BOOK REVIEWS

S.V. Emelyanov, S.K. Korovin, N.A. Bobylev, A.V. Bulatov, ***Homotopy of Extremal Problems***, Walter de Gruyter, Berlin - New York, 2007, 303 pp, ISBN 978-1-11-018942-1

The idea of homotopy appears in many branches of mathematics such as algebraic topology, differential topology, nonlinear analysis, variational calculus etc. Its importance comes from the invariance of some powerful tools, such as the *induced homomorphism* at the level of various homology/cohomology groups, the *degree*, the *Conley index* etc., on corresponding homotopy classes of maps or homotopically deformed spaces.

This book deals with the homotopy method applied in variational calculus and is structured in five chapters as follows:

The first chapter presents some classical facts on certain spaces of functions and their various topologies as well as on some special operators/functionals such as linear, nonlinear, monotone and potential operators as well as Lipschitzian and convex functionals. Among these facts we mention the presence of some necessary and sufficient conditions on a functional in order that either one of its critical points is a local minimizer or the functional itself is convex, strictly convex or strongly convex.

The second chapter starts with a sufficient condition, in finite dimensional context on a continuously differentiable deformation in order for a local minimizer of the initial function to be deformed into a local minimizer of the final one. This type of results are called *deformation principles for minimizers* and they are present all along the book. As a consequence one gets a sufficient condition, in terms of gradients, on two continuously differentiable functions of finitely many variables in order for a local minimizer of one of them to be a local minimizer of the other one. These type of results are then extended to the class of lipschitzian functions, in which case the role of the gradients is played by the generalized gradients, and even to the class of continuous functions. The chapter ends with a proof of the Hopf theorem on self maps of the N-sphere of zero degree, which are proved to be homotopic to each other, and with the Parusinski theorem. The last one concerns the gradient vector fields on the N-ball which are nondegenerate on the $(N-1)$-sphere and homotopic in the class of continuous vector fields, which are proved to be gradient homotopic.

The third chapter deals with problems similar to those treated in the second chapter, but in infinite dimensional setting.

The fourth chapter starts with some elementary facts on flows and then defines the Conley index of a set which is invariant with respect to a flow. The Conley index is proved to be invariant under a deformation of the flow and the initial invariant set. The isolated critical points of a differentiable function of finitely many variables are proved to be invariant sets with respect to the gradient flow and the Conley index of such a point is explicitly computed. Eventually, the Conley index is defined and studied in infinite dimensional context as well.

The fifth chapter is devoted to applications of the homotopy invariance of minimizers and of the Conley index. Among them we mention some deformation theorems and invariance of the global minimizers for the classical nonlinear programming problems, multicriteria problems, week minimizers problems, optimal control problems and bifurcation points problems. Stability of solutions of ordinary differential equations, focused on stability of gradient systems, stability of Hamiltonian systems and stability of dynamical systems, are also treated in this chapter.

The book is very well written and combines the power of homotopy methods with results coming from functional analysis, differential equations, variational calculus and other mathematical fields, either to prove some well known facts or to get relatively recent results.

It is useful for researchers in variational calculus and/or optimization desiring to be acquainted with the powerful tools of homotopy theory as well as for those working in homotopy theory, looking for applications.

Cornel Pintea

Beata Randrianatoanina and Narcisse Randrianatoanina (Editors) ***Banach Spaces and their Applications in Analysis*** - In Honor of Nigel Kalton's 60th Birhday, Walter de Gruyter • Berlin • New York, 2007, ix + 453 pp, ISBN: 978-3-11-019449-4

In recent years a lot of problems in analysis, apparently far from the theory of Banach spaces, were solved using Banach space methods. The aim of this conference was to bring together specialists who have been involved in these developments to honor the 60th birthday of Nigel Kalton. An excellent survey on Kalton's influential work in functional analysis and its applications is given in the introductory paper by Gilles Godefroy. It deals with quasi-Banach spaces and p-normed spaces (called *The Kalton zone*: $0 < p < 1$), non-linear geometry (mainly Lipschitz), isometric theory, interpolation and twisted sums, multipliers in spaces of vector functions. Although impressive, this survey covers only a part of the fundamental contributions Professor Kalton made in various areas of analysis.

The topics of the conference were:

1. Nonlinear theory (Lipschitz classification of Banach and metric spaces);
2. Isomorphism theory of Banach spaces (including connections with combinatorics and set theory);
3. Algebraic and homological methods in Banach spaces;
4. Approximation theory and algorithms in Banach spaces (greedy approximation, interpolation, abstract approximation theory);
5. Functional calculus and applications to partial differential equations.

The Conference was attended by over 160 mathematicians from around the world who delivered 15 plenary talk and 105 talks in specialized sessions. The present Proceedings reflect this situation - they contain 11 papers by plenary speakers and 18 specialized papers. In the following we shall mention some of them.

Concerning the first topic there are a survey paper by J. Lindenstrauss, D. Preis anf J. Tišer on the differentiability of Lipschitz functions on Banach spaces (a book dedicated to this topic is announced), J. Duda and O. Maleva (metric differentiability), A. Kaminska and A. M. Parrish (q-concavity and q-convexity in Lorentz spaces), R. Ni (fixed points of Φ-contractive mappings), T. Oikhberg (the Daugavet property), O. Brezhneva and A. Tretyakov (implicit function theorem for nonregular mappings in Banach spaces). Some papers dealing with the second theme , isompor-phic theory of Banach spaces, are those by V. Ferenczi and C. Rosendal (complexity and homogeneity in Banach spaces), E. Odell, Th. Schlumprecht, A. Zsák (a new infinite game in Banach spaces), G. Androulakis and F. Sanacory (equivalent norms on Hilbert space), M. Gonzales and M. Wójtowicz (quotients of $\ell_1(\Gamma)$), J. Talponen (asymptotically transitive Banach spaces).

Some approximation problems in Banach space setting are treated in the papers of Y. Brudnyi (multivariate functions of bounded variation), V. Temlyakov (greedy approximation in Banach spaces), P. Bandyopadhyay, B.-L. Lin and T. S. S. R. K. Rao (ball proximity in Banach spaces), R. Vershynin (numerical algorithms in asymptotic convex geometry).

There are some papers dealing with analysis of vector functions as, for instance, J. van Neerven, M. Veraar and Lutz Weis (stochastic integrability in UMD Banach spaces), M. D. Acosta, L. A. Morales (boundaries of spaces of holomorphic functions), T. Hytönen (a probabilistic Littlewood-Paley theory in Banach spaces).

Emphasizing connections between seemingly distant areas of analysis and illustrating the power and versatility of applications of Banach space theory, the volume will be of great interest to researchers in various domains of mathematics, especially to those interested in Banach space methods.

I. V. Şerb

Cédric Villani, ***Topics in Optimal Transportation***, American Mathematical Society, Graduate Studies in Mathematics, Volume 58, Providence, Rhode Island 2003, ISBN:0-8218-3312-X

The mass transportation problem (MTP) as posed initially in 1871 by Gaspard Monge in his paper *Mémoire sur la théorie des déblais et des remblais*, consists in finding an optimal volume-preserving map between two sets X, Y of equal volume. The optimality is evaluated by a cost function $c(x, y)$ representing the cost per unit mass for transporting from $x \in X$ to $y \in Y$, and one asks to minimize $I[T] = \int_X c(x, T(x))d\mu(x)$ over all transportation plans T. The functional $I[T]$ is non-linear in the transportation plan T and the set of admissible transportation plans is a nonconvex set, explaining the difficulty of this problem. A solution in the case $c(x, y) = |x - y|$ considered by Monge for - the Euclidean distance, was given only in 1979 by Sudakov in a 178 pages paper published as a volume of Trudy of the Steklov Institute. Recently some inaccuracies in Sudakov's paper were fixed by Alberti, Kircheim and Preis.

In 1942 L. V. Kantorovich proposed a new approach to the problem asking for the minimization of the functional $I[\pi] = \int_{X\times Y} c(x,y)d\pi(x,y)$ for $\pi \in \Pi(\mu,\nu)$. Here X, Y are Polish spaces (i.e., complete metrizable topological spaces), μ, ν regular probability measures on X and Y respectively, and $\Pi(\mu,\nu)$ denotes the set of all probability measures on $X \times Y$ with marginals μ, ν. In this way the nonlinear original Monge problem becomes a linear optimization problem over a convex sets of probability measures, allowing the use of the tools of linear programming and leading to the famous Kantorovich-Rubinshtein duality theorem. For this reason Kantorovich MTP is easy to solve that the original Monge MTP. At the same time it can be considered as a relaxation of Monge problems. It is worth to mention that Kantorovich contributions to the related problem of optimal allocation of resources earned him, jointly with Koopmans, the 1975 Nobel prize in economy.

It turned out that the MTP is a prototype for a class of problems arising in various fields as functional analysis, probability and statistics, linear and stochastic programming, differential geometry, with numerous applications to fluid mechanics, quantum physics and other domains. At the same time the solution of MTP requires tools, methods and results from these domains, explaining its beauty and the great appeal of MTP for mathematicians of various specialties.

The present book, based on a graduate course taught by the author at the Georgia Tech in the fall of 1999, is a carefully written introduction to various aspects of MTP.

The basic theory is developed in Chapters 1. *The Kantorovich duality*, 2. *Geometry of optimal transportation*, 4. *The Monge-Ampère equation*, and 7. *The metric side of the optimal transportation.* This part must be read by every graduate students to be acquainted with the basic results and tools of the theory. Here the proofs are given in detail, excepting Chapter 4 where the waste and difficult subject of regularity for fully nonlinear elliptic equations is only sketched. Chapter 3. *Brenier's polar factorization theorem*, present some of the motivations from fluid mechanics which led Brebier to his polar factorization theorem proved in 1987. As the author mention in the Preface, this give rise to a revival in the study of MTP "paving the way to a beautiful interplay between differential equations, fluid mechanics, geometry, probability theory and functional analysis". Chapter 5. *Displacement interpolation and displacement convexity,* is concerned with these two important notions, introduced by MvCann in 1994 and some applications.

In Chapter 6. *Geometric and Gaussian inequalities*, the author explains how mass transportation provides powerful tools to study some functional inequalities with geometric content, having as prototype the isoperimetric inequality - the Brunn-Minkowski inequality, the inequality of Prékopa-Leindler, Gaussian inequalities.

Chapters 8. *A differential point of view on optimal transportation*, and 9. *Entropy production and transportation*, are more advanced requiring some basic notions in partial differential equations and functional analysis.

There are a lot of exercises disseminated over the text and the last chapter, 10. *Problems,* gathers longer problems taken from recent research papers.

The book is clearly written and well organized and can be warmly recommended as an introductory text to this multidisciplinary area of research, both pure and applied - the mass transportation problem.

S. Cobzaş

György Darvas, ***Symmetry***, Cultural-Historical and Ontological Aspects of Science-Arts Relations; the Natural and Man-made World in an Interdisciplinary Approach, translated from the Hungarian by David Robert Evans 2007, XI, 508 pp. 420 illus., 66 in color., Softcover ISBN: 978-3-7643-7554-6, Birkhäuser 2007

As its subtitle shows ("Cultural-historical and ontological aspects of science-arts relations. The natural and man-made world in an interdisciplinary approach"), the book "Symmetry" by Darvas György is a wonderful voyage through different sciences and arts all connected by the universal concept of symmetry.

Symmetry (and the lack of it) is a fundamental phenomenon in physics, chemistry, mathematics, biology, psychology, architecture and all kind of arts, creating interesting interferences between these seemingly different subjects.

The book contains 15 chapters the first 4 introducing the basic notions and definitions related to symmetry and outlining its historical evolution. The rest of the chapters present the most typical applications of different appearances of symmetries in the sciences and the humanities. It is important to note the ontological ordering of these chapters: starting from the self-organization of the matter and the inanimate nature, through the formation of organic matter we end up investigating the human creativity. We also emphasize the huge number (350) of pictures and illustration making things much more accessible.

The book avoids difficult mathematical formalisms, however exceeds the limits of popular science being formulated at a university level. In this way it is highly recommended for every student and scientist interested in interdisciplinary interactions.

Cs. Szántó

A. Bensoussan, G. Da Prato, M. C. Delfour, S. K. Mitter, ***Representation and Control of Infinite Dimensional Systems***, Birkhäuser, Boston, 2007, 2nd ed., XXVI + 575 p. 5 illus., Series: Systems & Control: Foundations & Applications, ISBN 978-0-8176-4461-1

This reorganized, revised, and expanded edition is originated in a two-volume set: Representation and Control of Infinite Dimensional Systems (vol. I), Birkhäuser, Basel, 1992, 315 p., Series: Systems & Control: Foundations & Applications, ISBN 3-7643-3641-2 and Representation and Control of Infinite Dimensional Systems (vol. II), Birkhäuser, Boston, 1993, 372 p., Series: Systems & Control: Foundations & Applications, ISBN 978-0-8176-3642-5.

Since the publications of the two volumes in 1992-93 more sophisticated mathematical tools and approaches have been introduced in the field and a whole range of challenging applications appeared from new phenomenological, technological, and design developments. The two volumes have been recognized as key references in the field, hence a revised and corrected edition became desirable.

As the authors state in the Introduction to the book "the primary concern of this book is the control of linear infinite dimensional systems", systems whose state space is infinite dimensional and its evolution is typically described a linear differential equation, linear functional equation or linear integral equation.

Now we introduce the main parts of this impressive book.

Introduction. Part I. Finite dimensional linear control of dynamical systems. Control of linear differential systems. Controllability, observability, duality, stabilizability and detectability. Optimal control. Finite time horizon and infinite time horizon. Dissipative systems. Linear quadratic two-person zero-sum differential games.

Part II. Representation of infinite dimensional linear control dynamical systems. Semi-groups of operators and interpolation. Variational theory of parabolic systems. Semigroup methods for systems with unbounded control and observation operators. State space theory of differential systems with delays.

Part III. Qualitative properties of linear control dynamical systems. Controllability and observability for a class of infinite dimensional systems.

Part IV. Quadratic optimal control: finite time horizon. Bounded control operators: control inside the domain. Unbounded control operators: parabolic equations with control on the boundary. Unbounded control operators: hyperbolic equations with control on the boundary.

Part V. Quadratic optimal control: infinite time horizon. Bounded control operators: control inside the domain. Unbounded control operators: parabolic equations with control on the boundary. Unbounded control operators: hyperbolic equations with control on the boundary.

An isomorphism result is given in the Appendix A. Each part of the book is completed by important comments and/or references.

We mention some new material and original features of the second edition:

• Part I on finite dimensional controlled dynamical systems contains new material: an expanded chapter on the control of linear systems including a glimpse into H-infinity theory and dissipative systems, and a new chapter on linear quadratic two-person zero-sum differential games.

• A unique chapter on semigroup theory and interpolation of linear operators brings together advanced concepts and techniques that are usually treated independently.

• The material on delay systems and structural operators is not available elsewhere in book form.

Control of infinite dimensional systems has a wide range and growing number of challenging applications. This book is a key reference for anyone working on these applications, which arise from new phenomenological studies, new technological developments, and more stringent design requirements. It will be useful for mathematicians, graduate students, and engineers interested in the field and in the underlying conceptual ideas of systems and control.

This book represents a remarkable contribution to the development of this scientific field very useful for mathematicians, theoretical engineers, and, in general, for all the scientists interested in control of infinite dimensional systems.

The book ends with an extensively list of references and a useful index of notions and symbols.

We can state doubtless that in front of us there is a masterpiece on the topic of representation and control of infinite dimensional systems. Certainly this book will be included in many libraries all over the world.

Marian Mureşan

Dorothee D. Haroshke and Hans Trienbel, *Distributions, Sobolev Spaces, Elliptic Equations,* EMS Textbooks in Mathematics, European Mathematical Society, Zürich 2008, ix+294 pp, ISBN: 978-3-03719-042-5.

The book is based on two-semester courses taught several times over a period of ten years by the authors to graduate students and PhD students at the Friedrich Schiller University in Jena. Its aim is to give a gentle introduction to the basic results and techniques of the L_2 theory of elliptic differential operators of second order on bounded domains in $\mathbb{R}^n$. The prerequisites are calculus, measure theory and basic elements of functional analysis.

The book starts with the classical Laplace-Poisson equations and harmonic functions. The basic properties of distributions, including Fourier transform, are treated in the second chapter.

Chapters 3. *Sobolev spaces on* $\mathbb{R}^n$ *and* $\mathbb{R}^n_+$, and 4. *Sobolev spaces on domains,* constitute a self-contained introduction to the basic properties of Sobolev spaces - embeddings, extensions, traces.

The fifth chapter, *Elliptic operators in* L_2, is concerned with the L_2 theory of general elliptic operators on bounded domains Ω in $\mathbb{R}^n$, having as leading model the Laplacian studied in the first chapter. This study concerns: a priori estimates, homogeneous boundary problems, inhomogeneous boundary problems, smoothness theory, Green functions and Sobolev embeddings, degenerate elliptic operators. Chapter 6. *Spectral theory in Hilbert spaces and Banach spaces*, is a short introduction to spectral theory of self-adjoint operators in Hilbert space, approximation numbers, entropy numbers. This machinery is applied in the seventh chapter, *Compact embeddings, spectral theory of elliptic operators*, to the study of distribution of the eigenvalues and of the associated eigenelements of the self-adjoint operator $Au = -\sum_{j,k=1}^{n} \frac{\partial}{\partial x_j}\big(a_{j,k}(x)\frac{\partial u}{\partial x_k}\big) - a(x)u,\ \mathrm{dom}A = W^2_{2,0}(\Omega)$.

The book ends with six appendices: A. *Domains, basic spaces, and integral formulae,* B. *Orthonormal bases of trigonometric functions,* C. *Operator theory,* D. *Some integral inequalities,* E. *Function spaces,* collecting the basic notions and results used in the main text, or presenting more general function spaces (Appendix E), references to which were made in the Notes from the end of the chapters. A thorough and detailed presentation of these spaces is given in the recent books of the second-named author: *Theory of Function Spaces II,* Birkhäuser 1992, and *Theory of Function Spaces III,* Birkhäuser 2006.

Written by two leading experts in the area and including their teaching experience, the book is of great use for students and mathematicians looking for an accessible introduction to function spaces and partial differential equations. After its reading, more advanced and difficult texts on similar topics can be successfully approached with less effort.

S. Cobzaş

William Byers, ***How Mathematicians Think Using Ambiguity, Contradiction, and Paradox to Create Mathematics,*** Princeton University Press, 415 pages, ISBN-13:978-0-691-12738-5.

There are very much number of paper on the nature of mathematical thinking, on how mathematicians create mathematics. Here are some basic books on this direction:

- J. Hadamard, The Psychology of Invention in the Mathematical Field, Princeton University Press, 1949.
- H. Poincaré, Science and Hypothesis, Dover, New York, 1952.
- H. Weyl, Philosophy of Mathematics and Natural Science, Princeton University Press, 1949.
- P. Sergescu, Gândirea matematică (The mathematical thinking), Ed. Ardealul, Cluj, 1928 (Romanian).
- A. Froda, Eroare şi paradox în matematică, (Error and paradox in mathematics) Ed. Enciclopedică Română, Bucureşti, 1971 (Romanian).
- M. Kline, Mathematical Thought from Ancient to Modern Times, Oxford University Press, 1972.
- J. Dieudonné, Mathématique vides et mathématique significatives, Luxemburg, 1976.
- I. Lakatos, Proofs of Refutations, Cambridge University Press, 1976.
- R.L. Wilder, Mathematics as a Cultural System, Pergamon Press, New York, 1981.
- S. Mac Lane, Mathematics: Form and Function, Springer, New York, 1986.
- R. Penrose, The Emperor's New Mind, Oxford University Press, 1989 (Romanian translation: Ed. Tehnică, 1996).
- B. Heinz, Die Innenwelt der Mathematik, Springer, 2000.
- R. Hersch (Ed.), 18 Unconventional Essays on the Nature of Mathematics, Springer, 2005.

Byers's book provides a novel approach to many questions such as:

- Is mathematics objectively true?

- Is mathematics discovered and/or invented?
- Where does mathematical creativity come from?
- Is mathematical thought algorithmic in nature?

The book is divided into three sections: The light of ambiguity (Ambiguity in Mathematics, The Contradictory in Mathematics, Paradoxes and Mathematics: Infinity and the Real Numbers), The light as idea (The Idea as an Organizing Principle, Ideas, Logic and Paradox, Great Ideas) and The light and the eye of the beholder (The Truth of Mathematics, Conclusion: Is Mathematics Algorithmic or Creative?).

Well-organized and carefully written the present book is very useful to all who are interested in "How Mathematicians Think"! A related question could be: "Do mathematicians really think?"

Ioan A. Rus