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INFINITELY MANY SOLUTION FOR A NONLINEAR NAVIER BOUNDARY VALUE PROBLEM INVOLVING THE p-BIHARMONIC

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Abstract. The existence of infinitely many solutions is established for a class of nonlinear elliptic equations involving the p-biharmonic operator and under Navier boundary value conditions. The approach adopted is fully based on critical point theory.

1. **Introduction**

In this paper, we are interested in studying the existence of infinitely many solutions for the following nonlinear elliptic Navier boundary value problem involving the p-biharmonic

$$\begin{cases} \Delta(|\Delta u|^{p-2}\Delta u) = \lambda f(x,u) & \text{in } \Omega \\ u = \Delta u = 0 & \text{on } \partial\Omega, \end{cases} \tag{1.1}$$

where Ω is an open bounded subset of $\mathbf{R}^N$ with a smooth enough boundary $\partial\Omega$, $(N \geq 1)$, $p > \max\{1, N/2\}$, Δ is the usual Laplace operator, λ is a positive parameter and $f \in C^0(\bar{\Omega} \times \mathbf{R})$.

In these latest years, many authors looked for multiple solutions of boundary value problems involving biharmonic and p-biharmonic type operators, see for instance [5], [11], [12], [14] and the references cited therein.

More precisely, in [10], assuming that $f(x,\cdot)$ is odd and by using the Symmetric Mountain Pass Theorem of Ambrosetti-Rabinowitz, the existence of infinitely many solutions for nonlinear elliptic equations with a general p-biharmonic type operator and under either Navier or Dirichlet boundary conditions has been obtained. In [9], see also [13], requiring that the nonlinearity f is the sum of an odd term and a non-odd perturbation, via perturbation theory, the existence of infinitely many sign-changing solutions for problem (1.1) for $p = 2$ and $N \geq 5$ has been established.

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Moreover, in such frameworks, some additional suitable growth conditions, for example that f is p-sublinear at zero and p-superlinear at infinity, are supposed.

Here, we achieve our goal under different assumptions on f which turn out to be mutually independent with respect to those adopted on the above mentioned papers, see Examples 3.3 and 3.7. In particular, we obtain well precise intervals of parameters such that problem (1.1) admits either an unbounded sequence of solutions (Theorem 3.1) provided that f has a suitable behaviour at infinity or a sequence of non-zero solutions (Theorem 3.8) strongly converging to zero if a similar behaviour occurs at zero. Moreover, we explicitly observe that in the autonomous case (Theorem 3.4) our conclusions are sharped (Remark 3.5).

On the other hand, it is worth noticing that the results contained in [4], where the authors required that the nonlinearity changes sign in a suitable way, are included in the case $\alpha = 0$ and $\beta = \infty$ treated here (Remark 3.2), where the nonlinearity can also be nonnegative (Corollary 3.6). This is due to the fact that we use a more precise version of Ricceri's variational principle [7], given by Bonanno and Molica Bisci in [1]. Very recently, the same approach adopted here has also been followed in [3] to look for infinitely many solutions for a fourth order equation in the one dimensional case which, as particular case, contains problem (1.1) with $p = 2$.

For general references and for a complete and exhaustive overview on variational methods we refer the reader to the excellent monographs [6] and [8].

2. **Preliminaries**

Here and in the sequel Ω is an open bounded subset of $\mathbf{R}^N$ $(N \geq 1)$, $p > \max\{1, N/2\}$, while X denotes the space $W^{2,p}(\Omega) \cap W_0^{1,p}(\Omega)$ endowed with the norm

$$\|u\| = \left(\int_\Omega |\Delta u(x)|^p dx \right)^{1/p} \qquad \forall u \in X. \tag{2.1}$$

The Rellich Kondrachov Theorem assures that X is compactly imbedded in $C^0(\bar{\Omega})$, being

$$k := \sup_{u \in X \setminus \{0\}} \frac{\|u\|_{C^0(\bar{\Omega})}}{\|u\|} < +\infty. \tag{2.2}$$

Let $f \in C^0(\bar{\Omega} \times \mathbf{R})$ and let us put

$$F(x,t) := \int_0^t f(x,\xi) d\xi \quad \forall (x,t) \in \bar{\Omega} \times \mathbf{R}.$$

For our approach we will use the functionals $\Phi, \Psi : X \to \mathbf{R}$ defined by putting

$$\Phi(u) := \frac{1}{p}\|u\|^p, \quad \Psi(u) := \int_\Omega F(x, u(x))dx$$

for every $u \in X$. It is simple to verify that Φ and Ψ are well defined, as well as Gâteaux differentiable. Moreover, in view of the fact that Φ is continuous and convex, it turns out sequentially weakly lower semicontinuous, while, since Ψ has compact derivative, it results sequentially weakly continuous. In particular, one has

$$\Phi'(u)(v) = \int_\Omega |\Delta u(x)|^{p-2}\Delta u(x)\Delta v(x)dx, \quad \Psi'(u)(v) = \int_\Omega f(x,u(x))v(x)dx$$

for every $u, v \in X$.

We explicitly observe that, in view of (2.2), one has that, for every $r > 0$

$$\Phi^{-1}(]-\infty, r[) := \{u \in X :\ \Phi(u) < r\} \subseteq \{u \in C^0(\bar{\Omega}) :\ \|u\|_{C^0} < k(pr)^{1/p}\}. \quad (2.3)$$

Finally, if we recall that a weak solution of problem (1.1) is a function $u \in X$ such that

$$\int_\Omega |\Delta u(x)|^{p-2}\Delta u(x)\Delta v(x)dx - \lambda \int_\Omega f(x,u(x))v(x)dx = 0 \quad \forall v \in X,$$

it is obvious that our goal is to find critical points of the functional $\Phi - \lambda\Psi$. For this aim, our main tool is a general critical points theorem due to Bonanno and Molica Bisci (see [1]) that is a generalization of a previous result of Ricceri [7] and that here we state in a smooth version for the reader's convenience.

Theorem 2.1. *Let X be a reflexive real Banach space, let $\Phi, \Psi : X \to \mathbf{R}$ be two Gâteaux differentiable functionals such that Φ is sequentially weakly lower semicontinuous and coercive and Ψ is sequentially weakly upper semicontinuous. For every $r > \inf_X \Phi$, let us put*

$$\varphi(r) := \inf_{u \in \Phi^{-1}(]-\infty,r[)} \frac{\left(\sup_{v \in \Phi^{-1}(]-\infty,r[)} \Psi(v)\right) - \Psi(u)}{r - \Phi(u)}$$

and

$$\gamma := \liminf_{r \to +\infty} \varphi(r), \quad \delta := \liminf_{r \to (\inf_X \Phi)^+} \varphi(r).$$

Then, one has

(a) *for every $r > \inf_X \Phi$ and every $\lambda \in \left]0, \frac{1}{\varphi(r)}\right[$, the restriction of the functional $I_\lambda = \Phi - \lambda\Psi$ to $\Phi^{-1}(]-\infty, r[)$ admits a global minimum, which is a critical point (local minimum) of I_λ in X.*

(b) *If $\gamma < +\infty$ then, for each $\lambda \in \left]0, \frac{1}{\gamma}\right[$, the following alternative holds: either*

(b_1) I_λ possesses a global minimum,

or

(b_2) there is a sequence $\{u_n\}$ of critical points (local minima) of I_λ such that $\lim_{n\to+\infty} \Phi(u_n) = +\infty$.

(c) *If $\delta < +\infty$ then, for each $\lambda \in \left]0, \frac{1}{\delta}\right[$, the following alternative holds: either*

(c_1) *there is a global minimum of Φ which is a local minimum of I_λ,*
or
(c_2) *there is a sequence of pairwise distinct critical points (local minima) of I_λ which weakly converges to a global minimum of Φ.*

3. **Main results**

Fixed $x^0 \in \Omega$, let us pick $0 < s_1 < s_2$ such that $B(x^0, s_2) \subseteq \Omega$ and put

$$L := \frac{\Gamma\left(1 + \frac{N}{2}\right)}{\pi^{N/2}} \left(\frac{s_2^2 - s_1^2}{2Nk}\right)^p \frac{1}{s_2^N - s_1^N}, \tag{3.1}$$

where Γ denotes the Gamma function and k is defined in (2.2).

Theorem 3.1. *Assume that*

(i_1) *$F(x,t) \geq 0$ for every $(x,t) \in \Omega \times [0, +\infty[$;*
(i_2) *There exist $x^0 \in X$, $0 < s_1 < s_2$ as considered in (3.1) such that, if we put*

$$\alpha := \liminf_{t \to +\infty} \frac{\int_\Omega \max_{|\xi| \leq t} F(x,\xi) dx}{t^p}, \qquad \beta := \limsup_{t \to +\infty} \frac{\int_{B(x^0, s_1)} F(x,t) dx}{t^p},$$

one has

$$\alpha < L\beta. \tag{3.2}$$

Then, for every $\lambda \in \Lambda := \frac{1}{pk^p}\left]\frac{1}{L\beta}, \frac{1}{\alpha}\right[$ problem (1.1) admits an unbounded sequence of weak solutions.

Proof. With the purpose of applying Theorem 2.1, we begin observing that, for every $r > 0$, taking in mind (2.3), one has

$$\varphi(r) \leq \frac{\sup_{\Phi^{-1}(]-\infty, r[)} \Psi}{r} \leq \frac{\int_\Omega \max_{|\xi| \leq k(pr)^{1/p}} F(x,\xi) dx}{r}. \tag{3.3}$$

At this point, we consider a sequence $\{t_n\}$ of positive numbers such that $t_n \to +\infty$ and

$$\lim_{n \to +\infty} \frac{\int_\Omega \max_{|\xi| \leq t_n} F(x,\xi) dx}{t_n^p} = \alpha. \tag{3.4}$$

For every $n \in \mathbf{N}$ let us consider $r_n = \frac{1}{p}\left(\frac{t_n}{k}\right)^p$. Putting together (3.3), (3.4) and (3.2) one has

$$\gamma \leq \liminf_{n \to +\infty} \varphi(r_n) \leq pk^p \lim_{n \to +\infty} \frac{\int_\Omega \max_{|\xi| \leq t_n} F(x,\xi) dx}{t_n^p} < +\infty. \tag{3.5}$$

Moreover, we can also observe that, owing to (3.4) and (3.5),

$$\Lambda \subseteq \left]0, \frac{1}{\gamma}\right[.$$

Fix $\lambda \in \Lambda$ and claim that

$$\Phi - \lambda\Psi \ \text{ is unbounded from below.} \tag{3.6}$$

Indeed, since $\frac{1}{\lambda} < pk^p L\beta$, we can consider a sequence $\{\tau_n\}$ of positive numbers and $\eta > 0$ such that $\tau_n \to +\infty$ and

$$\frac{1}{\lambda} < \eta < pk^p L \frac{\int_{B(x^0,s_1)} F(x,\tau_n)dx}{\tau_n^p} \tag{3.7}$$

for every $n \in \mathbf{N}$ large enough. Let $\{w_n\}$ be a sequence in X defined by putting

$$w_n(x) = \begin{cases} \tau_n & \text{if } x \in B(x^0,s_1) \\ \frac{\tau_n}{s_2^2 - s_1^2}[s_2^2 - \sum_{i=1}^N (x_i - x_i^0)^2] & \text{if } x \in B(x^0,s_2) \setminus B(x^0,s_1) \\ 0 & \text{if } x \in \Omega \setminus B(x^0,s_2). \end{cases} \tag{3.8}$$

Fixed $n \in \mathbf{N}$, a simple computation shows that

$$\Phi(w_n) = \frac{1}{p}\left(\frac{2N\tau_n}{s_2^2 - s_1^2}\right)^p \frac{\pi^{N/2}}{\Gamma(1+N/2)}(s_2^N - s_1^N) = \frac{\tau_n^p}{pk^p L}. \tag{3.9}$$

On the other hand, thanks to assumption (i$_1$), one has

$$\Psi(w_n) = \int_\Omega F(x, w_n(x))dx \geq \int_{B(x^0,s_1)} F(x,\tau_n)dx. \tag{3.10}$$

According to (3.9), (3.10) and (3.7) we achieve

$$\Phi(w_n) - \lambda\Psi(w_n) \leq \frac{\tau_n^p}{pk^p L} - \lambda \int_{B(x^0,s_1)} F(x,\tau_n)dx < \frac{\tau_n^p}{pk^p L}(1 - \lambda\eta)$$

for every $n \in \mathbf{N}$ large enough. Hence, (3.6) holds.

The alternative of Theorem 2.1 (case (b)) assures the existence of an unbounded sequence $\{u_n\}$ of critical points of the functional $\Phi - \lambda\Psi$ and the proof is complete in view of the considerations made in the previous section. □

Remark 3.2. We explicitly observe that it is easier to verify assumption (3.2) provided that $\alpha = 0$ and $\beta = +\infty$ and of course in this case the interval Λ becomes $]0, +\infty[$. This situations occurs, for instance, in [4].

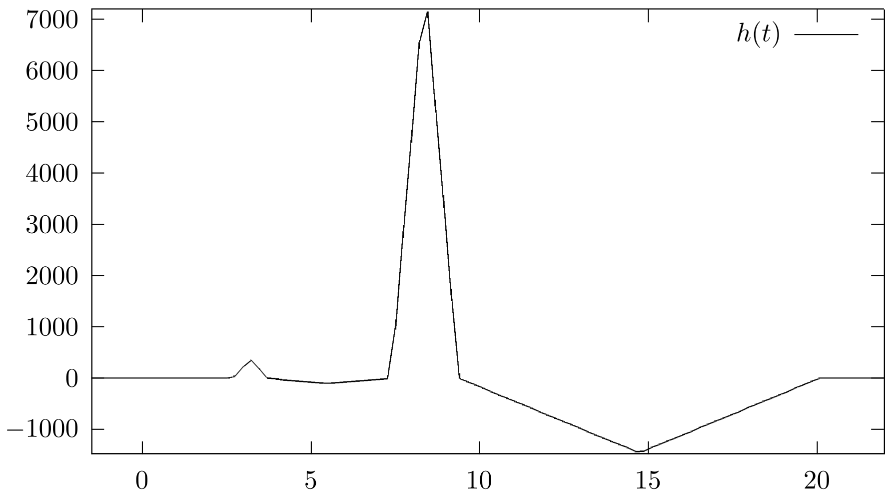
Example 3.3. Let Ω be an open, bounded subset of $\mathbf{R}^2$ and $g \in C^0(\bar{\Omega}) \setminus \{0\}$ a nonnegative function. Put

$$a_n := e^{n!}, \qquad b_n := e^{n!} + n, \qquad c_n := (a_n + b_n)/2, \qquad d_n := (b_n + a_{n+1})/2$$

for every $n \in \mathbf{N}^* := \mathbf{N} \setminus \{0\}$ and define the following function

$$h(t) := \begin{cases} \sum_{n\in\mathbf{N}^*} \mathbf{1}_{[a_n,b_n[} \frac{b_n^4}{c_n-a_n} \left(1 - \frac{|t-c_n|}{c_n-a_n}\right) & \text{if } t \in \cup_{n\in\mathbf{N}^*}[a_n, b_n[, \\ \sum_{n\in\mathbf{N}^*} \mathbf{1}_{[b_n,a_{n+1}[} \frac{-b_n^4}{d_n-b_n} \left(1 - \frac{|t-d_n|}{d_n-b_n}\right) & \text{if } t \in \cup_{n\in\mathbf{N}^*}[b_n, a_{n+1}[, \\ 0 & \text{otherwise ,} \end{cases}$$

where the symbol $\mathbf{1}_{[r,s[}$ denotes the characteristic function of the interval $[r, s[$. A qualitative graph of h is shown in the figure below.



Moreover, let us put

$$f(x, t) := g(x)h(t), \tag{3.11}$$

for every $(x, t) \in \bar{\Omega} \times \mathbf{R}$. Hence, one has that

$$F(x, t) = \int_0^t f(x, \xi)d\xi = g(x)H(t)$$

for every $(x, t) \in \bar{\Omega} \times \mathbf{R}$, where

$$H(t) = \int_0^t h(\tau)d\tau \qquad \forall t \in \mathbf{R}.$$

It is easy to verify that, for every $n \in \mathbf{N}^*$,

$$\int_{a_n}^{b_n} h(\tau)d\tau = b_n^4 \quad \text{and} \quad \int_{b_n}^{a_{n+1}} h(\tau)d\tau = -b_n^4.$$

From this, a simple computation gives

$$H(a_n) = 0, \quad \int_\Omega \max_{|\xi|\le a_{n+1}} F(x,\xi)dx = H(b_n)\int_\Omega g(x)dx = b_n^4 \int_\Omega g(x)dx.$$

Hence,

$$\alpha \le \int_\Omega g(x)dx \lim_{n\to+\infty} \frac{b_n^4}{a_{n+1}^3} = 0.$$

Moreover, let $x^0 \in \Omega$ such that $g(x^0) > 0$, fix $s_1 > 0$ such that $B(x^0, s_1) \subset \Omega$ and $g(x) > 0$ for every $x \in B(x^0, s_1)$, one has

$$\beta \ge \int_{B(x^0,s_1)} g(x)dx \lim_{n\to+\infty} \frac{H(b_n)}{b_n^3} = +\infty.$$

Applying Theorem 3.1, we can conclude that, for every $\lambda > 0$ the following problem

$$\begin{cases} \Delta(|\Delta u|\Delta u) = \lambda g(x)h(u) & \text{in } \Omega \\ u = \Delta u = 0 & \text{on } \partial\Omega, \end{cases}$$

admits an unbounded sequence of weak solutions.

In order to give the best formulation of the previous Theorem 3.1 in the autonomous case, let us observe that the function $s : \bar{\Omega} \to \mathbf{R}_0^+$ defined by

$$s(x) = d(x, \partial\Omega) \qquad \forall x \in \bar{\Omega}$$

is Lipschitz continuous. Hence, there exists $y^0 \in \Omega$ such that

$$\bar{s} = s(y^0) = \max_{x\in\Omega} s(x),$$

that is $\bar{s}$ is the biggest possible radius among all the balls contained in Ω.

Moreover, let $\bar{\mu} \in]0,1[$ be the point where the function $\frac{\mu^N(1-\mu^2)^p}{1-\mu^N}$ attains its maximum in $]0,1[$.

Theorem 3.4. *Let $h : \mathbf{R} \to \mathbf{R}$ be a continuous function such that:*

(i$_1$)$'$ $H(t) = \int_0^t h(\xi)dx \ge 0$ *for every* $t \in [0, +\infty[$*;*

(i$_2$)$'$ *Put*

$$\alpha' := \liminf_{t\to+\infty} \frac{\max_{|\xi|\le t} H(t)}{t^p}, \qquad \beta' := \limsup_{t\to+\infty} \frac{H(t)}{t^p},$$

one has

$$\alpha' < L'\beta',$$

where

$$L' = \frac{\bar{s}^{2p}}{(2Nk)^p|\Omega|} \; \frac{\bar{\mu}^N(1-\bar{\mu}^2)^p}{1-\bar{\mu}^N}. \tag{3.12}$$

Then, for every $\lambda \in \dfrac{1}{pk^p|\Omega|}\left]\dfrac{1}{L'\beta'}, \dfrac{1}{\alpha'}\right[$ *the following problem*

$$\begin{cases} \Delta(|\Delta u|^{p-2}\Delta u) = \lambda h(u) & \text{in } \Omega \\ u = \Delta u = 0 & \text{on } \partial\Omega, \end{cases} \tag{3.13}$$

admits an unbounded sequence of weak solutions.

Proof. Put $x^0 = y^0$, $s_2 = \bar{s}$, $s_1 = \bar{\mu}\bar{s}$ and $f(x,t) = h(t)$ for every $(t,x) \in \bar{\Omega} \times \mathbf{R}$. Obviously $(\mathrm{i}_1)'$ implies (i_1). Moreover,

$$\alpha = |\Omega|\alpha', \quad \beta = \frac{\pi^{N/2}}{\Gamma(1+N/2)}(\bar{s}\bar{\mu})^N\beta', \quad L = \frac{|\Omega|\Gamma(1+N/2)}{\pi^{N/2}(\bar{\mu}\bar{s})^N}L'.$$

Hence, in view of $(\mathrm{i}_2)'$, one has

$$\alpha < |\Omega|L'\beta' = L\beta,$$

that is (i_2) holds and the conclusion follows directly from Theorem 3.1. $\square$

Remark 3.5. It is worth noticing that in our framework, whenever $\beta' < +\infty$, taking in mind the properties of $\bar{s}$ and $\bar{\mu}$, the choice of such a L' is the best possible.

An immediate consequence of Theorem 3.4 is the following

Corollary 3.6. *Let* $h : \mathbf{R} \to \mathbf{R}$ *be a continuous and nonnegative function such that*

$$\liminf_{t\to+\infty} \frac{H(t)}{t^p} < L' \limsup_{t\to+\infty} \frac{H(t)}{t^p}, \tag{3.14}$$

being L' *defined in (3.12). Then, for every*

$$\lambda \in \Lambda' := \frac{1}{pk^p|\Omega|}\left]\frac{1}{L'\limsup_{t\to+\infty}\frac{H(t)}{t^p}}, \frac{1}{\liminf_{t\to+\infty}\frac{H(t)}{t^p}}\right[,$$

problem (3.13) admits an unbounded sequence of weak solutions.

Proof. It follows from Theorem 3.4 observing that, in view of the nonnegativity of h, $(\mathrm{i}_1)'$ holds and $\alpha' = \liminf_{t\to+\infty}\frac{H(t)}{t^p}$. $\square$

Example 3.7. Let $\Omega =]0,4[$, $p = 2$ and $h : \mathbf{R} \to \mathbf{R}$ be a function defined by putting

$$h(t) := \begin{cases} 2t\left(1 + 2\sin^2(\ln t) + 2\sin(\ln t)\cos(\ln t)\right) & \text{if } t \in]0,+\infty[, \\ 0 & \text{if } t \in]-\infty, 0]. \end{cases}$$

Obviously, h is continuous and nonnegative. Moreover,

$$H(t) = \int_0^t h(\xi)d\xi = \begin{cases} t^2\left(1 + 2\sin^2(\ln t)\right) & \text{if } t \in]0,+\infty[, \\ 0 & \text{if } t \in]-\infty, 0]. \end{cases}$$

Hence, putted $a_n = e^{n\pi}$ and $b_n = e^{\frac{2n+1}{2}\pi}$ for every $n \in \mathbf{N}$, one has that

$$\liminf_{t\to+\infty} \frac{H(t)}{t^p} \leq \lim_{n\to+\infty} \frac{H(a_n)}{a_n^2} = 1 \tag{3.15}$$

and

$$\limsup_{t\to+\infty} \frac{H(t)}{t^p} \geq \lim_{n\to+\infty} \frac{H(b_n)}{b_n^2} = 3. \tag{3.16}$$

In view of Proposition 2.1 of [2], one has that

$$k \leq \frac{1}{2\pi}. \tag{3.17}$$

Hence, from (3.12), (3.17) and the definition of $\bar{\mu}$, it follows that

$$L' = \frac{1}{k^2}\bar{\mu}(1-\bar{\mu})(1+\bar{\mu})^2 > \frac{9}{4}\pi^2. \tag{3.18}$$

Putting together (3.15), (3.16) and the definition of L', it is simple to verify that condition (3.14) holds, as well as

$$\left]\frac{2}{27}, \frac{\pi^2}{2}\right[\subset \Lambda'.$$

Finally, applying Corollary 3.6, one has that for every $\lambda \in \left]\frac{2}{27}, \frac{\pi^2}{2}\right[$ the following problem

$$\begin{cases} u^{iv} = \lambda h(u) & \text{in }]0,4[, \\ u(0) = u(4) = 0, \\ u''(0) = u''(4) = 0, \end{cases}$$

admits an unbounded sequence of weak solutions.

Similar reasonings assure the existence of infinitely many weak solutions to problem (1.1) converging at zero. More precisely, the following result holds.

Theorem 3.8. *Assume that* (i_1) *is satisfied. Suppose that*

(j_2) *There exist $x^0 \in X$, $0 < s_1 < s_2$ as considered in (3.1) such that, if we put*

$$\alpha^0 := \liminf_{t\to 0^+} \frac{\int_\Omega \max_{|\xi|\leq t} F(x,\xi)dx}{t^p}, \qquad \beta^0 := \limsup_{t\to 0^+} \frac{\int_{B(x^0,s_1)} F(x,t)dx}{t^p},$$

one has

$$\alpha^0 < L\beta^0. \tag{3.19}$$

Then, for every $\lambda \in \frac{1}{pk^p}\left]\frac{1}{L\beta^0}, \frac{1}{\alpha^0}\right[$ problem (1.1) admits a sequence $\{u_n\}$ of weak solutions such that $u_n \to 0$.

Proof. Once observed that $\min_X \Phi = \Phi(0) = 0$, let $\{t_n\}$ be a sequence of positive numbers such that $t_n \to 0^+$ and

$$\lim_{n\to+\infty} \frac{\int_\Omega \max_{|\xi|\le t_n} F(x,\xi)dx}{t_n^p} = \alpha^0 < +\infty. \tag{3.20}$$

Putting $r_n = \frac{1}{p}\left(\frac{t_n}{k}\right)^p$ for every $n \in \mathbf{N}$ and working as in the proof of Theorem 3.1, it follows that $\delta < +\infty$.

Fix now $\lambda \in \frac{1}{pk^p}\left]\frac{1}{L\beta^0}, \frac{1}{\alpha^0}\right[$ and claim that

$$\Phi - \lambda\Psi \;\text{ has not a local minimum at zero.} \tag{3.21}$$

Let $\{\tau_n\}$ be a sequence of positive numbers and $\eta > 0$ such that $\tau_n \to 0^+$ and

$$\frac{1}{\lambda} < \eta < pk^p L \frac{\int_{B(x^0,s_1)} F(x,\tau_n)dx}{\tau_n^p} \tag{3.22}$$

for every $n \in \mathbf{N}$ large enough. Let $\{w_n\}$ be the sequence in X defined in (3.8). Putting together (3.9), (3.10) and (3.22) we achieve

$$\Phi(w_n) - \lambda\Psi(w_n) < \frac{\tau_n^p}{pk^p L}(1-\lambda\eta) < 0 = \Phi(0) - \lambda\Psi(0)$$

for every $n \in \mathbf{N}$ large enough, that implies claim (3.21) in view of the fact that $\|w_n\| \to 0$.

The alternative of Theorem 2.1 (case (c)) completes the proof. □

Remark 3.9. In the same spirit of the previous Theorem 3.8, it could be possible to obtain suitable versions of Theorem 3.4, as well as Corollary 3.6, when the '$\liminf$' and the '$\limsup$' are considered for $t \to 0^+$, in order to assure the existence of arbitrarily small weak solutions of problem (3.13).

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