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Book reviews

Nonlinear analysis and optimization, Proceedings of the conference held in celebration of Alex Ioffe's 70th and Simeon Reich's 60th birthdays at the Technion, Haifa, June 18–24, 2008. Edited by *Arie Leizarowitz, Boris S. Mordukhovich, Itai Shafrir* and *Alexander J. Zaslavski*, Israel Mathematical Conference Proceedings. American Mathematical Society, Providence, RI; Bar-Ilan University, Ramat Gan, 2010.

I: **Nonlinear Analysis,** Contemporary Mathematics, vol. 513, 263 pp., ISBN-10: 0-8218-4834-8, ISBN-13: 978-0-8218-4834-0.

II: **Optimization**, Contemporary Mathematics, vol. 514, xx+290 pp., ISBN-10: 0-8218-4835-6, ISBN-13: 978-0-8218-4835-7.

A Conference on Nonlinear Analysis and Optimization took place on June 18–24, 2008, at the Technion - Israel Institute of Technology, Haifa, Israel. Continuing the tradition of several previous conferences on related topics, the present one was dedicated to the celebration of Alex Ioffe's 70th and Simeon Reich's 60th birthdays. Alex Ioffe is known for his important contributions to nonlinear analysis, optimization and variational analysis (over 130 publications) as well as for his book (written jointly with V. M. Tikhomirov) "Theory of Extremal Problems" , Moscow 1974 (in Russian), translated in English and German in 1979. Simeon Reich has published more than 300 research papers and two monographs into various areas of mathematics, related to nonlinear analysis and optimization. Taking into account the actuality of the topics as well as the prominence of the two celebrated personalities, the conference attracted a lot of eminent mathematicians from all over the world – more than 70 from 18 countries.

The present volumes collect the written and expanded versions of the contributions of some participants, as well as the contributions of some invited speakers who were unable to attend the conference. The contributions are divided into two volumes - I. *Nonlinear Analysis*, and II. *Optimization*, although it is difficult to trace a clear demarcation line between these two fields, taking into account their strong interdependence, a fact that can be seen by an examination of the titles in the volumes.

The first volume contains 14 contributed papers on various topics from nonlinear analysis such as fixed point theory (T. Domínguez Benavides and S. Phothi, W. Kaczor, T. Kuczumow, and M. Michalska, K. Goebel and B. Sims), nonexpansive mappings, monotone operators and Kirszbraun-Valentine extensions (H. H. Bauschke and X. Wang), nonexpansive operators and convex feasibility problems (A. Cegielski), iterative methods and algorithms for finding fixed points (T. Ibaraki, W. Takahashi, L. Leuştean, G. López, V. Martin-Márquez, and H.-K. Xu), random products of orthogonal projections (R. E. Bruck), Mosco stability for the generalized proximal mapping (D. Butnariu, E. Resmeriţa, and S. Sabach), control for Navier-Stokes equations (V. Barbu), Neumann problem for p-Laplacian (S. Aizicovici, N. S. Papageorgiou and V. Staicu). A paper on biology – an amphibian juvenile-adult model – by A. S. Ackleh, K. Deng and Q. Huang, is also included.

The second volume, on optimization, is concerned with several important topics of great interest in the current research in the area, such as regularity and calmness in nonsmooth analysis (A. Giannessi, A. Moldovan, L. Pellegrini and J.-P. Penot), quadratic optimal control (J. F. Bonnans, N. P. Osmolovskii, and V. Y. Glizer), transportation problems (J.-P. Aubin and S. Martin, G. Buttazzo and G. Carlier), subdifferential calculus (R. Baier and E. Farkhi, J. M. Borwein and S. Sciffer), constrained minimization problems (A. Zaslavski), isoperimetric problems in the calculus of variations (R. A. C. Ferreira and A. C. Torres), Kaldorian business fluctuations (T. Maruyama), time scales (D. Mozyrska and E. Pawluszewicz), Morse indexes for piecewise linear functions (D. Pallaschke and R. Urbanski).

Containing articles on leading themes of current research in nonlinear analysis and optimization, written by prominent researchers in these two areas, these two volumes bring the readers, advanced graduate students and researchers alike, to the frontline of research in important fields of mathematics. Undoubtedly, they must be on the desk of every one working in these areas.

S. Cobzaş

Jiři Matoušek, Thirty-three Miniatures. Mathematical and Algorithmic Applications of Linear Algebra, Student Mathematical Library, Volume 53, American Mathematical Society, Providence, Rhode Island, 2010, x+182 pp; ISBN-13: 978-0-8218-4977-4, ISBN-10: 0-8218-4977-8

This booklet contains a collection of succinct and clever applications of linear algebra to combinatorics, graph theory, geometry and algorithms. In this case, gem or jewel is a good synonym for miniature. Each miniature (chapter, lecture) is dense, concise, carefully polished and written in an attractive style. A motivation of the term is the complete exposition of the result, the length from two to ten pages, and the independence (with few exceptions) from all other chapters. Although Matoušek says in preface that nothing is original, the way of structuring, the selection of topics and the presentation is fully original. Each lecture has a short paragraph "Sources", containing annotated bibliographical references.

The text requires only a good undergraduate background in linear algebra and some knowledge in graph theory and geometry.

Among the topics treated, we mention (enumeration reflects the preferences of the reviewer) Fibonacci numbers, error-correcting codes, probabilistic checking of matrix multiplication, tiling a rectangle by squares, counting spanning trees, fast associativity testing, set pairs via exterior products. All topics, both the simple and more advanced, refer to beautiful results and have elegant proofs.

The impact of this book will be two-fold: first, provides the instructors and the students enroled to a linear algebra course with a rich set of examples; second, makes people interested in combinatorics and computer science aware of the existence of linear algebra tools and their strength.

Intended audience: undergraduates, graduate students and research mathematicians interested in combinatorics, graph theory, theoretical computer science and computational geometry, as well as lecturers who want to liven their courses.

Radu Trîmbiţaş

Elias M. Stein and Rami Shakarchi, Functional analysis. Introduction to further topics in analysis. Princeton Lectures in Analysis 4, Princeton University Press, Princeton, NJ, 2011. xviii+423 pp. ISBN: 978-0-691-11387-6.

This is the last of a four volume treatise on analysis published with Princeton University Press between 2003 and 2011 as the series *Princeton Lectures in Analysis.* The previous volumes, I. *Fourier analysis. An introduction* (2003), II. *Complex analysis,* (2003), III. *Real analysis. Measure theory, integration, and Hilbert spaces* (2005), got an enthusiastic reception from the mathematical community.

The aim of the treatise is to present some cornerstone results of analysis, with emphasis on their historical evolution and the interdependence existing between various parts. I think this is best illustrated by the authors in the preface:

> Our goal is to present the various sub-areas of analysis not as separate disciplines, but rather as highly interconnected. It is our view that seeing these relations and their resulting synergies will motivate the reader to attain a better understanding of the subject as a whole. With this outcome in mind, we have concentrated on the main ideas and theorems that have shaped the field (sometimes sacrificing a more systematic approach), and we have been sensitive to the historical order in which the logic of the subject developed.

In the same order of ideas, this last volume, on functional analysis, could be viewed as an illustration of the famous saying of Einar Hille that "a functional analyst is an analyst, first and foremost, and not a degenerate species of a topologist". In the first chapter, Ch. 1. *L^p spaces and Banach spaces*, the basic of Banach spaces (Hahn-Banach theorem, duality, separation by hyperplanes) are exposed in parallel with the corresponding properties of L^p spaces - completeness and dual spaces. In the Appendix the duals of $C(X)$, the space of all continuous functions on a compact metric space X, and of $C_b(X)$, the space of all bounded continuous functions on an arbitrary metric space X, are described.

The study of L^p spaces is continued in the second chapter, 2. *L^p spaces in harmonic analysis*, with Riesz interpolation theorem, the L^p theory of Hilbert transform, the Hardy spaces H_r^1 and the space BMO. Chapter 3. *Distributions: Generalized functions*, contains the basic properties of distributions, some important examples, as, for instance, the principal value of $1/x$ and its relation with the Hilbert transform, and applications to fundamental solutions of partial differential equations.

In Chapter 4. *Applications of the Baire category theorem*, one shows that some classes of singular functions as, e.g., continuous nowhere differentiable functions, continuous functions with divergent Fourier, are of second Baire category in the corresponding spaces (i.e., they form large subsets). The open mapping and the closed graph theorems are proved with application to Grothendieck's theorem on closed subspaces of L^p.

The basic of probability theory are presented in Ch. 5. *Rudiments of probability theory*, continued in Ch. 6. *An introduction to Brownian motion*, with stopping time, Markov property and applications to Dirichlet problem. Some basic results on analytic functions of several complex variables are considered in Chapter 7, *A glimpse into several complex variables*, while in the last chapter, 8. *Oscillatory integrals and Fourier analysis,* the presentation of this important area of harmonic analysis is based on the properties of the averaging operators, with applications to dispersion equation, Radon transform and to the counting of lattice points.

Each chapter ends with a consistent set of exercises (some of them with hints) related to and completing the main text. Other ones, more challenging, called Problems, are also included.

By completing this ambitious project, the authors, renowned specialists in harmonic analysis, have done a great service to the mathematical community. Undoubtedly that the volumes will become a standard reference for those interested in analysis understand in a large sense, but also for engineers or physicists needing tools from harmonic analysis in their research.

S. Cobzaş

Henri Anciaux, Minimal submanifolds in pseudo-Riemannian geometry, xv+167 pp., World-Scientific, London-Singapore-Beijing 2011, ISBN: 13 978-981-4291-24-8 and 10 981-4291-24-2.

Since the discovery by Lagrange in 1755 of the differential equation satisfied by a minimal surface in the Euclidean space (the catenoid in $\mathbb{R}^3$), the theory of minimal surfaces attracted the attention of many mathematicians, with successive generalizations, from the Euclidean space to Riemannian manifolds. After the introduction of pseudo-Riemannian manifolds as model of the space-time in the relativity theory, it became clear that this is the most general framework to treat this problem. The present book is the first devoted to a systematic exposition of the theory of minimal submanifolds in pseudo-Riemannian manifolds.

A pseudo-Riemannian manifold is a differentiable manifold $\mathcal{M}$ equipped with a smooth bilinear 2-form g, called the metric, which is non-degenerate in the following sense: $\forall Y \in T_x\mathcal{M},\ g(X,Y) = 0 \Rightarrow X = O$, for every vector X in the tangent space $T_x\mathcal{M}$ and every $x \in \mathcal{M}$. The pseudo-Riemannian space of relativity theory, called the Minkowski space and denoted by $\mathbb{R}^4_1$, is the space $\mathbb{R}^4$ equipped with the metric $\langle\cdot,\cdot\rangle_1 = -dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2$. Taking an orthonormal basis $(e_1, \dots, e_m)$ in the tangent space $T_x\mathcal{M}$ it turns out that p of these vectors are negative (i.e. $g(e_i, e_i) < 0$) and $m - p$ are positive (i.e. $g(e_i, e_i) > 0$), and the number p does not depend on the point x. The pair $(p, m-p)$ is called the signature of the pseudo-Riemannian space $\mathcal{M}$ (the Minkowski space has signature (1,3)). The basic results on pseudo-Riemannian manifolds, with emphasis on submanifolds and variation formulae for the volume functional, are developed in the first chapter of the book. The second one is devoted to the case of surfaces, meaning two-dimensional submanifolds, in pseudo-Euclidean space, including a variety of examples and the classification of ruled minimal surfaces, the first global result of the book. This discussion is continued in the third chapter, *Equivariant minimal hypersurfaces*, meaning submanifolds of codimension one, with the study of space forms which are the pseudo-Riemannian analogs of the Riemann round sphere.

Chapter 4, *Pseudo-Kähler manifolds*, is devoted to a special class of pseudo-Riemannian manifolds of even dimension, namely the pseudo-Kähler manifolds, which in the positive case yield the Kähler manifolds. The most natural example of a pseudo-Kähler manifold – the complex pseudo-Riemannian space forms – is presented and one proves that the tangent bundle of a pseudo-Kähler manifold admits a pseudo-Kähler structure.

In the fifth chapter, *Complex and Lagrangian submanifolds in pseudo-Kähler manifolds*, one describes several families of minimal submanifolds in pseudo-Kähler manifolds. The main question addressed in the last chapter of the book, 6, *Minimizing properties of minimal submanifolds*, is whether or not a minimal submanifold, which, by definition, is a critical point of the volume, is actually an extremum of the volume functional. After describing several submanifolds satisfying this requirement, one shows that a necessary condition is that the induced metric of both tangent and normal bundle be definite.

Previously pseudo-Riemannian manifolds were treated only in books directed to physical applications, the present one being the first devoted to an exposition of the basic results on pseudo-Riemannian manifolds and of their minimal submanifolds. Exposing in a clear, live and accessible style (the prerequisites are only familiarity with differentiable manifolds) the book will be of great help to young mathematicians and physicists interested in this topic. It can be used also as a base text for advanced graduate courses.

Cornel Pintea