Stud. Univ. Babeş-Bolyai Math. 57(2012), No. 1, 75–82

Strongly almost summable sequence spaces in 2-normed spaces defined by ideal convergence and an Orlicz function

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Abstract. In this paper we introduce some certain new sequence spaces via ideal convergence and an Orlicz function in 2-normed spaces and examine some properties of the resulting these spaces.

Mathematics Subject Classification (2010): 40A99, 40A05.

Keywords: 2-normed space, Orlicz function.

1. Introduction

Let X be a non-empty set, then a family of sets $I \subset 2^X$ (the class of all subsets of X) is called an ideal if and only if for each $A, B \in I$, we have $A \cup B \in I$ and for each $A \in I$ and each $B \subset A$, we have $B \in I$. A non-empty family of sets $F \subset 2^X$ is a filter on X if and only if $\varnothing \notin F$, for each $A, B \in F$, we have $A \cap B \in F$ and each $A \in F$ and each $A \subset B$, we have $B \in F$. An ideal I is called non-trivial ideal if $I \neq \varnothing$ and $X \notin I$. Clearly $I \subset 2^X$ is a non-trivial ideal if and only if $F = F(I) = \{X/A : A \in I\}$ is a filter on X. A non-trivial ideal $I \subset 2^X$ is called admissible if and only if $\{\{x\} : x \in X\} \subset I$. Further details on ideals of 2^X can be found in Kostyrko, et.al [3] . The notion was further investigated by Salat, et.al [4], Tripathy and Hazarika $[13-15]$, Tripathy and Mahanta [16] and others.

Recall in $[5,7]$ that an Orlicz function M is continuous, convex, nondecreasing function define for $x > 0$ such that $M(0) = 0$ and $M(x) > 0$. If convexity of Orlicz function is replaced by $M(x+y) \leq M(x) + M(y)$ then this function is called the modulus function and characterized by Ruckle [6] . An Orlicz function M is said to satisfy Δ_2*−condition* for all values u, if there exists $K > 0$ such that $M(2u) \leq KM(u), u \geq 0$. Subsequently, the notion of Orlicz function was used to defined sequence spaces by Altin et al [8] , Tripathy and Mahanta [9] , Et et al [10] , Tripathy et al [11], Tripathy and Sarma [12] and many others.

Lemma. *Let M be an Orlicz function which satisfies Δ_2-condition and let* $0<\delta<1$. *Then for each $t\geq\delta$, we have $M(t)<K\delta^{-1}M(2)$ for some constant $K>0$.*

A sequence space X is said to be solid or normal if $(\alpha_k x_k)\in X$, and for all sequences $\alpha=(\alpha_k)$ of scalars with $|\alpha_k|\leq 1$ for all $k\in\mathbb{N}$.

Let X be a real vector space of dimension d, where $2\leq d<\infty$. A 2-norm on X is a function $\|.,.\|:X\times X\rightarrow R$ which satisfies;

(i) $\|x,y\|=0$ if and only if x and y are linearly dependent,

(ii) $\|x,y\|=\|y,x\|$,

(iii) $\|\alpha x,y\|=|\alpha|\,\|x,y\|\,,\alpha\in\mathbb{R}$,

(iv) $\|x,y+z\|\leq\|x,y\|+\|x,z\|$.

The pair $(X,\|.,.\|)$ is called a 2-normed space [2]. As an example of a 2-normed space we may take $X=\mathbb{R}$ being equipped with the 2-norm $\|x,y\|=$*the area of paralelogram spanned by the vectors x and y*, which may be given explicitly by the formula

$$\|x,y\|_E=abs\left(\begin{vmatrix}x_{11} & x_{12}\\ y_{11} & y_{12}\end{vmatrix}\right).$$

2. Main results

In this section we introduce the notion of different types of I-convergent sequences.

Let I be an ideal of $2^{\mathbb{N}}$, M be an Orlicz function, $p=(p_k)$ be a bounded sequence of strictly positive real numbers and $(X,\|,.,\|)$ be an 2-normed space. Further $w(2-X)$ denotes X-valued sequence space. Now, we define the following sequence spaces:

$$\widehat{w}^I[M,p,\|,.,\|]_o=\left\{\begin{array}{c}x=(x_k)\in w(2-X):\forall\varepsilon>0,\left\{n\in\mathbb{N}:\frac{1}{n}\sum_{k=1}^n\left[M\left(\left\|\frac{t_{km}(x)}{\rho},z\right\|\right)\right]^{p_k}\geq\varepsilon\right\}\in I\\ \text{for some }\rho>0,\ m\in\mathbb{N}\text{ and each }z\in X\end{array}\right\},$$

$$\widehat{w}^I[M,p,\|,.,\|]=\left\{\begin{array}{c}x=(x_k)\in w(2-X):\forall\varepsilon>0,\left\{n\in\mathbb{N}:\frac{1}{n}\sum_{k=1}^n\left[M\left(\left\|\frac{t_{km}(x)}{\rho},z\right\|\right)\right]^{p_k}\geq\varepsilon\right\}\in I\\ \text{for some }\rho>0,L\in X,m\in\mathbb{N}\text{ and each }z\in X\end{array}\right\},$$

$$\widehat{w}^I[M,p,\|,.,\|]_\infty=\left\{\begin{array}{c}x=(x_k)\in w(2-X):\exists K>0,\left\{n\in\mathbb{N}:\frac{1}{n}\sum_{k=1}^n\left[M\left(\left\|\frac{t_{km}(x)}{\rho},z\right\|\right)\right]^{p_k}\geq K\right\}\in I\\ \text{for some }\rho>0,m\in\mathbb{N}\text{ and each }z\in X\end{array}\right\}$$

and

$$\widehat{w}[M,p,\|,.,\|]_\infty=\left\{\begin{array}{c}x=(x_k)\in w(2-X):\exists K>0,\frac{1}{n}\sum_{k=1}^n\left[M\left(\left\|\frac{t_{km}(x)}{\rho},z\right\|\right)\right]^{p_k}\leq K\\ \text{for some }\rho>0,m\in\mathbb{N}\text{ and each }z\in X\end{array}\right\},$$

where

$$t_{km}(x) = t_{km}(x_k) = \frac{1}{k+1}\sum_{i=0}^{k} x_{i+m}, m \in \mathbb{N}.$$

If $p_k = 1$ for all $k \in \mathbb{N}$, we denote

$$\widehat{w}^I[M, p, \|, ., \|] = \widehat{w}^I[M, \|, ., \|], \widehat{w}^I[M, p, \|, ., \|]_o = \widehat{w}^I[M, \|, ., \|]_o,$$

$$\widehat{w}^I[M, p, \|, ., \|]_\infty = \widehat{w}^I[M, \|, ., \|]_\infty$$

and

$$\widehat{w}[M, p, \|, ., \|]_\infty = \widehat{w}[M, \|, ., \|]_\infty$$

respectively.

If for $k = 0$, we get $t_{km}(x) = x_m$ for all $m \in \mathbb{N}$. We denote these three classes of sequences as $w^I[M, p, \|, ., \|], w^I[M, p, \|, ., \|]_o, w^I[M, p, \|, ., \|]_\infty$ and $w[M, p, \|, ., \|]_\infty$ respectively.

The following well-known inequality will be used for establishing some results of this article. If $0 \le \inf_k p_k (= h) \le p_k \le \sup_k (= H) < \infty$, $D = \max\left(1, 2^{H-1}\right)$, then

$$|x_k + y_k|^{p_k} \le D\left\{|x_k|^{p_k} + |y_k|^{p_k}\right\}$$

for all $k \in \mathbb{N}$ and $x_k, y_k \in \mathbb{C}$. Also $|x_k|^{p_k} \le \max\left(1, |x_k|^H\right)$ for all $x_k \in \mathbb{C}$.

Theorem 2.1. *The sets* $\widehat{w}^I[M, p, \|, ., \|]_o, \widehat{w}^I[M, p, \|, ., \|]$ *and* $\widehat{w}^I[M, p, \|, ., \|]_\infty$ *are linear spaces over the complex field* $\mathbb{C}$.

Proof. We will prove only for $\widehat{w}^I[M, p, \|, ., \|]_o$ and the others can be proved similarly. Let $x, y \in \widehat{w}^I[M, p, \|, ., \|]_o$ and $\alpha, \beta \in \mathbb{C}$. Then

$$\left\{n \in \mathbb{N} : \frac{1}{n}\sum_{k=1}^{n}\left[M\left(\left\|\frac{t_{km}(x)}{\rho_1}, z\right\|\right)\right]^{p_k} \ge \frac{\varepsilon}{2}\right\} \in I, \text{ for some } \rho_1 > 0$$

and

$$\left\{n \in \mathbb{N} : \frac{1}{n}\sum_{k=1}^{n}\left[M\left(\left\|\frac{t_{km}(y)}{\rho_2}, z\right\|\right)\right]^{p_k} \ge \frac{\varepsilon}{2}\right\} \in I, \text{ for some } \rho_2 > 0\ .$$

for all $m \in \mathbb{N}$. Since $\|, ., \|$ is a 2-norm and M is an Orlicz function, the following inequality holds:

$$\frac{1}{n}\sum_{k=1}^{n}\left[M\left(\left\|\frac{t_{km}(\alpha x + \beta y)}{|\alpha|\rho_1 + |\beta|\rho_2}, z\right\|\right)\right]^{p_k}$$

$$\le \frac{D}{n}\sum_{k=1}^{n}\left[\frac{|\alpha|}{|\alpha|\rho_1 + |\beta|\rho_2}M\left(\left\|\frac{t_{km}(x)}{\rho_1}, z\right\|\right)\right]^{p_k}$$

$$+ \frac{D}{n}\sum_{k=1}^{n}\left[\frac{|\beta|}{|\alpha|\rho_1 + |\beta|\rho_2}M\left(\left\|\frac{t_{km}(y)}{\rho_2}, z\right\|\right)\right]^{p_k}$$

$$\le \frac{D}{n}\sum_{k=1}^{n}\left[M\left(\left\|\frac{t_{km}(x)}{\rho_1}, z\right\|\right)\right]^{p_k} + \frac{D}{n}\sum_{k=1}^{n}\left[M\left(\left\|\frac{t_{km}(y)}{\rho_2}, z\right\|\right)\right]^{p_k}$$

for all $m \in \mathbb{N}$. From the above inequality we get

$$\left\{ n \in \mathbb{N} : \frac{1}{n} \sum_{k=1}^{n} \left[M \left(\left\| \frac{t_{km}(\alpha x + \beta y)}{|\alpha| \rho_1 + |\beta| \rho_2}, z \right\| \right) \right]^{p_k} \geq \varepsilon \right\}$$

$$\subset \left\{ n \in \mathbb{N} : \frac{DA}{n} \sum_{k=1}^{n} \left[M \left(\left\| \frac{t_{km}(x)}{\rho_1}, z \right\| \right) \right]^{p_k} \geq \frac{\varepsilon}{2} \right\}$$

$$\cup \left\{ n \in \mathbb{N} : \frac{DA}{n} \sum_{k=1}^{n} \left[M \left(\left\| \frac{t_{km}(y)}{\rho_2}, z \right\| \right) \right]^{p_k} \geq \frac{\varepsilon}{2} \right\}.$$

Two sets on the right hand side belong to I and this completes the proof.

It is also easy verify that the space $\widehat{w}[M, p, \|, ., \|]_\infty$ is also a linear space.

Theorem 2.2. *For fixed $n \in \mathbb{N}$, $\widehat{w}[M, p, \|, ., \|]_\infty$ paranormed space with respect to the paranorm defined by*

$$g(x) = \inf \left\{ \begin{array}{c} \rho^{\frac{p_n}{H}} > 0 : \left(\sup_{n,m} \frac{1}{n} \sum_{k=1}^{n} \left[M \left(\left\| \frac{t_{km}(x)}{\rho}, z \right\| \right) \right]^{p_k} \right)^{\frac{1}{H}} \leq 1, \\ for\ each\ z \in X \end{array} \right\}.$$

Proof. $g(\theta) = 0$ and $g(-x) = g(x)$ are easy to prove, so we omit them. Let us take $x, y \in \widehat{w}[M, p, \|, ., \|]_\infty$. Let

$$A(x) = \left\{ \rho > 0 : \sup_{n,m} \frac{1}{n} \sum_{k=1}^{n} \left[M \left(\left\| \frac{t_{km}(x)}{\rho}, z \right\| \right) \right]^{p_k} \leq 1, \forall z \in X \right\}$$

and

$$A(y) = \left\{ \rho > 0 : \sup_{n,m} \frac{1}{n} \sum_{k=1}^{n} \left[M \left(\left\| \frac{t_{km}(y)}{\rho}, z \right\| \right) \right]^{p_k} \leq 1, \forall z \in X \right\}.$$

Let $\rho_1 \in A(x)$ and $\rho_2 \in A(y)$. If $\rho = \rho_1 + \rho_2$, then we have

$$\sup_{n,m} \frac{1}{n} \sum_{k=1}^{n} \left[M \left(\left\| \frac{t_{km}(x+y)}{\rho}, z \right\| \right) \right]$$

$$\leq \frac{\rho_1}{\rho_1 + \rho_2} \sup_{n,m} \frac{1}{n} \sum_{k=1}^{n} \left[M \left(\left\| \frac{t_{km}(x)}{\rho_1}, z \right\| \right) \right]$$

$$+ \frac{\rho_2}{\rho_1 + \rho_2} \sup_{n,m} \frac{1}{n} \sum_{k=1}^{n} \left[M \left(\left\| \frac{t_{km}(y)}{\rho_1}, z \right\| \right) \right].$$

Thus

$$\sup_{n,m} \frac{1}{n} \sum_{k=1}^{n} \left[M \left(\left\| \frac{t_{km}(x+y)}{\rho_1 + \rho_2}, z \right\| \right) \right]^{p_k} \leq 1$$

and

$$g(x+y) = \inf \left\{ (\rho_1 + \rho_2)^{\frac{p_n}{H}} > 0 : \rho_1 \in A(x)\ and\ \rho_2 \in A(y) \right\}$$

$$\leq \inf\left\{(\rho_1)^{\frac{p_n}{H}} > 0 : \rho_1 \in A(x)\ \right\} + \inf\left\{(\rho_2)^{\frac{p_n}{H}} > 0 : \rho_2 \in A(y)\right\}$$
$$= g(x) + g(y).$$

Now, let $\lambda_k \to \lambda$, where $\lambda_k, \lambda \in \mathbb{C}$ and $g(x_k^u - x_k) \to 0$ as $u \to \infty$. We have to show that $g(\lambda_k x_k^u - \lambda x_k) \to 0$ as $u \to \infty$. Let

$$A(x^u) = \left\{\rho_u > 0 : \sup_{n,m} \frac{1}{n}\sum_{k=1}^{n}\left[M\left(\left\|\frac{t_{km}(x_k^u)}{\rho_u}, z\right\|\right)\right]^{p_{k,l}} \leq 1, \forall z \in X\right\}$$

and

$$A(x^u - x) = \left\{\rho_u^\imath > 0 : \sup_{n,m} \frac{1}{n}\sum_{k=1}^{n}\left[M\left(\left\|\frac{t_{km}(x^u - x)}{\rho_u^\imath}, z\right\|\right)\right]^{p_k} \leq 1, \forall z \in X\right\}.$$

If $\rho_u \in A(x^u)$ and $\rho_u^\imath \in A(x^u - x)$ then we observe that

$$M\left(\left\|\frac{t_{km}(\lambda_k x_k^u - \lambda x_k)}{\rho_u|\lambda_k - \lambda| + \rho_u^\imath|\lambda|}, z\right\|\right)$$
$$\leq M\left(\left\|\frac{t_{km}(\lambda_k x_k^u - \lambda x_k^u)}{\rho_u|\lambda_k - \lambda| + \rho_u^\imath|\lambda|}, z\right\| + \left\|\frac{t_{km}(\lambda x_k^u - \lambda x_k)}{\rho_u|\lambda_k - \lambda| + \rho_u^\imath|\lambda|}, z\right\|\right)$$
$$\leq \frac{\rho_u|\lambda_k - \lambda|}{\rho_u|\lambda_k - \lambda| + \rho_u^\imath|\lambda|} M\left(\left\|\frac{t_{km}(x_k^u)}{\rho_u}, z\right\|\right)$$
$$+ \frac{\rho_u^\imath|\lambda|}{\rho_u|\lambda_k - \lambda| + \rho_u^\imath|\lambda|} M\left(\left\|\frac{t_{km}(x_k^u - x_k)}{\rho_u^\imath}, z\right\|\right).$$

From this inequality, it follows that

$$\left[M\left(\left\|\frac{t_{km}(\lambda_k x_k^u - \lambda x_k)}{\rho_u|\lambda_k - \lambda| + \rho_u^\imath|\lambda|}, z\right\|\right)\right]^{p_k} \leq 1$$

and consequently

$$g(\lambda_k x_k^u - \lambda x_k) = \inf\left\{(\rho_u|\lambda_k - \lambda| + \rho_u^\imath|\lambda|)^{\frac{p_n}{H}} > 0 : \rho_u \in A(x^u)\ \textit{and}\ \rho_u^\imath \in A(x^u - x)\right\}$$
$$\leq (|\lambda_k - \lambda|)^{\frac{p_n}{H}} \inf\left\{(\rho_u)^{\frac{p_n}{H}} > 0 : \rho_u \in A(x^u)\ \right\}$$
$$+ (|\lambda|)^{\frac{p_n}{H}} \inf\left\{(\rho_u^\imath)^{\frac{p_n}{H}} > 0 : \rho_u^\imath \in A(x^u - x)\ \right\}$$
$$\leq \max\left\{|\lambda|, (|\lambda|)^{\frac{p_n}{H}}\right\} g(x_k^u - x_k).$$

Hence by our assumption the right hand side tends to zero as $u \to \infty$. This completes the proof.

Theorem 2.3. *Let M, M_1 and M_2 be Orlicz functions. Then we have*

(i) $\widehat{w}^I[M_1, p, \|,.,\|]_o \subset \widehat{w}^I[MoM_1, p, \|,.,\|]_o$ *provided that* $p = (p_k)$ *is such that* $h > 0$.

(ii) $\widehat{w}^I[M_1, p, \|,.,\|]_o \cap \widehat{w}^I[M_2, p, \|,.,\|]_o \subset \widehat{w}^I[M_1 + M_2, p, \|,.,\|]_o$.

Proof. (i). For given $\varepsilon > 0$, we first choose $\varepsilon_o > 0$ such that $\max\left\{\varepsilon_o^H, \varepsilon_o^{H_o}\right\} < \varepsilon$. Now using the continuity of M, choose $0 < \delta < 1$ such that $0 < t < \delta$ implies $M(t) < \varepsilon_o$.

Let $x \in \widehat{w}^I[M_1, p, \|,.,\|]_o$. Now from the definition of the space $\widehat{w}^I[M_1, p, \|,.,\|]_o$, for some $\rho > 0$

$$A(\delta) = \left\{ n \in \mathbb{N} : \frac{1}{n} \sum_{k=1}^{n} \left[M_1 \left(\left\| \frac{t_{km}(x)}{\rho}, z \right\| \right) \right]^{p_k} \geq \delta^H \right\} \in I, \ m \in \mathbb{N}$$

Thus if $n \notin A(\delta)$ then

$$\frac{1}{n} \sum_{k=1}^{n} \left[M_1 \left(\left\| \frac{t_{km}(x)}{\rho}, z \right\| \right) \right]^{p_k} < \delta^H$$

$$\Rightarrow \sum_{k=1}^{n} \left[M_1 \left(\left\| \frac{t_{km}(x)}{\rho}, z \right\| \right) \right]^{p_k} < n\delta^H,$$

$$\Rightarrow \left[M_1 \left(\left\| \frac{t_{km}(x)}{\rho}, z \right\| \right) \right]^{p_k} < \delta^H \text{ for all } k, m = 1, 2, ...,$$

$$\Rightarrow M_1 \left(\left\| \frac{t_{km}(x)}{\rho}, z \right\| \right) < \delta \text{ for all } k, m = 1, 2, ...$$

Hence from above inequality and using continuity of M, we must have

$$M \left(M_1 \left(\left\| \frac{t_{km}(x)}{\rho}, z \right\| \right) \right) < \varepsilon_o \text{ for all } k, m = 1, 2, ...$$

which consequently implies that

$$\sum_{k=1}^{n} \left[M \left(M_1 \left(\left\| \frac{t_{km}(x)}{\rho}, z \right\| \right) \right) \right]^{p_k} < n \max \left\{ \varepsilon_o^H, \varepsilon_o^{H_o} \right\} < n\varepsilon, \ m = 1, 2, ...,$$

$$\Rightarrow \frac{1}{n} \sum_{k=1}^{n} \left[M \left(M_1 \left(\left\| \frac{t_{km}(x)}{\rho}, z \right\| \right) \right) \right]^{p_k} < \varepsilon, \ m = 1, 2, ...$$

This shows that

$$\left\{ n \in \mathbb{N} : \frac{1}{n} \sum_{k=1}^{n} \left[M \left(M_1 \left(\left\| \frac{t_{km}(x)}{\rho}, z \right\| \right) \right) \right]^{p_k} \geq \varepsilon \right\} \subset A(\delta)$$

and so belongs to I. This completes the proof.

(ii) Let $x \in \widehat{w}^I[M_1, p, \|,.,\|]_o \cap \widehat{w}^I[M_2, p, \|,.,\|]_o$. Then the fact that

$$\frac{1}{n} \left[(M_1 + M_2) \left(\left\| \frac{t_{km}(x)}{\rho}, z \right\| \right) \right]^{p_k}$$

$$\leq \frac{D}{n} \left[M_1 \left(\left\| \frac{t_{km}(x)}{\rho}, z \right\| \right) \right]^{p_k} + \frac{D}{n} \left[M_2 \left(\left\| \frac{t_{km}(x)}{\rho}, z \right\| \right) \right]^{p_k}$$

gives us the result.

Theorem 2.4. *(i) If $0 < h \leq p_k < 1$, then $\widehat{w}^I[M, p, \|,.,\|]_o \subset \widehat{w}^I[M, \|,.,\|]_o$.*

(ii) If $1 \leq p_k \leq H < \infty$, then $\widehat{w}^I[M, \|,.,\|]_o \subset \widehat{w}^I[M, p, \|,.,\|]_o$.

(iii) If $0 < p_k < q_k < \infty$ and $\frac{q_k}{p_k}$ is bounded, then

$$\widehat{w}^I[M, p, \|,.,\|]_o \subset \widehat{w}^I[M, q, \|,.,\|]_o.$$

Proof. The proof is standard, so we omit it.

Theorem 2.5. *The sequence spaces* $\widehat{w}^I[M,p,\|,.,\|]_o$, $\widehat{w}^I[M,p,\|,.,\|]$, $\widehat{w}^I[M,p,\|,.,\|]_\infty$ *and* $\widehat{w}[M,p,\|,.,\|]_\infty$ *are solid.*

Proof. We give the proof for only $\widehat{w}^I[M,p,\|,.,\|]_o$. The others can be proved similarly. Let $x\in\widehat{w}^I[M_1,p,\|,.,\|]_o$ and $\alpha=(\alpha_k)$ be a sequence of scalars such that $|\alpha_k|\le 1$ for all $k\in\mathbb{N}$. Then we have

$$\left\{n\in\mathbb{N}:\ \frac{1}{n}\sum_{k=1}^{n}\left[M\left(\left\|\frac{t_{km}(\alpha_k x_k)}{\rho},z\right\|\right)\right]^{p_k}\le\varepsilon\right\}$$

$$\subset\left\{n\in\mathbb{N}:\ \frac{T}{n}\sum_{k=1}^{n}\left[M\left(\left\|\frac{t_{km}(x_k)}{\rho},z\right\|\right)\right]^{p_k}\le\varepsilon\right\}\in I,$$

where $T=\sup_k\left\{1,|\alpha_k|^H\right\}$. Hence $\alpha x\in\widehat{w}^I[M_1,p,\|,.,\|]_o$ for all sequences $\alpha=(\alpha_k)$ with $|\alpha_k|\le 1$ for all $k\in\mathbb{N}$ whenever $x\in\widehat{w}^I[M_1,p,\|,.,\|]_o$.

Acknowledgement. The author is grateful to the referee for corrections and suggestions, which have greatly improved the readability of the paper.

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