STUDIA UNIV. "BABEŞ–BOLYAI", MATHEMATICA, Volume **LIII**, Number 4, December 2008

LACUNARY STRONGLY ALMOST SUMMABLE SEQUENCES

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Abstract. The purpose of this paper is to introduce the concepts of $q-$ lacunary strongly almost convergence with respect to a modulus function and $q-$lacunary almost statistical convergence. We establish some connections between $q-$ lacunary strongly almost convergence and $q-$ lacunary almost statistical convergence. It is also shown that if a sequence is $q-$lacunary strongly almost convergent with respect to a modulus function then it is $q-$lacunary almost statistically convergent.

1. **Introduction**

Let w denote the set of all real sequences $x = (x_n)$. By ℓ_∞ and c, we denote respectively the Banach space of bounded and the Banach space of convergent sequences $x = (x_n)$, both normed by $\|x\| = \sup_n |x_n|$. A linear functional $\mathcal{L}$ on ℓ_∞ is said to be a Banach limit [1] if it has the properties

i) $\mathcal{L}(x) \geq 0$ if $x \geq 0$ (i.e. $x_n \geq 0$ for all n),

ii) $\mathcal{L}(e) = 1$, where $e = (1, 1, \ldots)$,

iii) $\mathcal{L}(Dx) = \mathcal{L}(x)$,

where the shift operator D is defined by $(Dx_n) = (x_{n+1})$.

Let $\mathfrak{B}$ be the set of all Banach limits on ℓ_∞. A sequence x is said to be almost convergent to a number L if $\mathcal{L}(x) = L$ for all $\mathcal{L} \in \mathfrak{B}$. Lorentz [11] has shown that x is almost convergent to L if and only if

$$t_{km} = t_{km}(x) = \frac{x_m + x_{m+1} + \ldots + x_{m+k}}{k+1} \to L \text{ as } k \to \infty, \text{ uniformly in } m.$$

Received by the editors: 15.01.2007.

2000 *Mathematics Subject Classification.* 40A05, 40C05, 46A45.

Key words and phrases. statistical convergence, lacunary sequence, modulus function, almost convergence.

Let $\hat{c}$ denote the set of all almost convergent sequences. Maddox [12] and (independently) Freedman et al. [8] have defined x to be strongly almost convergent to a number L if

$$\frac{1}{k+1}\sum_{i=0}^{k}\left|x_{i+m}-L\right|\rightarrow 0 \text{ as } k\rightarrow\infty, \text{ uniformly in } m.$$

Let $[\hat{c}]$ denote the set of all strongly almost convergent sequences. It is easy to see that $[\hat{c}]\subset\hat{c}\subset\ell_{\infty}$. Das and Sahoo [5] defined the sequence space

$$[w\left(p\right)]=\left\{x\in w:\frac{1}{n+1}\sum_{k=0}^{n}\left|t_{km}\left(x-L\right)\right|^{p_k}\rightarrow 0 \text{ as } n\rightarrow\infty, \text{ uniformly in } m\right\}$$

and investigated some of its properties.

The notion of statistical convergence was introduced by Fast [7] and Schoenberg [24] independently. Later on it was further investigated from sequence space point of view and linked with summability theory by Başarır [2], Fridy [9], Maddox [15], Nuray and Savaş [18], Tripathy ([20],[21]) and Salat [23]. Recently, statistical convergence has been studied by various authors ($cf.$ [3], [16], [17]).

The statistical convergence is depended on the density of subsets of $\mathbb{N}$, the set of natural numbers. A subset E of $\mathbb{N}$ is said to have density $\delta\left(E\right)$ if

$$\delta\left(E\right)=\lim_{n\rightarrow\infty}\frac{1}{n}\sum_{k=1}^{n}\chi_{E}\left(k\right) \text{ exists},$$

where χ_E is the characteristic function of E.

A sequence $x\in w$ is said to be statistically convergent to L if for every $\varepsilon>0$, $\delta\left(\left\{k\in\mathbb{N}:\left|x_k-L\right|\geq\varepsilon\right\}\right)=0$. In this case we write stat-$\lim x_k=L$.

Let $\theta=(k_r)$ be the sequence of positive integers such that $k_0=0$, $0<k_r<k_{r+1}$ and $h_r=k_r-k_{r-1}\rightarrow\infty$ as $r\rightarrow\infty$. Then θ is called a lacunary sequence. The intervals determined by θ will be denoted by $I_r=(k_{r-1},k_r]$ and the ratio k_r/k_{r-1} will be denoted by η_r.

Lacunary sequences have been studied in [4], [8], [10], [19].

We recall that a modulus f is a function from $[0,\infty)$ to $[0,\infty)$ such that

i) $f(x)=0$ if and only if $x=0$,

ii) $f(x+y)\leq f(x)+f(y)$ for $x,y\geq 0$,

iii) f is increasing,

iv) f is continuous from the right at 0.

It follows that f must be continuous everywhere on $[0, \infty)$. A modulus may be unbounded or bounded. Ruckle [22] and Maddox [13] used a modulus f to construct some sequence spaces.

A sequence space E is said to be solid (or normal) if $(\alpha_k x_k) \in E$ whenever $(x_k) \in E$ for all sequences (α_k) of scalars with $|\alpha_k| \leq 1$ for all $k \in \mathbb{N}$.

2. **Definitions and Preliminaries**

Let f be a modulus function, $p = (p_k)$ be a sequence of positive real numbers and X be a seminormed space over the field $\mathbb{C}$ of complex numbers with the seminorm q. $w(X)$ denotes the space of all sequences $x = (x_k)$, where $x_k \in X$. We define the following sequence spaces:

$$(w, \theta, f, p, q) = \{x \in w(X) : \lim_r \frac{1}{h_r} \sum_{k \in I_r} [f(q(t_{km}(x - L)))]^{p_k} = 0,$$

$$\text{uniformly in } m, \text{for some } L\},$$

$$(w, \theta, f, p, q)_0 = \{x \in w(X) : \lim_r \frac{1}{h_r} \sum_{k \in I_r} [f(q(t_{km}(x)))]^{p_k} = 0, \text{ uniformly in } m\},$$

$$(w, \theta, f, p, q)_\infty = \{x \in w(X) : \sup_{r,m} \frac{1}{h_r} \sum_{k \in I_r} [f(q(t_{km}(x)))]^{p_k} < \infty\}.$$

Throughout the paper Z denotes 0, 1 or ∞. We get the following sequence spaces from the above sequence spaces on giving particular values to θ, f and p.

i) If $p_k = 1$ for all $k \in \mathbb{N}$, then we shall write $(w, \theta, f, q)_Z$ instead of $(w, \theta, f, p, q)_Z$.

If $x \in (w, \theta, f, q)$ we say that x is $q-$lacunary almost strongly convergent with respect to the modulus function f.

ii) Taking $p_k = 1$ for all $k \in \mathbb{N}$ and $f(x) = x$, we denote the above sequence spaces by $(w, \theta, q)_Z$.

iii) In the case $\theta = (2^r)$, then we shall denote the above sequence spaces by $(w, f, p, q)_Z$.

Theorem 2.1 Let f be a modulus function, then $(w,\theta,f,p,q)_0 \subset (w,\theta,f,p,q) \subset (w,\theta,f,p,q)_\infty$.

Proof. The first inclusion is obvious. We establish the second inclusion. Let $x \in (w,\theta,f,p,q)$. By definition of f we have

$$\frac{1}{h_r}\sum_{k\in I_r}\left[f\left(q\left(t_{km}\left(x\right)\right)\right)\right]^{p_k} = \frac{1}{h_r}\sum_{k\in I_r}\left[f\left(q\left(t_{km}\left(x-L+L\right)\right)\right)\right]^{p_k}$$

$$\leq C\frac{1}{h_r}\sum_{k\in I_r}\left[f\left(q\left(t_{km}\left(x-L\right)\right)\right)\right]^{p_k} + C\frac{1}{h_r}\sum_{k\in I_r}\left[f\left(q\left(L\right)\right)\right]^{p_k}.$$

There exists a positive integer K_L such that $q\left(L\right) \leq K_L$. Hence we have

$$\frac{1}{h_r}\sum_{k\in I_r}\left[f\left(q\left(t_{km}\left(x\right)\right)\right)\right]^{p_k} \leq C\frac{1}{h_r}\sum_{k\in I_r}\left[f\left(q\left(t_{km}\left(x-L\right)\right)\right)\right]^{p_k} + \frac{C}{h_r}\left[K_L f(1)\right]^H h_r,$$

where $\sup_k p_k = H$ and $C = \max(1, 2^{H-1})$. Since $x \in (w,\theta,f,p,q)$, we have $x \in (w,\theta,f,p,q)_\infty$ and this completes the proof.

The following theorem can be proved using the same technique of Theorem 2.1 of Et [6], therefore we give without proof.

Theorem 2.2 Let the sequence (p_k) be bounded, then $(w,\theta,f,p,q)_Z$ are linear spaces over the set of complex numbers.

The proof of the following results are easy and thus omitted.

Theorem 2.3 Let $f,\ f_1,\ f_2$ be modulus function. For any two sequences $p = (p_k)$ and $t = (t_k)$ of strictly positive real numbers and any two seminorms q_1 , q_2 we have

i) $(w,\theta,f_1,q)_Z \subset (w,\theta,f\circ f_1,q)_Z$,

ii) $(w,\theta,f_1,p,q)_Z \cap (w,\theta,f_2,p,q)_Z \subset (w,\theta,f_1+f_2,p,q)_Z$,

iii) $(w,\theta,f,p,q_1)_Z \cap (w,\theta,f,p,q_2) \subset (w,\theta,f,p,q_1+q_2)$,

iv) If q_1 is stronger than q_2 then $(w,\theta,f,p,q_1)_Z \subset (w,\theta,f,p,q_2)_Z$,

v) If q_1 equivalent to q_2 then $(w,\theta,f,p,q_1)_Z = (w,\theta,f,p,q_2)_Z$,

vi) $(w,\theta,f,p,q)_Z \cap (w,\theta,f,t,q)_Z \neq \emptyset$.

The following result is a consequence of Theorem 2.3 (i).

Proposition 2.4 Let f be a modulus function. Then $(w,\theta,q)_Z \subset (w,\theta,f,q)_Z$.

Theorem 2.5 Let f be a modulus function, if $\lim \frac{f(t)}{t} = \beta > 0$, then $(w,\theta,q) = (w,\theta,f,q)$.

Proof. By Proposition 2.4, we need only show that $(w,\theta,f,q) \subset (w,\theta,q)$. Let $\beta > 0$ and

$x \in (w,\theta,f,q)$. Since $\beta > 0$, we have $f(t) \geq \beta t$ for all $t \geq 0$. Hence we have

$$\frac{1}{h_r}\sum_{k\in I_r} f\left(q\left(t_{km}\left(x-L\right)\right)\right) \geq \frac{\beta}{h_r}\sum_{k\in I_r} q\left(t_{km}\left(x-L\right)\right).$$

Therefore we have $x \in (w,\theta,q)$.

Theorem 2.6 Let $0 < p_k \leq t_k$ and $\left(\frac{t_k}{p_k}\right)$ be bounded, then $(w,\theta,f,t,q)_Z \subset (w,\theta,f,p,q)_Z$.

Proof. If we take $w_{km} = \left[f\left(q\left(t_{km}\left(x\right)\right)\right)\right]^{t_k}$ for all k,m and using the same technique of Theorem 5 of Maddox [14], it is easy to prove this Theorem.

Theorem 2.7 The sequence spaces $(w,\theta,f,p,q)_0$ and $(w,\theta,f,p,q)_\infty$ are not solid.

Proof. We give the proof only for $(w,\theta,f,p,q)_0$. For this let $p_k = 1$ for all $k \in \mathbb{N}$, $\theta = (2^r)$, $f(x) = x$ and $q(x) = |x|$. Consider the sequence $x_k = (-1)^k$ for all $k \in \mathbb{N}$ and (α_k) be defined as $\alpha_k = (-1)^k$ for all $k \in \mathbb{N}$. Then $(x_k) \in (w,\theta,f,p,q)_0$ but $(\alpha_k x_k) \notin (w,\theta,f,p,q)_0$. Hence $(w,\theta,f,p,q)_0$ is not solid.

Theorem 2.8 Let $\theta = (k_r)$ be a lacunary sequence. If $1 < \liminf_r \eta_r \leq \limsup_r \eta_r < \infty$ then for any modulus function f, we have $(w,f,p,q)_0 = (w,\theta,f,p,q)_0$.

Proof. Suppose $\liminf_r \eta_r > 1$ then there exist $\delta > 0$ such that $\eta_r = \left(\frac{k_r}{k_{r-1}}\right) \geq 1+\delta$ for all $r \geq 1$. Then for $x \in (w,f,p,q)_0$, we write

$$\frac{1}{h_r}\sum_{k\in I_r}\left[f\left(q\left(t_{km}\left(x\right)\right)\right)\right]^{p_k} = \frac{1}{h_r}\sum_{k=1}^{k_r}\left[f\left(q\left(t_{km}\left(x\right)\right)\right)\right]^{p_k} - \frac{1}{h_r}\sum_{k=1}^{k_{r-1}}\left[f\left(q\left(t_{km}\left(x\right)\right)\right)\right]^{p_k}$$

$$= \frac{k_r}{h_r}\left(k_r^{-1}\sum_{k=1}^{k_r}\left[f\left(q\left(t_{km}\left(x\right)\right)\right)\right]^{p_k}\right) - \frac{k_{r-1}}{h_r}\left(k_{r-1}^{-1}\sum_{k=1}^{k_{r-1}}\left[f\left(q\left(t_{km}\left(x\right)\right)\right)\right]^{p_k}\right).$$

Since $h_r = k_r - k_{r-1}$, we have

$$\frac{k_r}{h_r} \leq \frac{1+\delta}{\delta}$$

and

$$\frac{k_{r-1}}{h_r} \leq \frac{1}{\delta}.$$

The terms $k_r^{-1}\sum_{k=1}^{k_r}\left[f\left(q\left(t_{km}\left(x\right)\right)\right)\right]^{p_k}$ and $k_{r-1}^{-1}\sum_{k=1}^{k_{r-1}}\left[f\left(q\left(t_{km}\left(x\right)\right)\right)\right]^{p_k}$ both converge to zero, uniformly in m and it follows that

$$\frac{1}{h_r}\sum_{k\in I_r}\left[f\left(q\left(t_{km}\left(x\right)\right)\right)\right]^{p_k} \to 0,$$

as $r \to \infty$ uniformly in m, that is, $x \in \left(w,\theta,f,p,q\right)_0$.

If $\limsup_r \eta_r < \infty$, there exists $B > 0$ such that $\eta_r < B$ for all $r \geq 1$. Let $x \in \left(w,\theta,f,p,q\right)_0$ and $\varepsilon > 0$ be given. Then there exits $R > 0$ such that for every $j \geq R$ and all m

$$A_j = \frac{1}{h_j}\sum_{k\in I_j}\left[f\left(q\left(t_{km}\left(x\right)\right)\right)\right]^{p_k} < \varepsilon.$$

We can also find $K > 0$ such that $A_j < K$ for all $j = 1, 2, \ldots$. Now let t be any integer with $k_{r-1} < t \leq k_r$, where $r > R$. Then

$$t^{-1}\sum_{k=1}^{t}\left[f\left(q\left(t_{km}\left(x\right)\right)\right)\right]^{p_k} \leq k_{r-1}^{-1}\sum_{k=1}^{k_r}\left[f\left(q\left(t_{km}\left(x\right)\right)\right)\right]^{p_k}$$

$$= k_{r-1}^{-1}\sum_{k\in I_1}\left[f\left(q\left(t_{km}\left(x\right)\right)\right)\right]^{p_k} + k_{r-1}^{-1}\sum_{k\in I_2}\left[f\left(q\left(t_{km}\left(x\right)\right)\right)\right]^{p_k} +$$

$$\ldots + k_{r-1}^{-1}\sum_{k\in I_r}\left[f\left(q\left(t_{km}\left(x\right)\right)\right)\right]^{p_k}$$

$$= \frac{k_1}{k_{r-1}}k_1^{-1}\sum_{k\in I_1}\left[f\left(q\left(t_{km}\left(x\right)\right)\right)\right]^{p_k} + \frac{k_2-k_1}{k_{r-1}}\left(k_2-k_1\right)^{-1}\sum_{k\in I_2}\left[f\left(q\left(t_{km}\left(x\right)\right)\right)\right]^{p_k}$$

$$+ \ldots + \frac{k_R-k_{R-1}}{k_{r-1}}\left(k_R-k_{R-1}\right)^{-1}\sum_{k\in I_R}\left[f\left(q\left(t_{km}\left(x\right)\right)\right)\right]^{p_k} +$$

$$\ldots + \frac{k_r-k_{r-1}}{k_{r-1}}\left(k_r-k_{r-1}\right)^{-1}\sum_{k\in I_r}\left[f\left(q\left(t_{km}\left(x\right)\right)\right)\right]^{p_k}$$

$$= \frac{k_1}{k_{r-1}}A_1 + \frac{k_2 - k_1}{k_{r-1}}A_2 + \ldots + \frac{k_R - k_{R-1}}{k_{r-1}}A_R + \frac{k_{R+1} - k_R}{k_{r-1}}A_{R+1} + \ldots + \frac{k_r - k_{r-1}}{k_{r-1}}A_r$$

$$\leq \left(\sup_{j\geq 1} A_j\right)\frac{k_R}{k_{r-1}} + \left(\sup_{j\geq R} A_j\right)\frac{k_r - k_R}{k_{r-1}} \leq K\frac{k_R}{k_{r-1}} + \varepsilon B.$$

Since $k_{r-1} \to \infty$ as $t \to \infty$, it follows that $t^{-1}\sum_{k=1}^{t}\left[f\left(q\left(t_{km}\left(x\right)\right)\right)\right]^{p_k} \to 0$ uniformly in m and consequently $x \in (w, f, p, q)_0$.

3. $q-$ lacunary almost statistical convergence

In this section we give some relations between $q-$lacunary almost statistical convergence and $q-$lacunary strongly almost convergence with respect to the modulus functions f.

Definition 3.1 ([3]) Let θ be a lacunary sequence, then the sequence $x = (x_k)$ is said to be $q-$lacunary almost statistically convergent to the number L provided that for every $\varepsilon > 0$,

$$\lim_r \frac{1}{h_r}\left|\left\{k \in I_r : q\left(t_{km}\left(x - L\right)\right) \geq \varepsilon\right\}\right| = 0, \text{ uniformly in } m.$$

In this case we write $[S_\theta]_q - \lim x = L$ or $x_k \to L\left([S_\theta]_q\right)$ and we define

$$[S_\theta]_q = \left\{x \in w\left(X\right) : \ [S_\theta]_q - \lim x = L, \text{ for some } L\right\}.$$

In the case $\theta = (2^r)$, we shall write $[S]_q$ instead of $[S_\theta]_q$.

Theorem 3.2 Let f be a modulus function and $0 < h = \inf_k p_k \leq p_k \leq \sup_k p_k = H < \infty$. Then $(w, \theta, f, p, q) \subset [S_\theta]_q$.

Proof. Let $x \in (w, \theta, f, p, q)$ and $\varepsilon > 0$ be given. Then

$$\frac{1}{h_r}\sum_{k\in I_r}\left[f\left(q\left(t_{km}\left(x-L\right)\right)\right)\right]^{p_k} \geq \frac{1}{h_r}\sum_{\substack{k\in I_r \\ q(t_{km}(x-L))\geq\varepsilon}}\left[f\left(q\left(t_{km}\left(x-L\right)\right)\right)\right]^{p_k}$$

$$\geq \frac{1}{h_r} \sum_{\substack{k \in I_r \\ q(t_{km}(x-L)) \geq \varepsilon}} [f(\varepsilon)]^{p_k}$$

$$\geq \frac{1}{h_r} \sum_{\substack{k \in I_r \\ q(t_{km}(x-L)) \geq \varepsilon}} \min\left([f(\varepsilon)]^h, [f(\varepsilon)]^H\right)$$

$$\geq \frac{1}{h_r} |\{k \in I_r : q(t_{km}(x-L)) \geq \varepsilon\}| \min\left([f(\varepsilon)]^h, [f(\varepsilon)]^H\right).$$

Hence $x \in [S_\theta]_q$.

Theorem 3.3 Let f be bounded and $0 < h = \inf_k p_k \leq p_k \leq \sup_k p_k = H < \infty$. Then $[S_\theta]_q \subset (w, \theta, f, p, q)$.

Proof. Suppose that f is bounded. Then there exists an integer K such that $f(t) < K$, for all $t \geq 0$. Then

$$\frac{1}{h_r} \sum_{k \in I_r} [f(q(t_{km}(x-L)))]^{p_k} = \frac{1}{h_r} \sum_{\substack{k \in I_r \\ q(t_{km}(x-L)) \geq \varepsilon}} [f(q(t_{km}(x-L)))]^{p_k}$$

$$+\frac{1}{h_r} \sum_{\substack{k \in I_r \\ q(t_{km}(x-L)) < \varepsilon}} [f(q(t_{km}(x-L)))]^{p_k}$$

$$\leq \frac{1}{h_r} \sum_{\substack{k \in I_r \\ q(t_{km}(x-L)) \geq \varepsilon}} \max\left(K^h, K^H\right) + \frac{1}{h_r} \sum_{\substack{k \in I_r \\ q(t_{km}(x-L)) < \varepsilon}} [f(\varepsilon)]^{p_k}$$

$$\leq \max\left(K^h, K^H\right) \frac{1}{h_r} |\{k \in I_r : q(t_{km}(x-L)) \geq \varepsilon\}| + \max\left([f(\varepsilon)]^h, \ [f(\varepsilon)]^H\right).$$

Hence $x \in (w, \theta, f, p, q)$.

Theorem 3.4 $[S_\theta]_q = (w, \theta, f, p, q)$ if and only if f is bounded.

Proof. Let f be bounded. By the Theorem 3.2 and Theorem 3.3, we have $[S_\theta]_q = (w, \theta, f, p, q)$.

Conversely, suppose that f is unbounded. Then there exists a positive sequence (t_n) with $f(t_n) = n^2$, $n = 1, 2, \cdots$. If we choose

$$x_k = \begin{cases} t_n, & k = n^2,\ n = 1, 2, \ldots \\ 0, & \text{otherwise} \end{cases}$$

then we have

$$\frac{1}{n} \left| \{k \leq n : |x_k| \geq \varepsilon\} \right| \leq \frac{\sqrt{n}}{n} \to 0,\ n \to \infty.$$

Hence $x_k \to 0([S_\theta]_q)$ for $t_{0m}(x) = x_m$, $\theta = (2^r)$ and $q(x) = |x|$, but $x \notin (w, \theta, f, q)$. This contradicts to $[S_\theta]_q = (w, \theta, f, p, q)$.

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