STUDIA UNIV. "BABEŞ–BOLYAI", MATHEMATICA, Volume **LIV**, Number 3, September 2009

FIXED POINT THEORY FOR MULTIVALUED GENERALIZED CONTRACTION ON A SET WITH TWO b-METRICS

MONICA BORICEANU

Abstract. The purpose of this paper is to present some fixed point results for multivalued generalized contraction on a set with two b-metrics. The data dependence and the well-posedness of the fixed point problem are also discussed.

1. Introduction

The concept of b-metric space was introduced by Czerwik in [2]. Since then several papers deal with fixed point theory for singelvalued and multivalued operators in b-metric spaces (see [1], [2], [7]). In the first part of the paper we will present a fixed point theorem for Ćirić-type multivalued operator on b-metric space endowed with two b-metrics. Then, a strict fixed point result for multivalued generalized contraction in b-metric spaces is proved. The last part contains several conditions under which the fixed point problem for a multivalued operator in a b-metric space is well-posed and a data dependence result is given.

2. Preliminaries and auxiliary results

The aim of this section is to present some notions and symbols used in the paper.

We will first give the definition of a b-metric space.

Received by the editors: 01.10.2008.

2000 *Mathematics Subject Classification.* 47H10, 54H25.

Key words and phrases. Multivalued generalized contraction, fixed point, strict fixed point, b-metric space, data dependence, well-posedness.

This paper was presented at the 7-th Joint Conference on Mathematics and Computer Science, July 3-6, 2008, Cluj-Napoca, Romania.

Definition 2.1 (Czerwik [2]) Let X be a set and let $s \geq 1$ be a given real number. A function $d : X \times X \to \mathbb{R}_+$ is said to be a b-metric if and only if for all $x, y, z \in X$ the following conditions are satisfied:

1. $d(x, y) = 0$ if and only if $x = y$;
2. $d(x, y) = d(y, x)$;
3. $d(x, z) \leq s[d(x, y) + d(y, z)]$.

A pair (X, d) is called a b-metric space.

We give next some examples of b-metric spaces.

Example 2.2 (Berinde see [1])

The space $l_p (0 < p < 1)$,

$$l_p = \{(x_n) \subset \mathbb{R} | \sum_{n=1}^{\infty} |x_n|^p < \infty\},$$

together with the function $d : l_p \times l_p \to \mathbb{R}$,

$$d(x, y) = (\sum_{n=1}^{\infty} |x_n - y_n|^p)^{1/p},$$

where $x = (x_n), y = (y_n) \in l_p$ is a b-metric space.

By an elementary calculation we obtain: $d(x, z) \leq 2^{1/p}[d(x, y) + d(y, z)]$.

Hence $a = 2^{1/p} > 1$.

Example 2.3 (Berinde see[1])

The space $L_p (0 < p < 1)$ of all real functions $x(t)$, $t \in [0, 1]$ such that:

$$\int_0^1 |x(t)|^p dt, \infty,$$

is a b-metric space if we take

$$d(x, y) = (\int_0^1 |x(t) - y(t)|^p dt)^{1/p}, \text{ for each } x, y \in L_p,$$

The constant a is as in the previous example $2^{1/p}$.

We continue by presenting the notions of convergence, compactness, closedness and completeness in a b-metric space.

Definition 2.4 Let (X, d) be a b-metric space. Then a sequence $(x_n)_{n \in \mathbb{N}}$ in X is called:

(a) Cauchy if and only if for all $\varepsilon > 0$ there exists $n(\varepsilon) \in \mathbb{N}$ such that for each $n, m \geq n(\varepsilon)$ we have $d(x_n, x_m) < \varepsilon$.

(b) convergent if and only if there exists $x \in X$ such that for all $\varepsilon > 0$ there exists $n(\varepsilon) \in \mathbb{N}$ such that for all $n \geq n(\varepsilon)$ we have $d(x_n, x) < \varepsilon$. In this case we write $\lim\limits_{n\to\infty} x_n = x$.

Remark 2.5

1. The sequence $(x_n)_{n\in\mathbb{N}}$ is Cauchy if and only if $\lim\limits_{n\to\infty} d(x_n, x_{n+p}) = 0$, for all $p \in \mathbb{N}^*$.
2. The sequence $(x_n)_{n\in\mathbb{N}}$ is convergent to $x \in X$ if and only if $\lim\limits_{n\to\infty} d(x_n, x) = 0$.

Definition 2.6

1. Let (X, d) be a b-metric space. Then a subset $Y \subset X$ is called
   (i) compact if and only if for every sequence of elements of Y there exists a subsequence that converges to an element of Y.
   (ii) closed if and only if for each sequence $(x_n)_{n\in\mathbb{N}}$ in Y which converges to an element x, we have $x \in Y$.
2. The b-metric space is complete if every Cauchy sequence converges.

We consider next the following families of subsets of a b-metric space (X, d):

$$P(X) := \{Y \in \mathcal{P}(X) |\ Y \neq \emptyset\};$$

$$P_b(X) := \{Y \in P(X) |\ diam(Y) < \infty\},$$

where

$$diam : P(X) \to \mathbb{R}_+ \cup \{\infty\}, diam(Y) = sup\{d(a, b), a, b \in Y\}$$

is the generalized diameter functional;

$$P_{cp}(X) := \{Y \in P(X) |\ Y \text{ is compact}\};$$

$$P_{cl}(X) := \{Y \in P(X) |\ Y \text{ is closed}\};$$

$$P_{b,cl}(X) := P_b(X) \cap P_{cl}(X)$$

We will introduce the following generalized functionals on a b-metric space (X,d). Some of them were defined in [2].

1. $D : P(X) \times P(X) \to \mathbb{R}_+ \cup \{+\infty\}$,

$$D(A,B) = \inf\{d(a,b) | a \in A, b \in B\},$$

for any $A, B \subset X$.

D is called the gap functional between A and B. In particular, if $x_0 \in X$ then $D(x_0, B) := D(\{x_0\}, B)$.

2. $\delta : P(X) \times P(X) \to \mathbb{R}_+ \cup \{+\infty\}$,

$$\delta(A,B) = \sup\{d(a,b) |\ a \in A,\ b \in B\}.$$

3. $\rho : P(X) \times P(X) \to \mathbb{R}_+ \cup \{+\infty\}$,

$$\rho(A,B) = \sup\{D(a,B) | a \in A\},$$

for any $A, B \subset X$.

ρ is called the (generalized) excess functional.

4. $H : P(X) \times P(X) \to \mathbb{R}_+ \cup \{+\infty\}$,

$$H(A,B) = \max\left\{\sup_{x\in A} D(x,B), \sup_{y\in B} D(A,y)\right\},$$

for any $A, B \subset X$.

H is the (generalized) Pompeiu-Hausdorff functional.

Let (X,d) be a b-metric space. If $F : X \to P(X)$ is a multivalued operator, we denote by $FixF$ the fixed point set of F, i.e. $Fix(F) := \{x \in X | x \in F(x)\}$ and by $SFixF$ the strict fixed point set of F, i.e. $SFixF := \{x \in X | \{x\} = F(x)\}$.

Lemma 2.7 [4] *Let (X,d) be a b-metric space and let $A, B \in P(X)$. We suppose that there exists $\eta \in \mathbb{R}, \eta > 0$ such that:*

(i) for each $a \in A$ there is $b \in B$ such that $d(a,b) \le \eta$;

(ii) for each $b \in B$ there is $a \in A$ such that $d(a,b) \le \eta$.

Then

$$H(A,B) \le \eta.$$

Lemma 2.8 [4] *Let* (X, d) *be a b-metric space and let* $A \in P(X)$ *and* $x \in X$*. Then* $D(x, A) = 0$ *if and only if* $x \in \bar{A}$*.*

The following results are useful for some of the proofs in the paper.

Lemma 2.9 (Czerwik [2]) *Let* (X, d) *be a b-metric space. Then*

$$D(x, A) \leq s[d(x, y) + D(y, A)], \text{ for all } x, y \in X, A \subset X.$$

Lemma 2.10 (Czerwik [2]) *Let* (X, d) *be a b-metric space and let* $\{x_k\}_{k=0}^n \subset X$*. Then:*

$$d(x_n, x_0) \leq sd(x_0, x_1) + ... + s^{n-1}d(x_{n-2}, x_{n-1}) + s^{n-1}d(x_{n-1}, x_n).$$

Lemma 2.11 (Czerwik [2]) *Let* (X, d) *be a b-metric space and for all* $A, B, C \in X$ *we have:*

$$H(A, C) \leq s[H(A, B) + H(B, C)].$$

Lemma 2.12 (Czerwik [2])

(1) Let (X, d) *be a b-metric space and* $A, B \in P_{cl}(X)$*. Then for each* $\alpha > 0$ *and for all* $b \in B$ *there exists* $a \in A$ *such that:*

$$d(a, b) \leq H(A, B) + \alpha;$$

(2) Let (X, d) *be a b-metric space and* $A, B \in P_{cp}(X)$*. Then for all* $b \in B$ *there exists* $a \in A$ *such that:*

$$d(a, b) \leq sH(A, B).$$

3. **Main results**

The fist main result of this paper is a fixed point theorem.

Theorem 3.1 *Let* X *be a nonempty set,* d *and* ρ *two b-metrics on* X *with constants* $t > 1$ *and respectively* $s > 1$ *and let* $F : X \to P(X)$ *a multivalued operator. We suppose that:*

(i) *(X, d) is a complete b-metric space;*
(ii) *There exists $c > 0$ such that $d(x, y) \leq c \cdot \rho(x, y)$, for all $x, y \in X$;*
(iii) *$F : (X, d) \to (P(X), H_d)$ is closed;*

(iv) *There exists* $0 \le \alpha < \frac{1}{s}$ *such that*

$$H_\rho(F(x), F(y)) \le \alpha M_\rho^F(x,y),$$

for all $x, y \in X$, *where*

$$M_\rho^F(x,y) = max\left\{\rho(x,y), D_\rho(x,F(x)), D_\rho(y,F(y)), \frac{1}{2}\left[D_\rho(x,F(y)) + D_\rho(y,F(x))\right]\right\}.$$

Then we have:

1. $FixF \neq \emptyset$;
2. *For all* $x \in X$ *and each* $y \in F(x)$ *there exists* $(x_n)_{n\in\mathbb{N}}$ *such that:*
   (a) $x_0 = x, x_1 = y$;
   (b) $x_{n+1} \in F(x_n)$;
   (c) $d(x_n, x^*) \to 0$, *as* $n \to \infty$ *where* $x^* \in F(x^*)$;

Proof. Let $1 < q < \frac{1}{s\alpha}$ be arbitrary. For arbitrary $x_0 \in X$ and for $x_1 \in F(x_0)$ there exists $x_2 \in F(x_1)$ such that:

$$\rho(x_1, x_2) \le qH_\rho(F(x_0), F(x_1)) \le q\alpha M_\rho^F(x_0, x_1).$$

So we have

$$\rho(x_1,x_2) \le q\alpha \cdot max\left\{\rho(x_0,x_1), D_\rho(x_0,F(x_0)), D(x_1,F(x_1)), \frac{1}{2}\left[D_\rho(x_0,F(x_1)) + D_\rho(x_1,F(x_0))\right]\right\}.$$

Suppose that the $max = \rho(x_0, x_1)$. Then we have

$$\rho(x_1, x_2) \le q\alpha\rho(x_0, x_1).$$

Suppose that the $max = D_\rho(x_0, F(x_0))$. Then we have

$$\rho(x_1, x_2) \le q\alpha D_\rho(x_0, F(x_0)) \le q\alpha\rho(x_0, x_1).$$

Suppose that the $max = D_\rho(x_1, F(x_1))$. Then we have

$$\rho(x_1, x_2) \le q\alpha D_\rho(x_1, F(x_1)) \le q\alpha\rho(x_1, x_2).$$

So $\rho(x_1, x_2) = 0$ and thus $x_1 \in FixF$.

Suppose that the $max = \frac{1}{2}[D_\rho(x_0, F(x_1)) + D_\rho(x_1, F(x_0))]$. Then we have

$$\rho(x_1, x_2) \leq q\alpha\frac{1}{2}D_\rho(x_0, F(x_1)) \leq q\alpha\frac{1}{2}\rho(x_0, x_2)) \leq \frac{q\alpha}{2}s[\rho(x_0, x_1) + \rho(x_1, x_2)].$$

So we have $\rho(x_1, x_2) \leq \frac{q\alpha s}{2-q\alpha s}\rho(x_0, x_1)$.

For $x_2 \in F(x_1)$ there exists $x_3 \in F(x_2)$ such that:

$$\rho(x_2, x_3) \leq qH_\rho(F(x_1), F(x_2)) \leq q\alpha M_\rho^F\rho(x_1, x_2)$$

Suppose that the $max = \rho(x_1, x_2)$. Then we have

$$\rho(x_2, x_3) \leq q\alpha\rho(x_1, x_2) \leq (q\alpha)^2\rho(x_0, x_1).$$

Suppose that the $max = D_\rho(x_1, F(x_1))$. Then we have

$$\rho(x_2, x_3) \leq q\alpha D_\rho(x_1, F(x_1)) \leq q\alpha\rho(x_1, x_2) \leq (q\alpha)^2\rho(x_0, x_1).$$

Suppose that the $max = D_\rho(x_2, F(x_2))$. Then we have

$$\rho(x_2, x_3) \leq q\alpha D_\rho(x_2, F(x_2)) \leq q\alpha\rho(x_2, x_3).$$

So $\rho(x_2, x_3) = 0$ and thus $x_2 \in FixF$.

Suppose that the $max = \frac{1}{2}[D_\rho(x_1, F(x_2)) + D_\rho(x_2, F(x_1))]$. Then we have

$$\rho(x_1, x_2) \leq q\alpha\frac{1}{2}D_\rho(x_1, F(x_2)) \leq q\alpha\frac{1}{2}\rho(x_1, x_3)) \leq \frac{q\alpha}{2}s[\rho(x_1, x_2) + \rho(x_2, x_3)].$$

So we have

$$\rho(x_2, x_3) \leq \frac{q\alpha s}{2 - q\alpha s}\rho(x_1, x_2) \leq [\frac{q\alpha s}{2 - q\alpha s}]^2\rho(x_0, x_1).$$

We can construct by induction a sequence $(x_n)_{n\in\mathbb{N}}$ such that

$$\rho(x_n, x_{n+1}) \leq max\{(q\alpha)^n, [\frac{q\alpha s}{2 - q\alpha s}]^n\}\rho(x_0, x_1), \text{ for all } n \in \mathbb{N}.$$

We will prove next that the sequence $(x_n)_{n\in\mathbb{N}}$ is Cauchy, by estimating $\rho(x_n, x_{n+p})$.

We consider first that the maximum is $(q\alpha)^n$. So we have:

$$\begin{aligned}
\rho(x_n, x_{n+p}) &\leq s\rho(x_n, x_{n+1}) + s^2\rho(x_{n+1}, x_{n+2}) + ...+ \\
&+ s^{p-1}\rho(x_{n+p-2}, x_{n+p-1}) + s^{p-1}\rho(x_{n+p-1}, x_{n+p}) \\
&\leq s(q\alpha)^n\rho(x_0, x_1) + s^2(q\alpha)^{n+1}\rho(x_0, x_1) + ...+ \\
&+ s^{p-1}(q\alpha)^{n+p-2}\rho(x_0, x_1) + s^{p-1}(q\alpha)^{n+p-1}\rho(x_0, x_1) \\
&= s(q\alpha)^n\rho(x_0, x_1)[1 + sq\alpha + ... + (sq\alpha)^{p-2} + s^{p-2}(q\alpha)^{p-1}] \\
&\leq s(q\alpha)^n\rho(x_0, x_1)[1 + sq\alpha + ... + (sq\alpha)^{p-2} + s^{p-1}(q\alpha)^{p-1}] \\
&= s(q\alpha)^n\rho(x_0, x_1)\frac{1 - (sq\alpha)^p}{1 - sq\alpha}.
\end{aligned}$$

But $1 < q < \frac{1}{s\alpha}$. Hence we obtain that:

$$\rho(x_n, x_{n+p}) \leq s(q\alpha)^n\rho(x_0, x_1)\frac{1 - (sq\alpha)^p}{1 - sq\alpha} \to 0,$$

as $n \to \infty$. So $(x_n)_{n\in\mathbb{N}}$ is Cauchy and $x_n \to x \in X$.

We consider now the maximum $A := [\frac{q\alpha s}{2-q\alpha s}]^n$. So we have:

$$\begin{aligned}
\rho(x_n, x_{n+p}) &\leq s\rho(x_n, x_{n+1}) + s^2\rho(x_{n+1}, x_{n+2}) + ...+ \\
&+ s^{p-1}\rho(x_{n+p-2}, x_{n+p-1}) + s^{p-1}\rho(x_{n+p-1}, x_{n+p}) \\
&\leq sA^n\rho(x_0, x_1) + s^2A^{n+1}\rho(x_0, x_1) + ...+ \\
&+ s^{p-1}A^{n+p-2}\rho(x_0, x_1) + s^{p-1}A^{n+p-1}\rho(x_0, x_1) \\
&= sA^n\rho(x_0, x_1)[1 + sA + ... + (sA)^{p-2} + s^{p-2}A^{p-1}] \\
&\leq sA^n\rho(x_0, x_1)[1 + sA + ... + (sA)^{p-2} + s^{p-1}A^{p-1}] \\
&= sA^n\rho(x_0, x_1)\frac{1 - (sA)^p}{1 - sA}.
\end{aligned}$$

But $1 < q < \frac{1}{s\alpha}$ and we obtain that:

$$\rho(x_n, x_{n+p}) \leq sA^n\rho(x_0, x_1)\frac{1 - (sA)^p}{1 - sA} \to 0,$$

as $n \to \infty$. So $(x_n)_{n\in\mathbb{N}}$ is Cauchy in (X, ρ).

From (ii) it follows that the sequence is Cauchy in (X, d). Denote by $x^* \in X$ the limit of the sequence. From (i) and (iii) we get that $d(x_n, x^*) \to 0$, as $n \to \infty$ where $x^* \in F(x^*)$. The proof is complete. □

For the next results let us denote

$$N_\rho^F(x, y) = max\left\{\rho(x, y), D_\rho(y, F(y)), \frac{1}{2}\left[D_\rho(x, F(y)) + D_\rho(y, F(x))\right]\right\}.$$

The second main result of this paper is:

Theorem 3.2 *Let X be a nonempty set, d and ρ two b-metrics on X with constants $t > 1$ and respectively $s > 1$ and let $F : X \to P(X)$ a multivalued operator. We suppose that:*

(i) *(X, d) is a complete b-metric space;*

(ii) *There exists $c > 0$ such that $d(x, y) \le c \cdot \rho(x, y)$, for all $x, y \in X$;*

(iii) *$F : (X, d) \to (P(X), H_d)$ is closed;*

(iv) *There exists $0 \le \alpha < \frac{1}{s}$ such that*

$$H_\rho(F(x), F(y)) \le \alpha N_\rho^F(x, y),$$

for all $x, y \in X$;

(v) *$SFixF \neq \emptyset$.*

Then we have:

1. *$FixF = SFixF = \{x^*\}$;*
2. *$H_\rho(F^n(x), x^*) \le \alpha^n \rho(x, x^*)$, for all $n \in \mathbb{N}$ and for each $x \in X$;*
3. *$\rho(x, x^*) \le \frac{s}{1-s\alpha} H_\rho(x, F(x))$, for all $x \in X$;*
4. *The fixed point problem is well-posed for F with respect to D_ρ and with respect to H_ρ, too.*

Proof. 1. We suppose that $x^* \in SFixF$. Let $y \in SFixF$. Then we have

$$\begin{aligned}\rho(x^*, y) &= H_\rho(F(x^*), F(y)) \\ &\le \alpha \cdot max\{\rho(x^*, y), D_\rho(y, F(y)), \frac{1}{2}[D_\rho(x^*, F(y)) + D_\rho(y, F(x^*))]\} \\ &\le \alpha \cdot max\{\rho(x^*, y), \frac{1}{2}[\rho(x^*, y) + \rho(y, x^*)]\} = \alpha\rho(x^*, y),\end{aligned}$$

for all $x \in X$. So we have that $\rho(x^*, y) = 0$ and in conclusion $x^* = y$.

2. We take in the condition (iv) $y = x^*$. Then we have:

$$\begin{aligned} H_\rho(F(x), F(x^*)) \leq& \alpha \cdot max\{\rho(x, x^*), D_\rho(x^*, F(x^*)), \frac{1}{2}[D_\rho(x, F(x^*)) + D_\rho(x^*, F(x))]\} \\ &= \alpha \cdot max\{\rho(x, x^*), \frac{1}{2}[D_\rho(x, F(x^*)) + D_\rho(x^*, F(x))]\}. \end{aligned}$$

If the maximum is $\rho(x, x^*)$ we have that $H_\rho(F(x), x^*) \leq \alpha\rho(x, x^*)$.

If the maximum is $\frac{1}{2}[D_\rho(x, F(x^*)) + D_\rho(x^*, F(x))]$ we have that

$$\begin{aligned} H_\rho(F(x), x^*) &\leq \frac{\alpha}{2}[D_\rho(x, F(x^*)) + D_\rho(x^*, F(x))] \\ &= \frac{\alpha}{2}[D_\rho(x, F(x^*)) + H_\rho(F(x^*), F(x))] \\ &\leq \frac{\alpha}{2}[\rho(x, F(x^*)) + H_\rho(F(x^*), F(x))]. \end{aligned}$$

So we obtain $H_\rho(F(x^*), F(x)) \leq \frac{\alpha}{2-\alpha}\rho(x, x^*)$.

We take now $max\{\alpha, \frac{\alpha}{2-\alpha}\} = \alpha$ and obtain $H_\rho(F(x^*), F(x)) \leq \alpha\rho(x, x^*)$, for all $x \in X$.

By induction we obtain

$$H_\rho(F^n(x), x^*) \leq \alpha^n\rho(x, x^*), \text{ for all } x \in X.$$

Consider now $y^* \in FixF$. Then $\rho(y^*, x^*) \leq H_\rho(F(y^*), x^*) \leq \alpha^n\rho(y^*, x^*) \to 0$, as $n \to \infty$. Hence $y^* = x^*$.

3. $\rho(x, x^*) \leq s[H_\rho(x, F(x)) + H_\rho(F(x), x^*)] \leq sH_\rho(x, F(x)) + s\alpha\rho(x, x^*)$.

So we obtain

$$\rho(x, x^*) \leq \frac{s}{1 - s\alpha}H_\rho(x, F(x)).$$

4. Let (x_n) be such that $D_\rho(x_n, F(x_n)) \to 0$, as $n \to \infty$. We will prove that $\rho(x_n, x^*) \to 0$, as $n \to \infty$.

Estimating $\rho(x_n, x^*)$ we have

$$\rho(x_n, x^*) \leq s[\rho(x_n, y_n) + D_\rho(y_n, F(x^*))] \leq s[\rho(x_n, y_n) + H_\rho(F(x_n), F(x^*))],$$

for all $y_n \in F(x_n)$ and for each $n \in \mathbb{N}$.

Taking $\inf\limits_{y_n \in F(x_n)}$ we obtain

$$\rho(x_n, x^*) \leq s[D(x_n, F(x_n)) + H(F(x_n), F(x^*))] \leq sD(x_n, F(x_n)) + s\alpha\rho(x_n, x^*).$$

Hence we have $\rho(x_n, x^*) \leq \frac{s}{1-s\alpha} D(x_n, F(x_n)) \to$ as $n \to \infty$. So $x_n \to x^*$. $\square$

We will next give a data dependence result.

Theorem 3.3 *Let X be a nonempty set, d and ρ two b-metrics on X with constants $t > 1$ and respectively $s > 1$ and let $F, T : X \to P(X)$ two multivalued operators. We suppose that:*

(i) *(X, d) is a complete b-metric space;*

(ii) *There exists $c > 0$ such that $d(x, y) \leq c \cdot \rho(x, y)$, for all $x, y \in X$;*

(iii) *$F : (X, d) \to (P(X), H_d)$ is closed;*

(iv) *There exists $0 \leq \alpha < \frac{1}{s}$ such that*

$$H_\rho(F(x), F(y)) \leq \alpha N_\rho^T(x, y),$$

for all $x, y \in X$;

(v) *$SFixF \neq \emptyset$;*

(vi) *$FixT \neq \emptyset$;*

(vii) *There exists $\eta > 0$ such that $H_\rho(F(x), T(x)) \leq \eta$, for all $x \in X$.*

Then

$$H_\rho(FixF, FixT) \leq \frac{s\eta}{1 - s\alpha}.$$

Proof. Let $x^* \in SFixF$ and $y^* \in FixT$. We have that

$$\rho(y^*, x^*) \leq H_\rho(T(y^*), x^*) \leq s[H_\rho(T(y^*), F(y^*)) + H_\rho(F(y^*), x^*)]$$

$$\leq s[\eta + H_\rho(F(y^*), F(x^*))] \leq s[\eta + \alpha\rho(y^*, x^*)].$$

Hence we have $\rho(y^*, x^*) \leq \frac{s\eta}{1-s\alpha}$. $\square$

References

[1] Berinde, V., *Generalized contractions in quasimetric spaces*, Seminar on fixed point theory, Preprint no. **3**(1993), 3-9.

[2] Czerwik, S., *Nonlinear set-valued contraction mappings in B-metric spaces*, Atti Sem. Mat. Univ. Modena, **46**(1998), 263-276.

[3] Petruşel, A., Rus, I.A., *Fixed point theoy for multivalued operators on a set with two metrics*, Fixed Point Theory, **8**(2007), 97-104.

[4] Petruşel, A., Mot, G., Petruşel, G., *Topics in Nonlinear Analysis and Applications to Mathematical Economics*, Casa Cărţii de Ştiinţă, Cluj-Napoca, 2007.

[5] Rus, I.A., Petruşel, A., Sîntămărian, A., *Data dependence of the fixed points set of multivalued weakly Picard operators*, Studia. Univ. "Babeş-Bolyai", Mathematica, **46**(2001), 111-121.

[6] Rus, I.A., *Generalized Contractions and Applications*, Cluj University Press, Cluj-Napoca, 2001.

[7] Singh, S.L., Bhatnagar, C., Mishra, S.N., *Stability of iterative procedures for multivalued maps in metric spaces*, Demonstratio Math., **37**(2005), 905-916.

Department of Applied Mathematics,
Babeş-Bolyai University,
Cluj-Napoca, Romania
E-mail address: bmonica@math.ubbcluj.ro