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HEAT TRANSFER IN AXISYMMETRIC STAGNATION FLOW ON THIN CYLINDERS

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Abstract. The Navier-Stokes and energy equations for the steady laminar incompressible flow past a row of circular cylinders at constant temperature are solved numerically. Prime attention was focused on how heat transfer characteristics are affected by variation of Reynolds number. The study was limited to Reynolds number ranging from 1 to 100 and the Prandtl number has been fixed to a value equal to 1. For different values of above parameters streamlines, isotherms and the local Nusselt number has been determined and are shown on several graphs.

1. Introduction

Heat transfer from bodies of different geometries is one of the important problems that has received much attention due to its engineering applications, such as: heat transfer from rotating machinery, spinning projectiles, cooling of electronic devices, design of heat exchangers, theory of hot wire anemometer. According to the literature, the available information in these areas is limited to a few special cases. A general method for solving such problems is not easy available, not only due to the mathematical difficulties involved, but also due to the wide range of body shapes as well as the different characteristics of the velocity and thermal fields.

References related to this topic can be found in the books by: Bejan (1995), Postelnicu and Pop (1999), Pop and Ingham (2001), Khor and Pop (2005), White (2008). On the other hand the problem of the heat transfer between a circular cylinder

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and its surrounding stream of a viscous fluid is of great interest. A large number of experimental papers on this problem are available in the literature, see Jain and Goel (1976). It is well known that numerical solution of Navier-Stokes equations for fluid flow problems may give reliable information in a case when experimental measurements are difficult. This view is well supported by the numerical studies made by Collins and Dennis (1973), Ingham (1984), Nam (1990), etc. To our best knowledge, flow and the heat transfer through an array of cylinders has not been to much study. However, the natural convective heat transfer from a pair of horizontal cylinder of the same temperature placed one above the other in a vertical plane has been theoretically studied by Yuncu and Batta (1994).

In the present paper the problem of cooling by forced convection of a row of circular cylinders is numerically studied. Namely, flow and heat transfer characteristics are determined for different values of the Reynolds number, keeping the Prandtl number constant ($Pr = 1$). The value of the Reynolds number is consider to be in the range $1 \leq Re \leq 100$ and the viscous dissipation is negligible small.

2. **Basic Equations**

Consider the steady two-dimensional forced convection flow over a circular cylinders' row of radius R placed in a viscous fluid of ambient temperature T_∞ and velocity U_∞ (see Figure 1). It is assumed that the distance between the cylinders is $2R$ and that the temperature of the cylinders is constant T_w ($T_w > T_\infty$). The mathematical model is given by continuity, Navier-Stokes and energy equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{2.2}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \tag{2.3}$$

$$\rho c_P\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \tag{2.4}$$

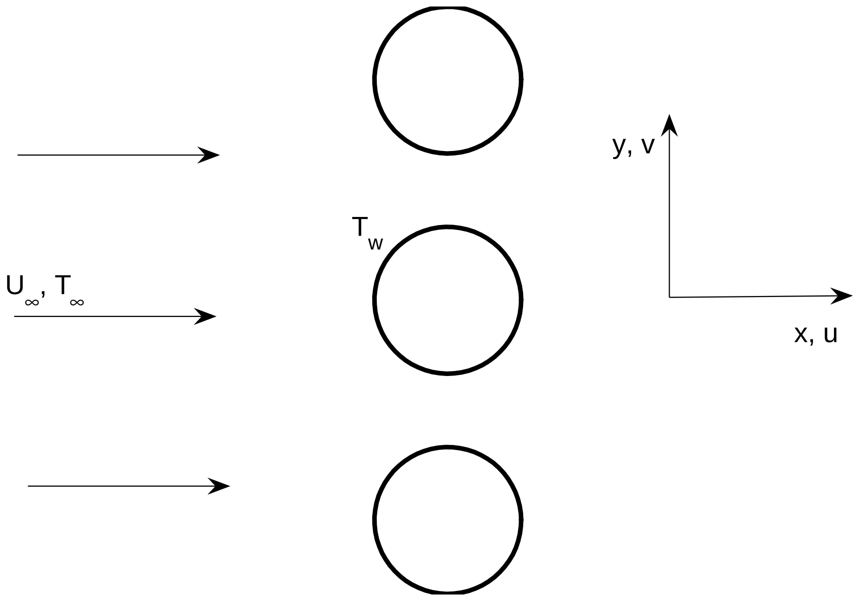

FIGURE 1. Geometry of the problem and the co-ordinate system

where x and y are the Cartesian co-ordinate along the horizontal and vertical direction, respectively, u and v are the velocity components along x and y -axes, p is the pressure, T is the temperature, k is the thermal diffusivity of the viscous fluid, ρ is the fluid density and c_p is the specific heat at constant pressure.

Further we use the following dimensionless variables for co-ordinate, velocity components and pressure:

$$X = \frac{x}{R}, \qquad Y = \frac{y}{R}, \qquad U = \frac{u}{U_\infty}, \qquad V = \frac{v}{U_\infty}, \qquad P = \frac{p - p_\infty}{\rho U_\infty^2} \tag{2.5}$$

Using (2.5) in Eqs.(2.1)-(2.4) we obtain:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{2.6}$$

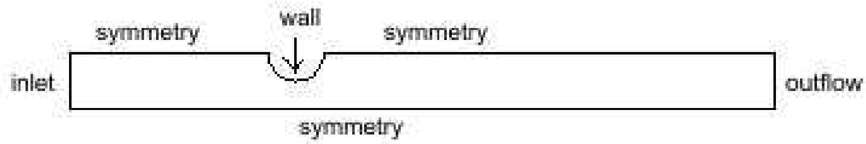

FIGURE 2. Domain of integration and boundary condition type

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\frac{1}{Re}\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right) \tag{2.7}$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\frac{1}{Re}\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right) \tag{2.8}$$

$$\left(U\frac{\partial T}{\partial X}+V\frac{\partial T}{\partial Y}\right)=\frac{1}{RePr}\left(\frac{\partial^2 T}{\partial X^2}+\frac{\partial^2 T}{\partial Y^2}\right) \tag{2.9}$$

where $Re = U_\infty R/\nu$ is the Reynolds number and $Pr = \nu/\alpha$ is the Prandtl number. Due to the software used to solve the model the temperature is mentained in the dimensional form. Due to the symmetry of the problem we consider a small domain to integrate equations (2.7)-(2.9), see Figure 2. The boundary conditions for the new domain can be expressed as:

$$-inlet : U_\infty = 1, T = T_\infty \tag{2.10}$$

$$-wall : U = 0, V = 0, T = T_w \tag{2.11}$$

$$-symmetry : \frac{\partial U}{\partial Y}=0, \frac{\partial V}{\partial Y}=0, \frac{\partial T}{\partial Y}=0 \tag{2.12}$$

$$-outflow : \frac{\partial V}{\partial Y}=0 \tag{2.13}$$

A quantity of interest is the local Nusselt number which express the ratio of the convective and conductive heat transfer. Using the energetic balance on the plate we deduce the convection heat transfer coefficient, h:

$$-k\left[\frac{\partial T}{\partial y}\right]_{y=0} = h(T_w - T_{fluid}) \tag{2.14}$$

and using the definition of the local Nusselt number, $Nu_w = \frac{hR}{k}$, one obtain:

$$Nu_w = \frac{-\frac{\partial T}{\partial \mathbf{n}}}{T_w - T_{fluid}} \tag{2.15}$$

where $\mathbf{n}$ is the outer normal vector to the cylindrical wall and T_{fluid} is the temperature of the fluid in the vicinity of the wall.

3. **Results and Discussions**

The full Navier-Stokes and energy equations (2.6) - (2.9), with the corresponding boundary conditions (2.10) - (2.13) were numerically solved using FLUENT. The model is solved for $T_\infty = 281$ K and $T_w = 400$ K. We use the following discretization: standard for pressure, SIMPLE for pressure-velocity coupling and power-law for momentum and energy equations.The stop residual were $1e-4$ for continuity and velocity while for energy the value $1e-6$ was used. Also the underrelaxation method has been used, the underrelaxation factor was 0.3 for the pressure and 0.7 for the momentum equation.

To examine the effect of the Reynolds number the streamlines, isotherms and local Nusselt number are presented in Figures 3 to 11. It is noticed that for larger values of the Reynolds number the vortex region increases, see Figures 3, 5 7 and 9. Further we notice that the maximum value of the streamline function increases with the increase of the Reynolds number.

On the other hand Figures 4, 6, 8 and 10 display the distribution of the temperature field for different values of the Reynolds number. These figures indicate that the temperature decrease with the increasing of the Reynolds number. Therefore, the cooling of the cylinders is more efficient for large values of the Reynolds number.

The variation of the local Nusselt number around the cylinder is shown in Figure 11 for several values of the Reynolds number. In this figure $\theta = 0^o$ correspond to the region of the forward stagnation point of the cylinder while $\theta = 180^o$ correspond

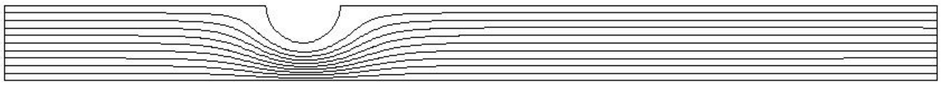

FIGURE 3. Streamlines for $Re = 1$, $\psi_{max} = 2.000003$

FIGURE 4. Isotherms for $Re = 1$

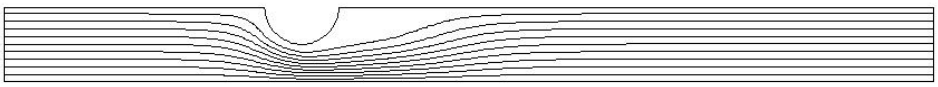

FIGURE 5. Streamlines for $Re = 10$, $\psi_{max} = 2.029994$

to the region of the rear stagnation point, respectively. It can be seen that the local Nusselt number sharply increases as the value of the Reynolds number increases, and then gradually decreases with the increases of the angle θ. In addition we notice that for $Re = 50$ and $Re = 100$ the graphs of the local Nusselt numbers change their shapes due to recirculation of the fluid.

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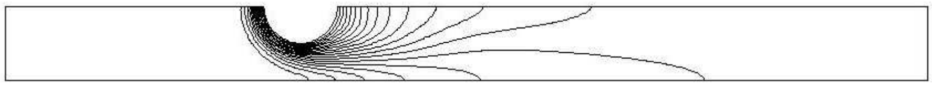

Figure 6. Isotherms for $Re = 10$

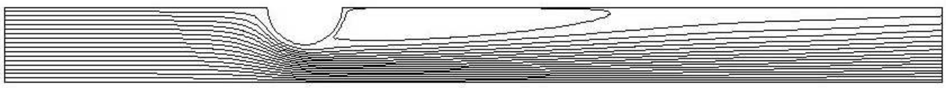

Figure 7. Streamlines for $Re = 50$, $\psi_{max} = 2.122836$

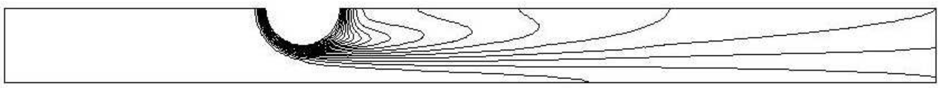

Figure 8. Isotherms for $Re = 50$

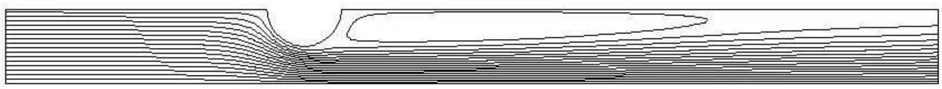

Figure 9. Streamlines for $Re = 100$, $\psi_{max} = 2.155566$

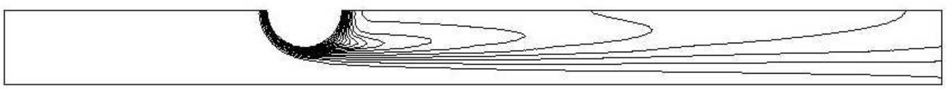

Figure 10. Isotherms for $Re = 100$

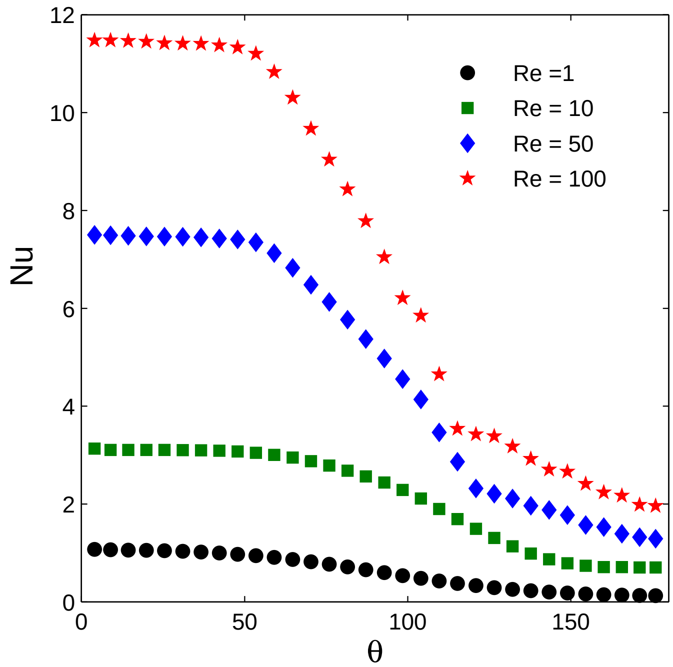

FIGURE 11. Variation of the local Nusselt number, Nu, for different values of the Reynold number Re

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