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CONVOLUTIONS OF UNIVALENT FUNCTIONS WITH NEGATIVE COEFFICIENTS USING A GENERALIZED SĂLĂGEAN OPERATOR

ADRIANA CĂTAŞ

Dedicated to Professor Grigore Ştefan Sălăgean on his 60^{th} *birthday*

Abstract. The object of this paper is to derive several interesting properties of the class $\mathcal{T}_j(n,\gamma,\alpha,\lambda)$ consisting of analytic and univalent functions with negative coefficients. Integral operators and modified Hadamard products of several functions belonging to the class $\mathcal{T}_j(n,\gamma,\alpha,\lambda)$ are studied.

1. Introduction and definitions

Let N denote the set of nonnegative integers $\{0,1,2,\dots,n,\dots\}$, $\mathbb{N}^*=\mathbb{N}\setminus\{0\}$ and let N_j, $j\in\mathbb{N}^*$, be the class of functions of the form

$$f(z)=z-\sum_{k=j+1}^{\infty}a_kz^k,\quad a_k\ge 0,\ k\ge j+1, \tag{1.1}$$

which are analytic in the open unit disc $U=\{z\in\mathbb{C}:\ |z|<1\}$.

We define the following generalized Sălăgean operator which has been introduced by Al-Oboudi in [1]

$$D^0f(z)=f(z) \tag{1.2}$$

$$D^1_\lambda f(z)=(1-\lambda)f(z)+\lambda zf'(z)=D_\lambda f(z),\quad \lambda>0 \tag{1.3}$$

$$D^n_\lambda f(z)=D_\lambda(D^{n-1}_\lambda f(z)). \tag{1.4}$$

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If f is given by (1.1), then (1.2), (1.3) and (1.4) yield to a convolution with the functions

$$\psi(n,\lambda) = z - \sum_{k=j+1}^{\infty} [1+(k-1)\lambda]^n z^k$$

$$D_\lambda^n f(z) = \psi(n,\lambda) * f(z) = z - \sum_{k=j+1}^{\infty} c_k(n,\lambda) z^k$$

where

$$c_k(n,\lambda) = [1+(k-1)\lambda]^n, \quad \lambda \geq 0,\ n = 0,1,2,\ldots \tag{1.5}$$

When $\lambda = 1$ we get Sălăgean differential operator [8].

Definition 1.1. [6] Let $\alpha, \gamma \in [0,1)$, $n \in \mathbb{N}$, $j \in \mathbb{N}^*$. A function f belonging to N_j is said to be in the class $\mathcal{T}_j(n,\gamma,\alpha,\lambda)$ if and only if

$$\mathrm{Re}\, \frac{D_\lambda^{n+1} f(z)/D_\lambda^n f(z)}{\gamma(D_\lambda^{n+1} f(z)/D_\lambda^n f(z)) + 1 - \gamma} > \alpha, \quad z \in U. \tag{1.6}$$

Remark 1.2. The class $\mathcal{T}_j(n,\gamma,\alpha,\lambda)$ is a generalization of the subclasses

i) $\mathcal{T}_1(0,0,\alpha,1) = \mathcal{T}^*(\alpha)$ and $\mathcal{T}_1(1,0,\alpha,1) = C(\alpha)$ defined and studied by Silverman [10] (these classes are the class of starlike functions of order α with negative coefficients and the class of convex functions of order α with negative coefficients respectively);

ii) $\mathcal{T}_j(0,0,\alpha,1)$ and $\mathcal{T}_j(1,0,\alpha,1)$ studied by Chatterjea [4] and Srivastava et al. [11];

iii) $\mathcal{T}_1(n,0,\alpha,1) = \mathcal{T}(n,\alpha)$ studied by Hur and Oh [7];

iv) $\mathcal{T}_1(0,\gamma,\alpha,1) = \mathcal{T}(\gamma,\alpha)$ and $\mathcal{T}_1(1,\gamma,\alpha,1) = C(\gamma,\alpha)$ studied by Altintaş and Owa [2];

v) $\mathcal{T}_1(n,\gamma,\alpha,1)$ studied by Aouf and Cho [3], [5].

Theorem 1.3. [6] *Let the function f be defined by (1.1). Then f belongs to the class $\mathcal{T}_j(n,\gamma,\alpha,\lambda)$ if and only if*

$$\sum_{k=j+1}^{\infty} [1+(k-1)\lambda]^n \{1+(k-1)\lambda - \alpha[1+\gamma(k-1)\lambda]\} a_k \leq 1-\alpha. \tag{1.7}$$

The result is sharp and the extremal functions are

$$f_k(z) = z - \frac{1-\alpha}{[1+(k-1)\lambda]^n\{1+(k-1)\lambda-\alpha[1+\gamma(k-1)\lambda]\}} \cdot z^k \qquad (1.8)$$

with $k \geq j+1$.

2. Main results

Let the functions f_i be defined for $i = 1, 2, \ldots, m$, by

$$f_i(z) = z - \sum_{k=j+1}^{\infty} a_{k,i} z^k, \quad a_{k,i} \geq 0,\ j \in \mathbb{N}^*, \quad z \in U. \qquad (2.1)$$

Theorem 2.1. *Let the functions* f_i *defined by (2.1) be in the class* $\mathcal{T}_j(n,\gamma,\alpha,\lambda)$, *for every* $i = 1, 2, \ldots, m$. *Then the functions* h *defined by*

$$h(z) = \sum_{i=1}^{m} d_i f_i(z), \quad d_i \geq 0 \qquad (2.2)$$

where

$$\sum_{i=1}^{m} d_i = 1, \qquad (2.3)$$

is also in the same class $\mathcal{T}_j(n,\gamma,\alpha,\lambda)$.

Proof. According to the definition of h, we can write

$$h(z) = z - \sum_{k=j+1}^{\infty} \left(\sum_{i=1}^{m} d_i a_{k,i} \right) z^k.$$

Further, since f_i are in the class $\mathcal{T}_j(n,\gamma,\alpha,\lambda)$ for every $i = 1, 2, \ldots, m$ we get

$$\sum_{k=j+1}^{\infty} c_k(n,\lambda)\{1+(k-1)\lambda-\alpha[1+\gamma(k-1)\lambda]\} a_{k,i} \leq 1-\alpha,$$

where $c_k(n,\lambda)$ is given by (1.5).

Hence we can see that

$$\sum_{k=j+1}^{\infty} c_k(n,\lambda)\{1+(k-1)\lambda-\alpha[1+\gamma(k-1)\lambda]\} \left(\sum_{i=1}^{m} d_i a_{k,i} \right) =$$

$$= \sum_{i=1}^{m} d_i \left(\sum_{k=j+1}^{\infty} c_k(n,\lambda)\{1+(k-1)\lambda-\alpha[1+\gamma(k-1)\lambda]\} a_{k,i} \right) \leq$$

$$\leq (1-\alpha)\sum_{i=1}^{m} d_i = 1-\alpha,$$

which implies that h is in $\mathcal{T}_j(n,\gamma,\alpha,\lambda)$. □

Theorem 2.2. *Let the function f defined by (1.1) be in the class $\mathcal{T}_j(n,\gamma,\alpha,\lambda)$ and let c be any real number such that $c > -1$. Then the function F defined by*

$$F(z) = \frac{c+1}{z^c}\int_0^z t^{c-1}f(t)dt \tag{2.4}$$

also belongs to the class $\mathcal{T}_j(n,\gamma,\alpha,\lambda)$.

Proof. From the representation (2.4) it follows that

$$F(z) = z - \sum_{k=j+1}^{\infty} b_k z^k$$

where

$$b_k = \left(\frac{c+1}{c+k}\right) a_k.$$

Therefore, we get

$$\sum_{k=j+1}^{\infty} c_k(n,\lambda)\{1+(k-1)\lambda - \alpha[1+\gamma(k-1)\lambda]\} b_k =$$

$$= \sum_{k=j+1}^{\infty} c_k(n,\lambda)\{1+(k-1)\lambda - \alpha[1+\gamma(k-1)\lambda]\}\left(\frac{c+1}{c+k}\right) a_k \leq$$

$$\leq \sum_{k=j+1}^{\infty} c_k(n,\lambda)\{1+(k-1)\lambda - \alpha[1+\gamma(k-1)\lambda]\} a_k \leq 1-\alpha.$$

Hence, by Theorem 1.3, $F \in \mathcal{T}_j(n,\gamma,\alpha,\lambda)$. □

Theorem 2.3. *Let c be a real number such that $c > -1$. If the function F belongs to the class $\mathcal{T}_j(n,\gamma,\alpha,\lambda)$ then the function f defined by (2.4) is univalent in $|z| < R^*$, where*

$$R^* = \inf_k \left[\frac{(c+1)c_k(n,\lambda)\{1+(k-1)\lambda - \alpha[1+\gamma(k-1)k]\}}{(1-\alpha)(c+k)k}\right]^{\frac{1}{k-1}} \tag{2.5}$$

and $c_k(n,\lambda)$ is given by (1.5). The result is sharp.

Proof. Let

$$F(z) = z - \sum_{k=j+1}^{\infty} a_k z^k, \quad a_k \geq 0.$$

It follows from (2.4) that

$$f(z) = \frac{z^{1-c}[z^c F(z)]'}{c+1} = z - \sum_{k=j+1}^{\infty} \left(\frac{c+k}{c+1}\right) a_k z^k.$$

In order to obtain the required result, it is sufficient to show that

$$|f'(z) - 1| < 1 \text{ whenever } |z| < R^*.$$

Now,

$$|f'(z) - 1| \leq \sum_{k=j+1}^{\infty} \frac{k(c+k)}{c+1} a_k |z|^{k-1}.$$

Thus, $|f'(z) - 1| < 1$ if

$$\sum_{k=j+1}^{\infty} \frac{k(c+k)}{c+1} a_k |z|^{k-1} < 1. \tag{2.6}$$

But, from Theorem 1.3 we have

$$\sum_{k=j+1}^{\infty} \frac{c_k(n,\lambda)\{1 + (k-1)\lambda - \alpha[1 + \gamma(k-1)\lambda]\}}{1-\alpha} a_k \leq 1. \tag{2.7}$$

Hence, by using (2.7), (2.6) will be satisfied if

$$\frac{k(c+k)}{c+1}|z|^{k-1} < \frac{c_k(n,\lambda)\{1 + (k-1)\lambda - \alpha[1 + \gamma(k-1)\lambda]\}}{1-\alpha}$$

that is

$$|z| < \left[\frac{(c+1)c_k(n,\lambda)\{1 + (k-1)\lambda - \alpha[1 + \gamma(k-1)\lambda]\}}{(1-\alpha)k(c+k)}\right]^{\frac{1}{k-1}}.$$

Therefore, f is univalent in $|z| < R^*$.

The sharpness follows if we take

$$f_k(z) = z - \frac{(1-\alpha)(c+k)}{(c+1)c_k(n,\lambda)\{1 + (k-1)\lambda - \alpha[1 + \gamma(k-1)\lambda]\}} z^k$$

$k \geq j+1$, $c_k(n,\lambda)$ is given by (1.5). □

Let the functions f_i, $(i=1,2)$ be defined by (2.1). The modified Hadamard product of f_1 and f_2 is defined here by

$$f_1 * f_2(z) = z - \sum_{k=j+1}^{\infty} a_{k,1} a_{k,2} z^k. \tag{2.8}$$

Theorem 2.4. *Let the function f_1 defined by (2.1) be in the class $\mathcal{T}_j(n,\gamma,\alpha,\lambda)$ and the function f_2 defined by (2.1) be in the class $\mathcal{T}_j(n,\gamma,\beta,\lambda)$. Then $f_1 * f_2$ belongs to the class $\mathcal{T}_j(n,\gamma,\delta,\lambda)$ where*

$$\delta = \delta(n,\gamma,\alpha,\beta,\lambda) = \tag{2.9}$$

$$= 1 - \frac{j\lambda(1-\gamma)(1-\alpha)(1-\beta)}{(1+j\lambda)^n[1+\lambda j - \alpha(1+\gamma j\lambda)][1+\lambda j - \beta(1+\gamma j\lambda)] - (1+\gamma j\lambda)(1-\alpha)(1-\beta)}.$$

The result is best possible for the functions

$$f_1(z) = z - \frac{1-\alpha}{[1+j\lambda - \alpha(1+j\lambda\gamma)](1+j\lambda)^n} z^{j+1} \tag{2.10}$$

and

$$f_2(z) = z - \frac{1-\beta}{[1+j\lambda - \beta(1+j\lambda\gamma)](1+j\lambda)^n} z^{j+1}. \tag{2.11}$$

Proof. Employing the technique used earlier by Schild and Silverman [9], we need to find the largest δ such that

$$\sum_{k=j+1}^{\infty} \frac{c_k(n,\lambda)\{1+(k-1)\lambda - \delta[1+\gamma(k-1)\lambda]\}}{1-\delta} a_{k,1} a_{k,2} \le 1.$$

Since

$$\sum_{k=j+1}^{\infty} \frac{c_k(n,\lambda)\{1+(k-1)\lambda - \alpha[1+\gamma(k-1)\lambda]\}}{1-\alpha} a_{k,1} \le 1 \tag{2.12}$$

and

$$\sum_{k=j+1}^{\infty} \frac{c_k(n,\lambda)\{1+(k-1)\lambda - \beta[1+\gamma(k-1)\lambda]\}}{1-\beta} a_{k,2} \le 1, \tag{2.13}$$

by the Cauchy-Schwarz inequality, we have

$$\sum_{k=j+1}^{\infty} c_k(n,\lambda)\sqrt{A(\gamma,\alpha,\lambda;k)B(\gamma,\beta,\lambda;k)} \cdot \sqrt{a_{k,1}a_{k,2}} \le 1 \tag{2.14}$$

where

$$A(\gamma,\alpha,\lambda;k) = \frac{1+(k-1)\lambda - \alpha[1+\gamma(k-1)\lambda]}{1-\alpha}$$

and

$$B(\gamma,\beta,\lambda;k)=\frac{1+(k-1)\lambda-\beta[1+\gamma(k-1)\lambda]}{1-\beta}.$$

Thus it is sufficient to show that

$$\sqrt{a_{k,1}a_{k,2}}\le\frac{(1-\delta)\sqrt{A(\gamma,\alpha,\lambda;k)B(\gamma,\beta,\lambda;k)}}{1+(k-1)\lambda-\delta[1+\gamma(k-1)\lambda]}.$$

Note that

$$\sqrt{a_{k,1}a_{k,2}}\le\frac{1}{c_k(n,\lambda)\sqrt{A(\gamma,\alpha,\lambda,k)B(\gamma,\beta,\lambda,k)}}.$$

Consequently, we need only to prove that

$$\frac{1}{c_k(n,\lambda)\sqrt{A(\gamma,\alpha,\lambda;k)B(\gamma,\beta,\lambda;k)}}\le\frac{(1-\delta)\sqrt{A(\gamma,\alpha,\lambda;k)B(\gamma,\beta,\lambda;k)}}{1+(k-1)\lambda-\delta[1+\gamma(k-1)\lambda]}$$

which is equivalent to

$$\delta\le1-\frac{\lambda(k-1)(1-\gamma)(1-\alpha)(1-\beta)}{c_k(n,\lambda)E_\alpha(\gamma,\lambda;k)E_\beta(\gamma,\lambda;k)-[1+\gamma(k-1)\lambda](1-\alpha)(1-\beta)}$$

where

$$E_\alpha(\gamma,\lambda;k)=1+(k-1)\lambda-\alpha[1+\gamma(k-1)\lambda] \tag{2.15}$$

and

$$E_\beta(\gamma,\lambda,k)=1+(k-1)\lambda-\beta[1+\gamma(k-1)\lambda]. \tag{2.16}$$

If we denote

$$S(n,\gamma,\alpha,\beta,\lambda;k)= \tag{2.17}$$

$$=1-\frac{\lambda(k-1)(1-\gamma)(1-\alpha)(1-\beta)}{c_k(n,\lambda)E_\alpha(\gamma,\lambda;k)E_\beta(\gamma,\lambda;k)-[1+\gamma(k-1)\lambda](1-\alpha)(1-\beta)}$$

one obtains that $S(n,\gamma,\alpha,\beta,\lambda,k)$ is an increasing function of k, $k\ge j+1$. Letting $k=j+1$ in (2.17), we obtain

$$\delta\le S(n,\gamma,\alpha,\beta,\lambda;j+1).$$

This completes the proof of Theorem 2.4. □

Theorem 2.5. *Let the function f_i, $(i = 1, 2)$ defined by (2.1) be in the class $\mathcal{T}_j(n, \gamma, \alpha, \lambda)$. Then $f_1 * f_2(z)$ belongs to the class $\mathcal{T}_j(n, \gamma, \beta, \lambda)$ where*

$$\beta = \beta(n, \gamma, \alpha, \lambda) = \tag{2.18}$$

$$= 1 - \frac{j\lambda(1-\alpha)^2(1-\gamma)}{(1+j\lambda)^n[1+j\lambda-\alpha(1+\gamma j\lambda)]^2 - (1-\alpha)^2(1+\gamma j\lambda)}.$$

The result is sharp.

Proof. Employing the technique used earlier by Schild and Silverman [9], we need to find the largest β such that

$$\sum_{k=j+1}^{\infty} c_k(n, \lambda)\{1 + (k-1)\lambda - \beta[1 + \gamma(k-1)\lambda]\}a_{k,1}a_{k,2} \leq 1 - \beta.$$

The proof is the same as in the previous theorem.

Finally, by taking the functions f_i, given by

$$f_i(z) = z - \frac{1-\alpha}{[1+j\lambda-\alpha(1+j\lambda\gamma)](1+j\lambda)^n} z^{j+1}, \quad i = 1, 2 \tag{2.19}$$

we can see that the result is sharp. □

Corollary 2.6. *For f_1 and f_2 as in Theorem 2.4, the function*

$$h(z) = z - \sum_{k=j+1}^{\infty} \sqrt{a_{k,1}a_{k,2}}\, z^k \tag{2.20}$$

belongs to the class $\mathcal{T}_j(n, \gamma, \alpha, \lambda)$. The result is sharp.

Proof. This result follows from the Cauchy-Schwarz inequality. It is sharp for the same function as in Theorem 2.4. □

Corollary 2.7. *Let the functions f_i, $(i = 1, 2, 3)$ defined by (2.1) be in the class $\mathcal{T}_j(n, \gamma, \alpha, \lambda)$. Then $f_1 * f_2 * f_3$ belongs to the class $\mathcal{T}_j(n, \gamma, \eta, \lambda)$ where*

$$\eta = \eta(n, \gamma, \alpha, \lambda) = \tag{2.21}$$

$$= 1 - \frac{j\lambda(1-\alpha)^3(1-\gamma)}{(1+j\lambda)^{2n}[1+j\lambda-\alpha(1+\gamma j\lambda)]^3 - (1+j\gamma\lambda)(1-\alpha)^3}.$$

The result is best possible for the functions

$$f_i(z) = z - \frac{1-\alpha}{[1+j\lambda-\alpha(1+j\lambda\gamma)](1+j\lambda)^n} z^{j+1}, \quad i = 1, 2, 3. \tag{2.22}$$

Proof. From Theorem 2.5 one obtains that $f_1 * f_2$ belongs to the class $\mathcal{T}_j(n, \gamma, \beta, \lambda)$ where β is given by (2.18). By using Theorem 2.4 we get $f_1 * f_2 * f_3$ belongs to the class $\mathcal{T}_j(n, \gamma, \eta, \lambda)$ where

$$\eta = \eta(n, \gamma, \alpha, \beta, \lambda) =$$
$$= 1 - \frac{j\lambda(1-\gamma)(1-\alpha)(1-\beta)}{(1+j\lambda)^n E_\alpha(\gamma, \lambda; j+1) E_\beta(\gamma, \lambda; j+1) - (1+\gamma j\lambda)(1-\alpha)(1-\beta)}$$

and $E_\alpha(\gamma, \lambda; j+1)$, $E_\beta(\gamma, \lambda; j+1)$ are given as in (2.15) and (2.16).

Hence, Corollary 2.7 follows at once. □

Theorem 2.8. *Let the function f_i, $(i = 1, 2)$ defined by (2.1) be in the class $\mathcal{T}_j(n, \gamma, \alpha, \lambda)$. Then the function*

$$h(z) = z - \sum_{k=j+1}^{\infty} (a_{k,1}^2 + a_{k,2}^2) z^k, \tag{2.23}$$

belongs to the class $\mathcal{T}_j(n, \gamma, \eta, \lambda)$ where

$$\eta = \eta(n, \gamma, \alpha, \lambda) = \tag{2.24}$$
$$= 1 - \frac{2j\lambda(1-\alpha)^2(1-\gamma)}{(1+j\lambda)^n[1+j\lambda-\alpha(1+\gamma j\lambda)]^2 - 2(1-\alpha)^2(1+\gamma j\lambda)}.$$

The result is sharp for the functions f_i, $(i = 1, 2)$ defined by (2.19).

Proof. By virtue of Theorem 1.3, one obtains

$$\sum_{k=j+1}^{\infty} \left[\frac{c_k(n,\lambda)\{1+(k-1)\lambda-\alpha[1+\gamma\lambda(k-1)]\}}{1-\alpha}\right]^2 a_{k,i}^2 \leq$$

$$\leq \left[\sum_{k=j+1}^{\infty} \frac{c_k(n,\lambda)\{1+(k-1)\lambda-\alpha[1+\gamma(k-1)\lambda]\}}{1-\alpha} a_{k,i}\right]^2 \leq 1, \quad i = 1, 2.$$

It follows that

$$\sum_{k=j+1}^{\infty} \frac{1}{2} \left[\frac{c_k(n,\lambda)\{1+(k-1)\lambda-\alpha[1+\gamma(k-1)\lambda]\}}{1-\alpha}\right]^2 (a_{k,1}^2 + a_{k,2}^2) \leq 1.$$

Therefore, we need to find the largest η such that

$$\frac{c_k(n,\lambda)\{1+(k-1)\lambda-\eta[1+\gamma(k-1)\lambda]\}}{1-\eta} \leq$$

$$\leq \frac{1}{2} \left[\frac{c_k(n,\lambda)\{1+(k-1)\lambda-\alpha[1+\gamma(k-1)\lambda]\}}{1-\alpha}\right]^2$$

that is

$$\eta \leq 1 - \frac{2\lambda(1-\alpha)^2(k-1)(1-\gamma)}{c_k(n,\lambda)\{1+(k-1)\lambda-\alpha[1+\gamma(k-1)\lambda]\}^2-2(1-\alpha)^2[1+\gamma(k-1)\lambda]}.$$

Since

$$F(n,\gamma,\alpha,\lambda;k) =$$

$$= 1 - \frac{2\lambda(1-\alpha)^2(k-1)(1-\gamma)}{c_k(n,\lambda)\{1+(k-1)\lambda-\alpha[1+\gamma(k-1)\lambda]\}^2-2(1-\alpha)^2[1+\gamma(k-1)\lambda]}$$

is an increasing function of k, $(k \geq j+1)$ we get

$$\eta \leq F(n,\gamma,\alpha,\lambda;j+1)$$

and Theorem 2.8 follows at once. □

Theorem 2.9. *Let the function*

$$f_1(z) = z - \sum_{k=j+1}^{\infty} a_{k,1}z^k, \quad a_{k,1} \geq 0$$

be in the class $\mathcal{T}_j(n,\gamma,\alpha,\lambda)$ and

$$f_2(z) = z - \sum_{k=j+1}^{\infty} |a_{k,2}|z^k,$$

with $|a_{k,2}| \leq 1$. Then $f_1 * f_2$ belongs to the class $\mathcal{T}_j(n,\gamma,\alpha,\lambda)$.

Proof. Since

$$\sum_{k=j+1}^{\infty} c_k(n,\lambda)\{1+(k-1)\lambda-\alpha[1+\gamma(k-1)\lambda]\}|a_{k,1}a_{k,2}| =$$

$$= \sum_{k=j+1}^{\infty} c_k(n,\lambda)\{1+(k-1)\lambda-\alpha[1+\gamma(k-1)\lambda]\}a_{k,1}|a_{k,2}| \leq$$

$$\leq \sum_{k=j+1}^{\infty} c_k(n,\lambda)\{1+(k-1)\lambda-\alpha[1+\gamma(k-1)\lambda]\}a_{k,1} \leq 1-\alpha$$

by Theorem 1.3, one obtains that $f_1 * f_2$ belongs to the class $\mathcal{T}_j(n,\gamma,\alpha,\lambda)$. □

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University of Oradea
Faculty of Science, Department of Mathematics
1 University Street, 410087, Oradea, Romania
E-mail address: acatas@gmail.com