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Book reviews

Barry Simon, Szegö's Theorem and Its Descendants – Spectral Theory for L^2 Perturbations of Orthogonal Polynomials, Princeton University Press, Princeton and Oxford 2011, x + 650 pp. ISBN-13: 978-0-691-14704-8.

This volume, dealing with orthogonal polynomials on the real line (OPRL), can be considered as complementary to the monumental two volume treatise of the author on orthogonal polynomials on the unit circle $\mathbb{D}$ (OPUC), published by the American Mathematical Society, AMS Colloquium Series, volumes 54.1 and 54.2, Providence RI, 2005. As the author does mention in the Preface, although there are some inevitable overlap between them (mainly in Chapters 2 and 3), the present one is concentrated on topics not contained there. The focus is on sum rules for OPRL, but some results existing at the time when the OPUC volumes were written but of which the author was not aware, are also included. In fact, as remarked Szegö, using the transformations $E : \mathbb{D} \to \mathbb{C} \cup \{\infty\}$, $E(z) = z + z^{-1}$, and $Q = E|_{\partial\mathbb{D}}$, $Q(e^{i\theta}) = 2\sin\theta$, some results on OPUC can be translated to OPRL.

The main goal of the book is to emphasize the deep connections between spectral theory and topics from classical analysis related to OPRL. The author calls a *gem of spectral theory* a theorem putting in one-to-one correspondence a class of spectral data with a class of objects.

The general framework is that of orthogonal polynomials $(P_n)_{n=0}^{\infty}$ with respect to a finite positive measure $d\rho$ on $\mathbb{R}$ having finite moments: $\int |x^n| d\rho(x) < \infty$ for all n. The measure $d\rho$ is called trivial if supp $(d\rho)$ is finite (equivalently, $L^2(\partial\mathbb{D}, d\rho)$ is finite dimensional), and nontrivial otherwise. If $d\rho$ is nontrivial, then $0 < \int |P(x)| d\rho(x) < \infty$ for every polynomial P. The OPRL satisfy a recursion relation $xP_n(x) = P_{n+1}(x) + b_{n+1}P_n(x) + a_n^2 P_{n-1}(x)$, with $P_{-1}(x) \equiv 0$. The first gems of spectral theory presented in the book are the Blumenthal-Weil theorem asserting that for an infinite Jacobi matrix J with coefficients satisfying the conditions (1): $a_n \to 1$ and $b_n \to 0$, the essential spectrum of J is $\sigma_{\text{ess}}(J) = [-2, 2]$, and the converse, the theorem of Denisov and Rakhmanov, asserting that, under some supplementary hypotheses, the equality $\sigma_{\text{ess}}(J) = [-2, 2]$ implies the conditions (1).

Szegö's theorem, published in 1915, alluded to in the title of the book, solves positively a conjecture of Pólya asserting that $\lim_{n\to\infty} D_n(w)^{1/n} = \exp\left(\int \log(w(\theta) \frac{d\theta}{2\pi}\right)$, where $w > 0$ is a weight function and $D_n(w)$ is the determinant of the Toeplitz matrix of order n associated with the moments

$c_k = \int e^{-ik\theta} w(\theta) \frac{d\theta}{2\pi}$, $k \geq 0$. In fact, Szegö proved a stronger result, namely that D_{n+1}/D_n has as limit the written quantity. A detailed proof of this theorem, some extensions and detours are given in Chapter 2, while in Chapter 3, *The Killip-Simon theorem: Szegö theorem for OPRL*, one gives a proof of Szegö theorem for OPRL whose essential support is $[-2, 2]$. Matrix orthogonal polynomials on the real line are discussed in Chapter 4 and periodic OPRL in Chapter 5. Chapter 6, *Toda flows and symplectic structures,* is concerned with the close relations between periodic Jacobi matrices and Toda lattices of dynamical systems. Chapter 7, *Right limits,* contains some results needed for the proofs in Chapter 8 of Szegö and Killip-Simon theorems for periodic OPRL, and in Chapter 9 of Szegö theorem for finite gap OPRL. The last chapter of the book, Chapter 10, *A.C. spectrum for Bethe-Cayley trees,* is concerned with the sum rules in the study of perturbed Laplacians, called Bethe lattices by physicists and Cayley trees by mathematicians. The author adopted a mixed terminology.

The book is clearly written and very well organized - each section ends with a paragraph called *Remarks and Historical Notes,* containing references to bibliography as well as some pertinent and cute remarks of the author. The bibliography counts 465 items with specifications of the pages where each one is cited in the text.

The author is a reputed specialist in the area, most of his contributions and of his students appearing here for the first time in book form.

Presenting a modern approach to some classical problems, relating classical analysis and spectral theory, but with some problems in physics as well, this book together with the AMS volumes on orthogonal polynomials on the unit circle, will become standard references in the field and an invaluable source for further research.

Mirela Kohr

A. Ya. Helemskii, Quantum Functional Analysis. A Non-Coordinate Approach, University Lecture Series, Vol. 56, American Mathematical Society, Providence, Rhode Island 2010, xvii+241 pp, ISBN:978-0-8218-5254-5

The term "quantum space" used in the book is synonym to that of "abstract operator space". As the author explains in the Introduction the aim of the present book is to introduce the "pedestrian" reader to this fascinating area of investigation, being based on the difficulties he encountered when reading the classical texts of the "founding fathers" of the theory - E. G. Effros, G. Pisier, V. Paulsen, U. Haagerup. a.o. The term "quantum" or "non-commutative" means that at an early stage, in some crucial definitions, some commutative objects, functions or scalars, are replaced by "non-commutative" ones, meaning matrices or operators. In the case of a linear space E, written as $\mathbb{C} \otimes E$, the scalars $\mathbb{C}$ are replaced by some "good" operator algebras, and the usual norm by a "quantum norm" (synonym for "operator space structure"). In the present book as good algebras one takes the algebra $\mathcal{F}(L)$ of bounded finite rank operators on a fixed separable Hilbert space L or the algebra $\mathcal{B}(L)$ of bounded operators on L. A natural core notion is that of completely bounded map which makes the whole machinery to work properly. In this way a far reaching generalization of classical functional analysis is obtained and, at a same time, it led to spectacular solutions of some long standing problems in operator theory and in other areas, to quote only the negative solution given by Pisier in 1997 (see, G. Pisier, *Similarity problem and completely bounded maps*, 2nd, Expanded Edition, Lect. Notes. Math. vol. 1618, Springer, Berlin 2001) to the problem of the similarity of polynomially bounded operators, posed by Halmos in his famous paper "Ten problems in Hilbert space", Bulletin AMS 76 (1970).

Although, as the author does mention, in essence a matter of test, the non-coordinate approach (i.e., based on operators) adopted in this book has some advantages over the matrix (coordinate) based approach, or at least it can be an alternative for the presentation given in most of the books on operator spaces.

Some fundamental results of the theory such as Ruan's representation theorem (every abstract operator space can be realized as a concrete operator space), the Hahn-Banach type theorem of Arveson and Wittstock and the decomposition theorem of Paulsen are enounced in Chapter 0. *Three basic definitions and three principal theorems.* Their proofs, rather long, deep and intricate, requiring considerable effort from the reader are postponed to Part III. *Principal theorems revisited in earnest,* of the book.

The presentation is made axiomatically, based on Ruan's axioms, allowing a quick access to some fundamental constructions (as quantum tensor products and duality theory) as well as the presentation of some illuminating examples. This is done in Part I. *The beginning: Spaces and operators,* containing the chapters 1. *Preparing the stage*, 2. *Abstract operator (=quantum) spaces*, 3. *Completely bounded operators*, 4. *The completion of abstract operator spaces*, and Part II. *Bilinear operators, tensor products and duality*, with the chapters 5. *Strongly and weakly completely bounded bilinear operators*, 6. *New preparations: Classical tensor products*, 7. *Quantum tensor products*, and 8. *Quantum duality.*

The author is well-known for his results on the homology of Banach algebras (see the book, A. Ya. Helemskii, *The homology of Banach and topological algebras*, Kluwer A.P., Dordrecht, 1989), as well as for his expository texts as, for instance, *Lectures and exercises on functional analysis*, AMS , Providence, RI, 2005.

Written in a didactic manner, the book contains a very clear presentation of the basic results of quantum functional analysis, accessible to a reader with few experience in the area - the prerequisites are basic functional analysis (for instance, as exposed by ordinary print in author's book mentioned above), some results on modules and bimodules over an algebra and some basic facts on C^*-algebras (in fact, this last field can be avoided by considering them as the algebra of all bounded operators on a Hilbert space).

S. Cobzaş

Peter Kosmol and Dieter Müller-Wichards, Optimization in Function Spaces - with stability considerations in Orlicz spaces, Series in Nonlinear Analysis and Applications, Vol. 13, Walter de Gruyter, Berlin - New York, 2011, xiv + 377 pages, ISBN: 978-3-11-025020-6, e-ISBN 978-3-11-025021-3, ISSN 0941-813X.

The book is concerned with convex optimization in Banach spaces, with emphasis on Orlicz spaces.

The first two chapters, 1. *Approximation in Orlicz spaces,* and 2. *Polya algorithms in Orlicz spaces,* deal with Haar subspaces and Chebyshev alternation theorem in $C(T)$, their extensions to Orlicz spaces and with Polya algorithm for discrete Chebyshev approximation - convergence and stability.

Chapter 3. *Convex sets and convex functions,* contains a fairly complete presentation of basic results about convex functions, including continuity, differentiability (Gâteaux and Fréchet), Fenchel-Moreau duality with applications to optimization problems (existence, characterization, Lagrange multipliers). The fact that the Gâteaux differential of a continuous convex function is demi-continuous is used later (in Chapter 8) to prove that a reflexive and Gâteaux smooth Orlicz space is Fréchet smooth.

The fourth chapter contains an overview of some numerical methods for non-linear and optimization problems (secant and Newton-type methods), a detailed treatment being given in an other monograph by P. Kosmol, *Methoden zur numerischen Behandlung nichtlinearer Gleichungen und Optimierungaufgaben,* B. G. Teubner Studienbücher, Stuttgart, 1993, 2nd ed.

The main tools used in Chapter 5, *Stability and two-stage optimization problems*, are some uniform boundedness results for families of convex functions and convex operators, proved by the first author, and extending the well-known Banach-Steinhaus principle.

Orlicz spaces are studied in chapters 6. *Orlicz spaces*, 7. *Orlicz norms and duality*, and 8. *Differentiability and convexity in Orlicz spaces.* This study includes the Orlicz spaces L^Φ and M^Φ equipped with Luxemburg or Orlicz norms, duality, reflexivity, as well as geometric properties of Orlicz spaces - rotundity and smoothness, Efimov-Stechkin property, with applications to best approximation and optimization, Tikhonov regularization, Ritz method and greedy algorithms.

In the last chapter of the book, 9. *Variational calculus*, one considers minimization with respect to both the state variables x and $\dot{x}$ in some minimization problems. The fundamental theorem of the Calculus of Variations (the Euler-Lagrange equation) is obtained by using some quadratic supplements making the Lagrangian convex in the vicinity of the solution, avoiding in this way the usage of field theory and of Hamilton-Jacobi equations. As application, a detailed treatment of the isoperimetric problem, called by the authors the Dido problem, is included.

The authors are authoritative voices in the area, known for their papers and books (e.g., P. Kosmol, *Optimierung und Approximation*, de Gruyter, Berlin-New York, 2010, 2nd ed, and the book quoted above).

Written in a clear and accessible manner (only mathematical analysis, linear algebra and familiarity with measure theory and functional analysis is required), the book can serve as a base for second half undergraduate or master courses on linear and nonlinear functional analysis, dealing with themes as convex functions and optimization, Orlicz spaces and their geometry, variational calculus.

W. W. Breckner

Siu-Ah Ng, Nonstandard methods in functional analysis - lectures and notes, World Scientific, London - Singapore - Beijing, xxii + 316 pages, ISBN: 13 978-981-4287-54-8 and 10 981-4287-54-7.

The nonstandard analysis has its origins in the 60s in the work of Abraham Robinson in his attempt to put on a rigorous basis Leibniz's differential calculus based on infinitesimal quantities (called monads). The construction, based on techniques from model theory for the first order logic, was presented for the first time in book form by A. Robinson, *Non-standard analysis*, North Holland 1966. Although, at the beginning, the idea was to present nonstandard proofs of known results, soon new results were obtained by nonstandard methods, the most striking being the solution to the invariant space problem for polynomially compact operators obtained by A. R. Bernstein and A. Robinson in 1966. In the same year P. Halmos gave a standard proof to this result. A presentation of the result is given in the book by M. Davis, *Applied nonstandard analysis*, J. Wiley 1977. Some spectacular results in measure theory with applications to probability theory, based on nonstandard methods, were obtained in 1975 by P. Loeb. The methods of nonstandard analysis require some preliminary effort from the newcomer, for which, as remarked A. Uspenski in the authoritative Preface written for the Russian edition (Mir, Moskva 1982) of the above mentioned book of M. Davis, some nonstandard reasonings can look as strange as "*a description of the endocrine systems of griffons and unicorns or of the chemical reactions between the philosophical stone and phlogiston*".

The aim of the present book is to disprove this impression and to show that, once acquainted, the methods of nonstandard analysis lead to more transparent and intuitive proofs of known results in functional analysis, and to new results as well.

The first chapter contains an overview of the basic methods and tools of nonstandard analysis (extensions and ultraproduct techniques) with applications to elementary calculus, topology and measure theory. In the second chapter, *Banach spaces,* the author passes to the presentation of basic results on normed spaces - nonstandard hulls, linear operators, Hahn-Banach theorem, weak compactness, reflexivity. Two topics that fit very well with nonstandard methods are finite representability and superreflexivity. The exposure continues in Chapter 3 with a presentation of basic results on Banach algebras. The last chapter of the book, Chapter 4, *Selected research topics*, is concerned with fixed points, noncommutative Loeb measures, Hilbert space-valued integrals.

The book is a good introduction to nonstandard methods in functional analysis and can serve as a base text for master courses or for self-study.

S. Cobzaş

Kenneth Kuttler, Calculus - Theory and applications,
World Scientific, London - Singapore - Beijing, 2011
Volume I, xii + 480 pages, ISBN: 13 978-981-4329-69-9 (pbk) and 10 981-4329-69-X (pbk).
Volume II, xii + 410 pages, ISBN: 13 978-981-4329-70-5 (pbk) and 10 981-4329-70-3(pbk).

This is a comprehensive course on Calculus of functions of one and of several variables. To make the book self-contained (as much as possible), the author has included in the first chapter of the first volume, *A short review of the precalculus*, some supplementary material, mainly from linear algebra and geometry (some trigonometry is included as well). This volume contains also the elements of calculus of functions of one variable - limits, continuity, derivatives, antiderivatives, some elementary differential equations, the Riemann integral and infinite series.

Chapters 12, *Fundamentals*, and 13, *Vector products*, are devoted to vector calculus in $\mathbb{R}^n$ including the dot and the cross products. The last chapter of the first volume, 13, S*ome curvilinear coordinate systems*, beside some results on curvilinear coordinates (as, e.g., graphs and area in polar coordinates) contains also an exposition of Kepler's laws on the planetary motion, completed in Appendix B with a presentation of Newton's laws of motion. Appendix A is devoted to some results in plane geometry and trigonometry. This topic is considered again in Appendix F of the second volume with the study of Christoffel symbols, curl and cross products in curvilinear coordinates

The second volume is devoted to the calculus of functions of several variables. As the linear algebra is the skeleton of the n-dimensional calculus and at the same time furnishes a lot of useful tools, the first three chapters 1, *Matrices and linear transformations*, 2, *Determinants*, and 3, *Spectral theory* (a study of eigenvalues and eigenvectors), are devoted to this topic, completed in Appendix A, *The mathematical theory of determinants*, with rigorous proofs of the properties of determinants and applications to the diagonalization of matrices.

Chapter 4, *Vector valued functions*, is concerned with limits and continuity properties. A special chapter (Chapter 5) is devoted to vector functions of one real variable (derivatives and integrals) with applications in Chapter 6, *Motion on a space curve*, to spatial curves and their geometry.

The differential calculus of functions of several variables is developed in Chapters 7, *Functions of many variables*, and 8, *The derivatives of functions of many variables*, with applications, in Chapter 9, *Optimization*, to local and conditioned extrema. This study is completed in Appendix B with a proof of the implicit function theorem with applications to local structure of differentiable functions.

The basic notions and tools of the Riemann integral in $\mathbb{R}^n$ are given in chapters 10, *The Riemann integral* and 11, *The integral in other coordinates*, at an informal level, the rigorous proofs (including a proof of the change of variables formula) being postponed to Appendices C, *The theory of Riemann integral*, and D, *Volumes in higher dimensions* (the function Gamma and the volume of balls in $\mathbb{R}^n$).

The integral on two dimensional surfaces in $\mathbb{R}^3$ is treated in Chapter 12, while Chapter 13 is concerned with the calculus of vector fields. Other physical applications (as, e.g., the Coriolis acceleration of the rotating earth) are given in Appendix E.

The book is written in a very didactic manner reflecting the teaching experience of the author - one starts with examples and particular cases before presenting the general case and rigorous proofs (most being given in appendices).

There are a lot of interesting concrete examples (some of them included in the exercises) from physics, mechanics, astronomy, economics, some of them presented in an entertaining amazing way. Each chapter ends with a set of well chosen exercises, many having answers or hints at the end of each volume. The most challenging are marked by *. A web page, *http://www.byu.edu/ klkuttle/CalculusMaterial*, contains supplementary material and routine exercises.

The author is well known for his books on calculus and linear algebra, from which *Modern Calculus*, CRC Press, Boca Raton 1998, contains more advanced topics.

These volumes, written in a live and accessible but at the same time rigorous style, can be used for basic courses in calculus (or linear algebra), at beginning or advanced levels.

Tiberiu Trif

I. Meghea, Ekeland variational principle: with generalizations and variants, Old City Publishing, Philadelphia and Éditions des Archives Contemporaines, Paris, 2009, iv+524 pp. ISBN: 978-1-933153-08-7; 978-2-914610-96-4

The variational principle discovered by Ivar Ekeland in 1972 (called in what follows EkVP) turned out to be a powerful and versatile tool in many branches of mathematics – Banach space geometry, optimization, economics, etc. This was masterly illustrated by Ekeland in the survey paper from the Bulletin of the American Mathematical Society 39 (2002), no. 2, pp. 207265, and proved by subsequent developments. In fact, EkVP is a paradigm of a maximality principle used by E. Bishop and R. Phelps in the proof of their famous result on the density of support functionals and is also related to the Brezis-Browder maximality principle. Nowadays there are a lot of variational principles originating from EkVP: smooth variational principles, vector variational principles, perturbed variational principles. After more than 30 years since the discovery of EkVP, the present monograph demonstrates that this principle could be considered as the landmark of modern variational calculus.

The first chapter, I. *Ekeland variational principle*, is devoted to the presentation of the original Ekeland variational principle in complete metric spaces and of some equivalent results – the drop and flower petal theorems, the Bishop-Phelps theorem, Caristi-Kirk fixed point theorem. Applications to minimax-type theorems in Banach space and on Finsler manifolds and to Clarke's subdifferential calculus for locally Lipschitz functions are included. This chapter contains also some extensions of EkVP – vector variants of EkVP, EkVP in locally convex spaces, in uniform spaces and in probabilistic metric spaces.

The second chapter II. *Smooth variational principles*, is concerned with Borwein-Preiss and Deville-Godefroy-Zizler smooth variational principles and Ghoussoub-Maurey perturbed variational principle. The chapter ends with a variational principle, proved by Yongxin Li and Shuzhong Shi (2000), unifying and generalizing both EkVP and Borwein-Preiss variational principle. As it is shown in the book EkVP cannot be recovered in its full generality from Borwein-Preiss variational principle.

Beside the bibliography referred to in the main text, the book contains also an Additional Bibliography with brief presentations of some results appeared after the book was written or which do not fit the general context.

Collecting the essential results on variational principles and presenting them in a coherent and didactic way (there are a lot of improvements of the results taken from various sources and corrections of the proofs, belonging to the author), the book is a very useful reference for researchers in this area as well as for those interested in applications. By the detailed presentation of the subject the book can be used also by the newcomers or as a support for an advanced course in nonlinear analysis.

S. Cobzaş