDR_\lambda^{m+1} f(z,\zeta)}{DR_\lambda^m f(z,\zeta)}\right)'_z, \quad z \in U,\ \zeta \in \overline{U}, \tag{2.10}$$

then

$$q(z,\zeta) \prec\prec \frac{DR_\lambda^{m+1} f(z,\zeta)}{DR_\lambda^m f(z,\zeta)}, \quad z \in U,\ \zeta \in \overline{U},$$

where $q(z,\zeta) = \frac{1}{nz^{\frac{1}{n}}} \int_0^z h(t,\zeta) t^{\frac{1}{n}-1} dt$. The function q is convex and it is the best subordinant.

Proof. Consider

$$p(z,\zeta) = \frac{DR_\lambda^{m+1} f(z,\zeta)}{DR_\lambda^m f(z,\zeta)} = \frac{z + \sum_{j=n+1}^{\infty} C_{m+j}^{m+1} [1 + (j-1)\lambda]^{m+1} a_j^2(\zeta) z^j}{z + \sum_{j=n+1}^{\infty} C_{m+j-1}^{m} [1 + (j-1)\lambda]^{m} a_j^2(\zeta) z^j}$$

$$= \frac{1 + \sum_{j=n+1}^{\infty} C_{m+j}^{m+1} [1 + (j-1)\lambda]^{m+1} a_j^2(\zeta) z^{j-1}}{1 + \sum_{j=n+1}^{\infty} C_{m+j-1}^{m} [1 + (j-1)\lambda]^{m} a_j^2(\zeta) z^{j-1}}.$$

Evidently $p \in \mathcal{H}^*[1,n,\zeta]$.
We have

$$p'_z(z,\zeta) = \frac{\left(DR_\lambda^{m+1} f(z,\zeta)\right)'_z}{DR_\lambda^m f(z,\zeta)} - p(z,\zeta) \cdot \frac{\left(DR_\lambda^m f(z,\zeta)\right)'_z}{DR_\lambda^m f(z,\zeta)}.$$

Then

$$p(z,\zeta) + zp'_z(z,\zeta) = \left(\frac{zDR_\lambda^{m+1} f(z,\zeta)}{DR_\lambda^m f(z,\zeta)}\right)'_z.$$

Then (2.10) becomes

$$h(z,\zeta) \prec\prec p(z,\zeta) + zp'_z(z,\zeta), \quad z \in U,\ \zeta \in \overline{U}.$$

By using Lemma 1.8 for $\gamma = 1$, we have

$$q(z,\zeta) \prec\prec p(z,\zeta),\ z \in U,\ \zeta \in \overline{U}, \quad \text{i.e.} \quad q(z,\zeta) \prec \frac{DR_\lambda^{m+1} f(z,\zeta)}{DR_\lambda^m f(z,\zeta)},\ z \in U,\ \zeta \in \overline{U},$$

where $q(z,\zeta) = \frac{1}{nz^{\frac{1}{n}}} \int_0^z h(t,\zeta)\, t^{\frac{1}{n}-1} dt$. The function q is convex and it is the best subordinant. □

Corollary 2.11. *Let $h(z,\zeta) = \frac{1+(2\beta-\zeta)z}{1+z}$ be a convex function in $U \times \overline{U}$, where $0 \le \beta < 1$. Let $\lambda \ge 0$, $m,n \in \mathbb{N}$, $f(z,\zeta) \in \mathcal{A}^*_{n\zeta}$ and suppose that $\left(\frac{zDR_\lambda^{m+1} f(z,\zeta)}{DR_\lambda^m f(z,\zeta)}\right)'_z$ is univalent, $\frac{DR_\lambda^{m+1} f(z,\zeta)}{DR_\lambda^m f(z,\zeta)} \in \mathcal{H}^*[1,n,\zeta] \cap Q^*$. If*

$$h(z,\zeta) \prec\prec \left(\frac{zDR_\lambda^{m+1} f(z,\zeta)}{DR_\lambda^m f(z,\zeta)}\right)'_z, \quad z \in U,\ \zeta \in \overline{U}, \tag{2.11}$$

then

$$q(z,\zeta) \prec\prec \frac{DR_\lambda^{m+1} f(z,\zeta)}{DR_\lambda^m f(z,\zeta)}, \quad z \in U,\ \zeta \in \overline{U},$$

where q is given by $q(z,\zeta) = 2\beta - \zeta + \frac{1+\zeta-2\beta}{nz^{\frac{1}{n}}} \int_0^z \frac{t^{\frac{1}{n}-1}}{1+t} dt$, $z \in U$, $\zeta \in \overline{U}$. The function q is convex and it is the best subordinant.

Proof. Following the same steps as in the proof of Theorem 2.10 and considering

$$p(z,\zeta) = \frac{DR_\lambda^{m+1} f(z,\zeta)}{DR_\lambda^m f(z,\zeta)},$$

the strong differential superordination (2.11) becomes

$$h(z,\zeta) = \frac{1+(2\beta-\zeta)z}{1+z} \prec\prec p(z,\zeta) + zp'_z(z,\zeta), \quad z \in U,\ \zeta \in \overline{U}.$$

By using Lemma 1.8 for $\gamma = 1$, we have $q(z,\zeta) \prec\prec p(z,\zeta)$, i.e.

$$q(z,\zeta) = \frac{1}{nz^{\frac{1}{n}}} \int_0^z h(t,\zeta)\, t^{\frac{1}{n}-1} dt = \frac{1}{nz^{\frac{1}{n}}} \int_0^z t^{\frac{1}{n}-1} \frac{1+(2\beta-\zeta)t}{1+t} dt$$

$$= 2\beta - \zeta + \frac{1+\zeta-2\beta}{nz^{\frac{1}{n}}} \int_0^z \frac{t^{\frac{1}{n}-1}}{1+t} dt \prec\prec \frac{DR_\lambda^{m+1} f(z,\zeta)}{DR_\lambda^m f(z,\zeta)}, \quad z \in U,\ \zeta \in \overline{U}.$$

The function q is convex and it is the best subordinant. □

Theorem 2.12. *Let $q(z,\zeta)$ be convex in $U \times \overline{U}$ and let h be defined by*

$$h(z,\zeta) = q(z,\zeta) + zq'_z(z,\zeta).$$

If $\lambda \ge 0$, $m,n \in \mathbb{N}$, $f(z,\zeta) \in \mathcal{A}^*_{n\zeta}$, suppose that $\left(\frac{zDR_\lambda^{m+1} f(z,\zeta)}{DR_\lambda^m f(z,\zeta)}\right)'_z$ is univalent, $\frac{DR_\lambda^{m+1} f(z,\zeta)}{DR_\lambda^m f(z,\zeta)} \in \mathcal{H}^*[1,n,\zeta] \cap Q^*$ and satisfies the strong differential superordination

$$h(z,\zeta) = q(z,\zeta) + zq'_z(z,\zeta) \prec\prec \left(\frac{zDR_\lambda^{m+1} f(z,\zeta)}{DR_\lambda^m f(z,\zeta)}\right)'_z, \quad z \in U,\ \zeta \in \overline{U}, \tag{2.12}$$

then

$$q(z,\zeta) \prec\prec \frac{DR_\lambda^{m+1} f(z,\zeta)}{DR_\lambda^m f(z,\zeta)}, \qquad z \in U,\ \zeta \in \overline{U},$$

where $q(z,\zeta) = \frac{1}{nz^{\frac{1}{n}}} \int_0^z h(t,\zeta)\, t^{\frac{1}{n}-1} dt$. *The function* q *is the best subordinant.*

Proof. Let

$$p(z,\zeta) = \frac{DR_\lambda^{m+1} f(z,\zeta)}{DR_\lambda^m f(z,\zeta)} = \frac{z + \sum_{j=n+1}^{\infty} C_{m+j}^{m+1} \left[1+(j-1)\lambda\right]^{m+1} a_j^2(\zeta) z^j}{z + \sum_{j=n+1}^{\infty} C_{m+j-1}^{m} \left[1+(j-1)\lambda\right]^{m} a_j^2(\zeta) z^j}$$

$$= \frac{1 + \sum_{j=n+1}^{\infty} C_{m+j}^{m+1} \left[1+(j-1)\lambda\right]^{m+1} a_j^2(\zeta) z^{j-1}}{1 + \sum_{j=n+1}^{\infty} C_{m+j-1}^{m} \left[1+(j-1)\lambda\right]^{m} a_j^2(\zeta) z^{j-1}}.$$

Evidently $p \in \mathcal{H}^*[1,n,\zeta]$.

Differentiating with respect to z, we obtain

$$p(z,\zeta) + zp_z'(z,\zeta) = \left(\frac{zDR_\lambda^{m+1} f(z,\zeta)}{DR_\lambda^m f(z,\zeta)}\right)_z', \quad z \in U,\ \zeta \in \overline{U},$$

and (2.12) becomes

$$q(z,\zeta) + zq_z'(z,\zeta) \prec\prec p(z,\zeta) + zp_z'(z,\zeta), \qquad z \in U,\ \zeta \in \overline{U}.$$

Using Lemma 1.9 for $\gamma = 1$, we have

$$q(z,\zeta) \prec\prec p(z,\zeta), \quad z \in U,\ \zeta \in \overline{U}, \quad \text{i.e.}$$

$$q(z,\zeta) = \frac{1}{nz^{\frac{1}{n}}} \int_0^z h(t,\zeta)\, t^{\frac{1}{n}-1} dt \prec\prec \frac{DR_\lambda^{m+1} f(z,\zeta)}{DR_\lambda^m f(z,\zeta)}, \ z \in U, \zeta \in \overline{U},$$

and q is the best subordinant. □

Theorem 2.13. *Let* $h(z,\zeta)$ *be a convex function,* $h(0,\zeta) = 1$. *Let* $\lambda \geq 0$, $m,n \in \mathbb{N}$, $f(z,\zeta) \in \mathcal{A}_{n\zeta}^*$ *and suppose that* $\frac{m+1}{(m\lambda+1)z} DR_\lambda^{m+1} f(z,\zeta) - \frac{m(1-\lambda)}{(m\lambda+1)z} DR_\lambda^m f(z,\zeta)$ *is univalent and* $\left(DR_\lambda^m f(z,\zeta)\right)_z' \in \mathcal{H}^*[1,n,\zeta] \cap Q^*$. *If*

$$h(z,\zeta) \prec\prec \frac{m+1}{(m\lambda+1)z} DR_\lambda^{m+1} f(z,\zeta) - \frac{m(1-\lambda)}{(m\lambda+1)z} DR_\lambda^m f(z,\zeta), \qquad z \in U,\ \zeta \in \overline{U}, \tag{2.13}$$

then

$$q(z,\zeta) \prec\prec \left(DR_\lambda^m f(z,\zeta)\right)_z', \qquad z \in U,\ \zeta \in \overline{U},$$

where $q(z,\zeta) = \frac{m\lambda+1}{n\lambda z^{\frac{m\lambda+1}{n\lambda}}} \int_0^z h(t,\zeta)\, t^{\frac{(m-n)\lambda+1}{n\lambda}} dt$. *The function* q *is convex and it is the best subordinant.*

Proof. With notation

$$p(z,\zeta) = \left(DR_{\lambda}^{m} f(z,\zeta)\right)'_{z} = 1 + \sum_{j=n+1}^{\infty} C_{m+j-1}^{m} \left[1 + (j-1)\lambda\right]^{m} j a_j^2(\zeta) z^{j-1}$$

and $p(0,\zeta) = 1$, we obtain for $f(z,\zeta) = z + \sum_{j=n+1}^{\infty} a_j(\zeta) z^j$,

$$p(z,\zeta) + z p'_z(z,\zeta)$$

$$= \frac{m+1}{\lambda z} DR_{\lambda}^{m+1} f(z,\zeta) - \left(m - 1 + \frac{1}{\lambda}\right) \left(DR_{\lambda}^{m} f(z,\zeta)\right)'_{z} - \frac{m(1-\lambda)}{\lambda z} DR_{\lambda}^{m} f(z,\zeta)$$

and

$$p(z,\zeta) + \frac{\lambda}{m\lambda+1} z p'_z(z,\zeta) = \frac{m+1}{(m\lambda+1)z} DR_{\lambda}^{m+1} f(z,\zeta) - \frac{m(1-\lambda)}{(m\lambda+1)z} DR_{\lambda}^{m} f(z,\zeta).$$

Evidently $p \in \mathcal{H}^*[1,n,\zeta]$.

Then (2.13) becomes

$$h(z,\zeta) \prec\prec p(z,\zeta) + \frac{\lambda}{m\lambda+1} z p'_z(z,\zeta), \qquad z \in U,\ \zeta \in \overline{U}.$$

By using Lemma 1.8 for $\gamma = m + \frac{1}{\lambda}$, we have

$$q(z,\zeta) \prec\prec p(z,\zeta),\ z \in U,\ \zeta \in \overline{U},\ \text{i.e. } q(z,\zeta) \prec\prec \left(DR_{\lambda}^{m} f(z,\zeta)\right)'_{z},\ z \in U,\ \zeta \in \overline{U},$$

where $q(z,\zeta) = \frac{m\lambda+1}{n\lambda z^{\frac{m\lambda+1}{n\lambda}}} \int_0^z h(t,\zeta) t^{\frac{(m-n)\lambda+1}{n\lambda}} dt$. The function q is convex and it is the best subordinant. □

Corollary 2.14. *Let $h(z,\zeta) = \frac{1+(2\beta-\zeta)z}{1+z}$ be a convex function in $U \times \overline{U}$, where $0 \le \beta < 1$. Let $\lambda \ge 0$, $m,n \in \mathbb{N}$, $f(z,\zeta) \in \mathcal{A}^*_{n\zeta}$ and suppose that $\frac{m+1}{(m\lambda+1)z} DR_{\lambda}^{m+1} f(z,\zeta) - \frac{m(1-\lambda)}{(m\lambda+1)z} DR_{\lambda}^{m} f(z,\zeta)$ is univalent, $\left(DR_{\lambda}^{m} f(z,\zeta)\right)'_{z} \in \mathcal{H}^*[1,n,\zeta] \cap Q^*$. If*

$$h(z,\zeta) \prec\prec \frac{m+1}{(m\lambda+1)z} DR_{\lambda}^{m+1} f(z,\zeta) - \frac{m(1-\lambda)}{(m\lambda+1)z} DR_{\lambda}^{m} f(z,\zeta), \quad z \in U,\ \zeta \in \overline{U}, \tag{2.14}$$

then

$$q(z,\zeta) \prec\prec \left(DR_{\lambda}^{m} f(z,\zeta)\right)'_{z}, \quad z \in U,\ \zeta \in \overline{U},$$

where q is given by $q(z,\zeta) = 2\beta - \zeta + \frac{(1+\zeta-2\beta)(m\lambda+1)}{\lambda n z^{\frac{m\lambda+1}{\lambda n}}} \int_0^z \frac{t^{\frac{m\lambda+1}{\lambda n}-1}}{1+t} dt$, $z \in U$, $\zeta \in \overline{U}$. The function q is convex and it is the best subordinant.

Proof. Following the same steps as in the proof of Theorem 2.13 and considering $p(z,\zeta) = \left(DR_{\lambda}^{m} f(z,\zeta)\right)'_{z}$, the strong differential superordination (2.14) becomes

$$h(z,\zeta) = \frac{1+(2\beta-\zeta)z}{1+z} \prec\prec p(z,\zeta) + z p'_z(z,\zeta), \qquad z \in U,\ \zeta \in \overline{U}.$$

By using Lemma 1.8 for $\gamma = \frac{m\lambda+1}{\lambda}$, we have $q(z,\zeta) \prec\prec p(z,\zeta)$, i.e.

$$q(z,\zeta)=\frac{m\lambda+1}{n\lambda z^{\frac{m\lambda+1}{n\lambda}}}\int_0^z h(t,\zeta)\, t^{\frac{(m-n)\lambda+1}{n\lambda}}dt = \frac{m\lambda+1}{n\lambda z^{\frac{m\lambda+1}{n\lambda}}}\int_0^z t^{\frac{(m-n)\lambda+1}{n\lambda}}dt\frac{1+(2\beta-\zeta)\,t}{1+t}dt$$

$$= 2\beta-\zeta+\frac{(1+\zeta-2\beta)(m\lambda+1)}{\lambda n z^{\frac{m\lambda+1}{\lambda n}}}\int_0^z \frac{t^{\frac{m\lambda+1}{\lambda n}-1}}{1+t}dt \prec\prec \left(DR_\lambda^m f(z,\zeta)\right)_z', \ z\in U,\ \zeta\in\overline{U}.$$

The function q is convex and it is the best subordinant. □

Theorem 2.15. *Let $q(z,\zeta)$ be convex in $U\times\overline{U}$ and let h be defined by*

$$h(z,\zeta) = q(z,\zeta)+\frac{\lambda}{m\lambda+1}zq_z'(z,\zeta), \ \lambda\geq 0,\ m,n\in\mathbb{N}.$$

If $f(z,\zeta)\in\mathcal{A}_{n\zeta}^*$, suppose that $\frac{m+1}{(m\lambda+1)z}DR_\lambda^{m+1}f(z,\zeta)-\frac{m(1-\lambda)}{(m\lambda+1)z}DR_\lambda^m f(z,\zeta)$ is univalent and $\left(DR_\lambda^m f(z,\zeta)\right)_z'\in\mathcal{H}^*[1,n,\zeta]\cap Q^*$ and satisfies the strong differential superordination

$$h(z,\zeta)=q(z,\zeta)+\frac{\lambda}{m\lambda+1}zq_z'(z,\zeta)\prec\prec \tag{2.15}$$
$$\frac{m+1}{(m\lambda+1)z}DR_\lambda^{m+1}f(z,\zeta)-\frac{m(1-\lambda)}{(m\lambda+1)z}DR_\lambda^m f(z,\zeta),\ z\in U,\zeta\in\overline{U},$$

then

$$q(z,\zeta)\prec\prec\left(DR_\lambda^m f(z,\zeta)\right)_z',\quad z\in U,\ \zeta\in\overline{U},$$

where $q(z,\zeta) = \frac{m\lambda+1}{n\lambda z^{\frac{m\lambda+1}{n\lambda}}}\int_0^z h(t,\zeta)\,t^{\frac{(m-n)\lambda+1}{n\lambda}}dt$. The function q is the best subordinant.

Proof. Let $p(z,\zeta)=\left(DR_\lambda^m f(z,\zeta)\right)' = 1+\sum_{j=n+1}^{\infty} C_{m+j-1}^m\left[1+(j-1)\lambda\right]^m ja_j^2(\zeta)\, z^{j-1}$.

Differentiating, we obtain

$$p(z,\zeta)+zp_z'(z,\zeta)$$
$$=\frac{m+1}{\lambda z}DR_\lambda^{m+1}f(z,\zeta)-\left(m-1+\frac{1}{\lambda}\right)\left(DR_\lambda^m f(z,\zeta)\right)_z'-\frac{m(1-\lambda)}{\lambda z}DR_\lambda^n f(z,\zeta)$$

and

$$p(z,\zeta)+\frac{\lambda}{m\lambda+1}zp_z'(z,\zeta)$$
$$=\frac{m+1}{(m\lambda+1)z}DR_\lambda^{m+1}f(z,\zeta)-\frac{m(1-\lambda)}{(m\lambda+1)z}DR_\lambda^m f(z,\zeta),\ z\in U,\ \zeta\in\overline{U},$$

and (2.15) becomes

$$q(z,\zeta)+\frac{\lambda}{m\lambda+1}zq_z'(z,\zeta)\prec\prec p(z,\zeta)+\frac{\lambda}{m\lambda+1}zp_z'(z,\zeta),\quad z\in U,\ \zeta\in\overline{U}.$$

Using Lemma 1.9 for $\gamma = m+\frac{1}{\lambda}$, we have $q(z,\zeta)\prec\prec p(z,\zeta)$, $z\in U$, $\zeta\in\overline{U}$, i.e.

$$q(z,\zeta)=\frac{m\lambda+1}{n\lambda z^{\frac{m\lambda+1}{n\lambda}}}\int_0^z h(t,\zeta)\,t^{\frac{(m-n)\lambda+1}{n\lambda}}dt\prec\prec\left(DR_\lambda^m f(z,\zeta)\right)_z',\quad z\in U,\ \zeta\in\overline{U},$$

and q is the best subordinant. □

References

[1] Alb Lupaş, A., *Certain strong differential subordinations using Sălăgean and Ruscheweyh operators*, Advances in Applied Mathematical Analysis, **6**(2011), no. 1, 27-34.

[2] Alb Lupaş, A., *A note on strong differential subordinations using a generalized Sălăgean operator and Ruscheweyh operator*, submitted 2011.

[3] Alb Lupaş, A., *A note on strong differential subordinations using Sălăgean and Ruscheweyh operators*, Libertas Mathematica, **31**(2011), 15-21.

[4] Alb Lupaş, A., Oros, G.I., Oros, Gh., *On special strong differential subordinations using Sălăgean and Ruscheweyh operators*, Journal of Computational Analysis and Applications, Vol. 14, 2012 (to appear).

[5] Alb Lupaş, A., *On special strong differential subordinations using a generalized Sălăgean operator and Ruscheweyh derivative*, Journal of Concrete and Applicable Mathematics, **10**(2012), no. 1-2, 17-23.

[6] Alb Lupaş, A., *On special strong differential superordinations using Sălăgean and Ruscheweyh operators*, Archimedes Journal of Mathematics, **1**(2011) (to appear).

[7] Alb Lupaş, A., *Certain strong differential superordinations using Sălăgean and Ruscheweyh operators*, Acta Universitatis Apulensis, submitted 2011.

[8] Alb Lupaş, A., *A note on strong differential superordinations using Sălăgean and Ruscheweyh operators*, Journal of Applied Functional Analysis, submitted 2011.

[9] Oros, G.I., *Strong differential superordination*, Acta Universitatis Apulensis, **19**(2009), 101-106.

[10] Oros, G.I., *On a new strong differential subordination*, (to appear).

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