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A MONOTONY METHOD IN QUASISTATIC PROCESSES FOR VISCOPLASTIC MATERIALS

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Abstract. In this paper, we study a quasistatic problem for semi-linear rate-type viscoplastic models with two parameters χ, θ; χ may be interpreted as the absolute temperature or an internal state variable. The existence and uniqueness of the solution is proved using monotony arguments followed by a Cauchy-Lipschitz technique.

1. **Introduction**

Throughout the paper, Ω is a bounded in $IR^N (N = 1, 2, 3)$ with a smooth boundary $\partial\Omega = \Gamma$ and Γ_1 is an open subset of Γ such that $meas\Gamma_1 > 0$. We denote $\Gamma_2 = \Gamma \ -\overline{\Gamma}_1$. Let ν be the outward unit normal vector on Γ and S_N the set of second order symmetric tensors on IR^N. Let T be a real positive constant.LET us the mixed problem.

$$\dot{\sigma} = \mathcal{E}(\varepsilon(\dot{u}), \theta, \chi) + F(\sigma, \varepsilon(u), \theta) \quad \text{in } \Omega \times (0, T) \tag{1}$$

$$Div\ \sigma + f = 0 \quad \text{in } \Omega \times (0, T) \tag{2}$$

$$u = g \quad \text{on } on\Gamma_1 \times (0, T) \tag{3}$$

$$\sigma\nu = h \quad \text{on } \Gamma_2 \times (0, T) \tag{4}$$

$$u(0) = u_0, \quad \sigma(0) = \sigma_0 \quad \text{in } \Omega \tag{5}$$

in which the unknowns are the displacement function $u : \Omega \times [0, T] \rightarrow R^N$, the stress function $\sigma : \Omega \times [0, T] \rightarrow S_N$ This problem represents a quasistatic problem for rate-type models of the form (1) in with ε is a nonlinear function depending on

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$\varepsilon(\dot{u})$, θ and χ ,are parameters and $\varepsilon(u) : \Omega \times [0,T] \rightarrow S_N$ is the small strain tensor (i.e. $\varepsilon(u) = \frac{1}{2}\nabla u + \nabla^t u$). In (1) $\mathcal{E}$ and F are given constitutive function .

In (2) $Div\ \sigma$ represent the divergence of vector valued function σ and f represents the given body force, g and h are the given bounded data and, finally, u_0, σ_0 are the initial data.

In the case when ε depends only on χ, existence and uniqueness results for problems of the form (1)-(5) was obtained by Sofonea (1991) reducing the studied problem to an ordinary differential equation in a Hilbert space. In the case when $\mathcal{E}$ is a nonlinear function depending only on $\varepsilon(\dot{u})$ and χ existence and uniqueness results for problems of the form (1)-(5) was obtained by Djabi (1993) using monotony arguments followed by a Cauchy-Lipschitz technique.

The purpose of this paper is to give a now proof for the existence and uniqueness of the solution for the problem (1)-(5) there based only on monotony arguments followed by a Cauchy-Lipschitz technique (theorem 3.1).

2. Notations and preliminaries

Everywhere in this paper we utilize the following notations: ".“ the inner product on the spaces $\mathbb{R}^N$, $\mathbb{R}^M$ and S_N and $|\cdot|$ are the Euclidean norms on these spaces.

$$H \ = \{\ v = (v_i) \mid v_i \in L^2(\Omega),\ i = \overline{1,N}\ \},$$

$$H_1 = \{\ v = (v_i) \mid v_i \in H^1(\Omega),\ i = \overline{1,N}\ \},$$

$$\mathcal{H} = \{\ \tau = (\tau_{ij}) \mid \tau_{ij} = \tau_{ji} \in L^2(\Omega),\ i,j = \overline{1,N}\ \},$$

$$\mathcal{H}_1 = \{\ \tau = (\tau_{ij}) \mid Div\ \tau \in H\ \},$$

$$Y = \{\ \kappa = (\kappa_i) \mid \kappa_i \in L^2(\Omega),\ i = \overline{1,M}\ \}.$$

The spaces H, H_1, $\mathcal{H}$, $\mathcal{H}_1$ and Y are real Hilbert spaces endowed with the canonical inner products denoted by $< \cdot,\cdot >_H$, $< \cdot,\cdot >_{H_1}$, $< \cdot,\cdot >_{\mathcal{H}}$, $< \cdot,\cdot >_{\mathcal{H}_1}$ and $< \cdot,\cdot >_Y$ respectively.

Let $H_\Gamma = [H^{\frac{1}{2}}(\Gamma)]^N$ and $\gamma : H_1 \to H_\Gamma$ be the trace map. We denote by

$$V = \{\ u \in H_1 \mid \gamma u = 0 \text{ on } \Gamma_1\ \}$$

and let E be the subspace of H_Γ defined by

$$E = \gamma(V) = \{\ \xi \in H_\Gamma \mid \xi = 0 \text{ on } \Gamma_1\}. \tag{6}$$

Let $H'_\Gamma = [H^{-\frac{1}{2}}(\Gamma)]^N$ be the strong dual of the space H_Γ and let $<\cdot,\cdot>$ denote the duality between H'_Γ and H_Γ. If $\tau \in \mathcal{H}_1$ there exists an element $\gamma_\nu \tau \in H'_\Gamma$ such that

$$<\gamma_\nu \tau, \gamma v> = <\tau, \varepsilon(v)>_{\mathcal{H}} + <Div\ \tau, v>_H \ \text{ for all } v \in H_1. \tag{7}$$

By $\tau\nu$ we shall understand the element of E' (the strong dual of E) that is the projection of $\gamma_\nu \tau$ on E.

Let us now denote by $\mathcal{V}$ the following subspace of $\mathcal{H}_1$.

$$\mathcal{V} = \{\ \tau \in \mathcal{H}_1 \mid \ Div\ \tau = 0 \text{ in } \Omega, \ \ \tau\nu = 0 \text{ on } \ \Gamma_2\ \}$$

Using (7), it may be proved that $\varepsilon(V)$ is the orthogonal complement of $\mathcal{V}$ in $\mathcal{H}$, hence

$$<\tau, \varepsilon(v)>_{\mathcal{H}} = 0, \text{ for all } v \in V,\ \tau \in \mathcal{V}. \tag{8}$$

Finally, for every real Hilbert space X we denote by $|\cdot|_X$ the norm on X and by $C^j(0,T,X)(j = 0,1)$ the spaces defined as follows:

$$C^0(0,T,X) = \{z : [0,T] \to X \mid z \text{ is continuous }\}.$$

Let us recall that if $C^j(0,T,X)$ are real Banach spaces endowed with the norms

$$C^1(0,T,X) = \{z : [0,T] \to X \mid \text{there exists } \dot{z} \text{ the derivate of } z \text{ and } \dot{z} \in C^0(0,T,X)\}.$$

$$|z|_{0,T,X} = \max_{t\in[0,T]} |z(t)|_X \tag{9}$$

and

$$|z|_{1,T,X} = |z|_{0,T,X} + |\dot{z}|_{0,T,X}$$

respectively.

Let us recall that if K is a convex closed non empty set of X and $P : X \to K$ is the projector map on K, we have

$$y = Px \text{ if only if } y \in K \text{ and } < y - x,\ z - x >_X \geq 0 \text{ for all } z \in K. \tag{10}$$

3. **An existence and uniqueness result**

In the study of the problem (1)-(5), we consider the following assumptions:

$$\left\{\begin{array}{l} \mathcal{E} : \Omega \times S_N \times L^2(\Omega)^p \times L^2(\Omega)^M \to S_N \text{ and} \\ \text{(a) there exists } \ m > 0 \text{ such that} \\ < \mathcal{E}(\varepsilon_1, \theta, \chi) - \mathcal{E}(\varepsilon_2, \theta, \chi), \varepsilon_1 - \varepsilon_2 > \geq \\ \geq m|\varepsilon_1 - \varepsilon_2|^2 \text{ for all } \varepsilon_1, \varepsilon_2 \in S_N, , \theta \in L^2(\Omega)^p, \chi \in L^2(\Omega)^M a.e. \text{ in } \Omega, \\ \text{(b) there exists } L' > 0 \text{ such that} \\ |\mathcal{E}(\varepsilon_1, \theta_1, \chi_1) - \mathcal{E}(\varepsilon_2, \theta_2, \chi_2)| \leq L'|\varepsilon_1 - \varepsilon_2| + |\theta_1 - \theta_2| + |\chi_1 - \chi_2| \\ \text{for all } \varepsilon_1, \varepsilon_2 \in S_N, a.e. \text{ in } \Omega, \\ \text{(c) } x \to \mathcal{E}(x, \varepsilon, \theta, \chi) \text{ is a measurable function with respect to} \\ \text{the lebesgue measure in } \Omega \text{ for all } \varepsilon \in \mathrm{S}_N \ , \\ \text{(d) } x \to \mathcal{E}(x, 0, 0, 0) \in \mathcal{H} \end{array}\right. \tag{11}$$

$$\left\{\begin{array}{l} F : \Omega \times S_N \times S_N \times L^2(\Omega)^p \times L^2(\Omega)^M \to S_N \text{ and} \\ \text{a) there exists } L > 0 \text{such that} \\ \qquad |F(x, \sigma_1, \varepsilon_1, \theta_1, \chi_1) - F(x, \sigma_2, \varepsilon_2, \theta_2, \chi_2)| \leq \\ \quad \leq \ L\left(|\sigma_1 - \sigma_2| + |\varepsilon_1 - \varepsilon_2| + |\theta_1 - \theta_2| + |\chi_1 - \chi_2|\right) \\ \text{(b) } x \to F(x, \sigma, \varepsilon, \theta, \chi) \text{ is a measurable function with respect to} \\ \text{the Lebesgue measure on } \Omega, \text{ for all } \sigma, \varepsilon \in S_N, \kappa \in \mathbb{R}^M, \theta \in \mathbb{R}^P, \\ \text{(c) } x \to F(x, 0, 0, 0, 0) \in \mathcal{H}. \end{array}\right. \tag{12}$$

$$f \ \in C^1\left(0, T, H\right), \qquad g \in \left(0, T, H_\Gamma\right), \qquad h \in C^1\left(0, T, E'\right) \tag{13}$$

$$u_0 \in H_1, \qquad \sigma_0 \in \mathcal{H}_1 \tag{14}$$

$$Div\ \sigma_0 + f\left(0\right) = 0 \text{ in } \Omega, \quad u_0 = g\left(0\right) \text{ on } \Gamma_1, \quad \sigma_0 \nu = h\left(0\right) \text{ on } \Gamma_2. \tag{15}$$

$$\theta \in C^0\left(0, T, L^2\left(\Omega\right)^P\right) .\chi \in C^0\left(0, T, L^2\left(\Omega\right)^M\right) \tag{16}$$

The main result of this section is as follows.

Theorem 3.1. Let (11)-(16) hold. Then there exists a unique solution $u \in C^1(0,T,H_1), \sigma \in C^1(0,T,\mathcal{H}_1)$ of the problem (1)-(5). In order to prove theorem 3.1, we need some preliminaries.

Let $\tilde{u} \in C^1(0,T,H_1), \tilde{\sigma} \in C^1(0,T,\mathcal{H}_1)$be two functions such that

$$Div\ \tilde{\sigma} + f = 0 \text{ in } \Omega \times (0,T) \tag{17}$$

$$\tilde{u} = g \text{ on } \Gamma_1 \times (0,T) \tag{18}$$

$$\tilde{\sigma}\nu = h \text{ on } \Gamma_2 \times (0,T) \tag{19}$$

(the existence of this couple follows from (13) and the properties of the trace maps).

Considering the functions defined by

$$\bar{u} = u - \tilde{u}, \quad \bar{\sigma} = \sigma - \tilde{\sigma}, \tag{20}$$

$$\bar{u}_0 = u_0 - \tilde{u}_0, \quad \bar{\sigma}_0 = \sigma_0 - \tilde{\sigma}_0, \tag{21}$$

it easy to see that the triplet $(u,\sigma) \in C^1(0,T,H \times \mathcal{H}_1)$is a solution of the problem (1)-(5) if and only if

$$(\bar{u},\bar{\sigma}) \in C^1(0,T,V \times \mathcal{V}) \tag{22}$$

$$\dot{\sigma} = \mathcal{E}(\varepsilon(\dot{u}) + \varepsilon(\dot{\tilde{u}}),\theta,\chi) + F(\bar{\sigma}+\tilde{\sigma},\varepsilon(\bar{u})+\varepsilon(\tilde{u}),\theta,\chi) - \dot{\tilde{\sigma}} \text{ in } \Omega \times (0,T) \tag{23}$$

$$\bar{u}(0) = \bar{u}_0, \quad \bar{\sigma}(0) = \bar{\sigma}_0 \text{ in } \Omega \tag{24}$$

hence we may write (22)-(24) in the form

$$\dot{y}(t) = \mathcal{G}(\theta(t),\chi(t),x(t),y(t),\dot{x}(t)) \tag{25}$$

$$x(0) = x_0, \quad y(0) = y_0 \tag{26}$$

In which the unknowns are the function $x : [0.T] \to X$ and $y : [0.T] \to Y$ $\mathcal{G}$:$L^2(\Omega)^p \times L^2(\Omega)^M \times X \times Y \times H \to H$ is a nonlinear operator, and $X : [0.T] \to L^2(\Omega)^M, \theta : [0.T] \to L^2(\Omega)^p$ are parameters, where H is a real Hilbert space, X, Y, are two orthogonal subspaces of H such that $\mathsf{H} = X \oplus Y$ and $L^2(\Omega)^M, L^2(\Omega)^p$, are real normed space.

Hence (22)-(24) may be written in the form (25)-(26) where

$$y = \bar{\sigma},\ x = \varepsilon(\bar{u}), \dot{x} = \varepsilon\left(\dot{\bar{u}}\right)$$

and replacing the spaces $\varepsilon(V), \mathcal{V}, \mathcal{H}$, by X, Y, H respectively.

For resolving the problem (22)-(24), we consider the product Hilbert space $Z = \varepsilon(V) \times V$ which $H{=}\varepsilon(V) \oplus V$,and the problem $\mathcal{G}$ defined by

$$\mathcal{G} : L^2(\Omega)^p{\times}L^2(\Omega)^M \times \varepsilon(V) \times v \times H{\rightarrow}H$$

$$\mathcal{G}(\theta, \chi, x, y, q) = \mathcal{E}\left(q + \varepsilon(\dot{\tilde{u}}), \dot{\theta}(t), \chi(t))\right) + F(y + \tilde{\sigma}(t), x + \varepsilon(\tilde{u}), \theta(t), \chi(t)) - \dot{\tilde{\sigma}}(t) \quad (27)$$

We have the following result.

Lemma 3.1. Let $\theta(t) \in L^2(\Omega)^P$, $\chi(t) \in L^2(\Omega)^M$ $x \in X, y \in Y$ and $t \in [0.T]$. Then there exists a unique element $z = (\varepsilon(v), \tau) \in Z$ such that

$$\tau = \mathcal{G}(\theta, \chi, x, y, \varepsilon(v)) \quad (28)$$

Proof. The uniqueness part is a consequence of (11); indeed, if

$$z_1 = (\varepsilon(v_1), \tau_1), \quad z_2 = (\varepsilon(v_2), \tau_2) \in Z$$

are such that

$$\tau_1 = \mathcal{G}(\theta, \chi, x, y, \varepsilon(v_1))$$

$$\tau_2 = \mathcal{G}(\theta, \chi, x, y, \varepsilon(v_2)),$$

using (11-a) we have

$$\langle \tau_1 - \tau_2, \, \varepsilon(v_1) - \varepsilon(v_2) \rangle_{\mathcal{H}} =$$

$$\left\langle \mathcal{E}(\varepsilon(v_1) + \varepsilon(\dot{\tilde{u}}(t)), \theta(t), \chi(t))) - \mathcal{E}(\varepsilon(v_2) + \varepsilon(\dot{\tilde{u}}(t)), \theta(t), \chi(t)), \, \varepsilon(v_1) - \varepsilon(v_2) \right\rangle_{\mathcal{H}}$$

$$\geq m \left|\varepsilon(v_1) - \varepsilon(v_2)\right|_{\mathcal{H}}$$

Using now the orthogonality in H of $(\tau_1 - \tau_2) \in \mathcal{V}$ and $(\varepsilon(v_1) - \varepsilon(v_2)) \in \varepsilon(V)$,we deduce that $\varepsilon(v_1) = \varepsilon(v_2)$, which implies $\tau_1 = \tau_2$.

For the existence part,let us consider the operator $S : \varepsilon(V) \rightarrow \varepsilon(V)$ given by $S = P \circ \mathcal{G}$, where P is the projector map $\varepsilon(V)$.

Using now the hypothesis $\mathcal{E}$, F and the properties of the projectors, we can prove for θ, χ, x, y fixed, the following inequalities:

$$\begin{cases} \langle S(\theta, \chi, x, y, q_1) - S(\theta, \chi, x, y, q_2), q_1 - q_2 \rangle_{\mathcal{H}} \geq \\ \geq \langle \mathcal{G}(\theta, \chi, x, y, q_1) - \mathcal{G}(\theta, \chi, x, y, q_2), q_1 - q_2 \rangle_{\mathcal{H}} \geq \\ \geq m \left| q_1 - q_2 \right|_{\mathcal{H}}^2 . \end{cases} \tag{29}$$

Moreover, from (11), (12), and the properties of the projectors, we get

$$\begin{cases} \left| S(\theta, \chi, x, y, q_1) - S(\theta, \chi, x, y, q_2) \right|_{\mathcal{H}} \leq \\ \leq \left| \mathcal{G}(\theta, \chi, x, y, q_1) - \mathcal{G}(\theta, \chi, x, y, q_2) \right|_{\mathcal{H}} \leq \\ \leq L' \left| q_1 - q_2 \right|_{\mathcal{H}}^2 . \end{cases} \tag{30}$$

Hence $S(\theta, x, y, .) : \varepsilon(V) \to \varepsilon(V)$ is a strongly monotone Lipschitz operator. Using now Browder's surjectivity theorem we get that there exists $\varepsilon(v) \in \varepsilon(V)$such that $S(\theta, \chi, x, y, \varepsilon(v)) = 0_{\varepsilon(V)}$. It results that the element $\mathcal{G}(\theta, \chi, x, y, \varepsilon(v))$ belongs to $\mathcal{V}$ and we finish the proof using $z = (\varepsilon(v), \tau)$ where

$$\tau = \mathcal{G}(\theta, \chi, x, y, \varepsilon(v)).$$

The previous lemma allows to consider the operator $B : L^2(\Omega)^P \times L^2(\Omega)^M \times Z \to Z$ defined as follows:

$$\begin{cases} B(\theta, \chi, \omega) = z \\ \omega = (x, y), z = (\varepsilon(v), \tau) \\ \tau = \mathcal{G}(\theta, \chi, x, y, \varepsilon(v)). \end{cases} \tag{31}$$

Moreover we have

Lemma 3.2. For all $\theta \in L^2(\Omega)^P$ and $\chi \in L^2(\Omega)^M$ $\omega_1, \omega_2 \in Z$, the operator $L^2(\Omega)^P \times L^2(\Omega)^M \times Z \to Z$ is continuous and there exists $C > 0$ such that

$$|B(\theta, \chi, \omega_1) - B(\theta, \chi, \omega_2)|_Z \leq C|\omega_1 - \omega_2|_Z \tag{32}$$

for all $\theta \in L^2(\Omega)^P$ and $\chi \in L^2(\Omega)^M \omega_1, \omega_2 \in Z$.

Proof. Let $\theta_i \in L^2(\Omega)^P$, $\omega_i = (x_i, y_i) \in Z$ and

$$z_i = (\varepsilon(v_i), \tau_i) = B(\theta_i, \chi_i, \omega_i) \ , \ i = 1, 2.$$

Using (32)

$$\tau_i = \mathcal{G}(\theta_i, \chi_i, x_i, y_i, \varepsilon(v_i))), \quad i = 1, 2 \tag{33}$$

which implies

$$S(\theta_i, \chi_i, x_i, y_i, \varepsilon(v_i)) = 0_{\varepsilon(V)}\ ,\ i = 1, 2. \tag{34}$$

Using the hypothesis on $\mathcal{E}$, F, and the properties of the projectors, we get:

$$m|\varepsilon(v_1) - \varepsilon(v_1)_2|^2_{\mathcal{H}} \leq < S(\theta_1, \chi_1, x_1, y_1, \varepsilon(v_1))$$

$$-S(\theta_1, \chi_1, x_1, y_1, \varepsilon(v_2)), \varepsilon(v_1) - \varepsilon(v_2) >_{\mathcal{H}}$$

$$=< S(\theta_2, \chi_2, x_2, y_2, \varepsilon(v_2)) - S(\theta_1, \chi_1, x_1, y_1, \varepsilon(v_2)), \varepsilon(v_1) - \varepsilon(v_2) >^2_{\mathcal{H}} \leq$$

$$\leq |\mathcal{G}(\theta_2, \chi_2, x_2, y_2, \varepsilon(v_2)) - \mathcal{G}(\theta_1, \chi_1, x_1, y_1, \varepsilon(v_2))|_{\mathcal{H}} \times |\varepsilon(v_1) - \varepsilon(v_2)|^2_{\mathcal{H}}$$

which implies

$$|\varepsilon(v_1) - \varepsilon(v_2)|_{\mathcal{H}} \leq \frac{1}{m} \times |\mathcal{G}(\theta_2, \chi_2, x_2, y_2, \varepsilon(v_2)) - \mathcal{G}(\theta_1, \chi_1, x_1, y_1, \varepsilon(v_2))|_{\mathcal{H}}\ . \tag{35}$$

Using now (12), (34) we get

$$\left\{ \begin{array}{c} |\tau_1 - \tau_2|_{\mathcal{H}} \leq L'|\varepsilon(v_1) - \varepsilon(v_2|_{\mathcal{H}} + \\ |\mathcal{G}(\theta_1, \chi_1, x_1, y_1, \varepsilon(v_2)) - \mathcal{G}(\theta_2, \chi_2, x_2, y_2, \varepsilon(v_2)|_{\mathcal{H}} \end{array} \right. \tag{36}$$

Hence by (36) it result

$$\left\{ \begin{array}{l} \quad |\tau_1 - \tau_2|_{\mathcal{H}} \leq \\ \leq (\frac{L'}{m} + 1)|\mathcal{G}(\theta_1, \chi_1, x_1, y_1, \varepsilon(v_2)) - \mathcal{G}(\theta_2, \chi_2, x_2, y_2, \varepsilon(v_2)|_{\mathcal{H}} \end{array} \right. \tag{37}$$

Using now (11)-(12)(27) and the fact that $\bar{\sigma}, \dot{\bar{\sigma}}$ are continuous, we get that

$$|\mathcal{G}(\theta_1, \chi_1, x_1, y_1, \varepsilon(v_2)) - \mathcal{G}(\theta_2, \chi_2, x_2, y_2, \varepsilon(v_2)|_{\mathcal{H}} \to 0$$

When $\theta_1 \to \theta_2, in\ L^2(\Omega)^P x_1 \to x_2\ in\ X,\ y_1 \to y_2\ in\ Y$ it follows that B is continuous operator. Taking $\theta_1 = \theta_2$ and X_1=X_2 from (37) we get (33).

Proof of theorem 3.1. Let $A : [0.T] \times Z \to Z$ and z_0 be defined by:

$$\{A(t, z) = B(\theta(t), \chi(t), z) \text{ for all t} \in [0.T]\ and\ z \in Z \tag{38}$$

$$z_0 = (x_0, y_0) = \varepsilon((u_0), \bar{\sigma}_0)\,.$$

Using the definition of operator B, we get that

$$x = \varepsilon(\dot{\bar{u}}) \in C^1(0, T, \varepsilon(V)) \in C^1(0, T, Z'), y = \bar{\sigma} \in C^1(0, T, \mathcal{V})$$

is solution to (22)-(24), if and only

$$\dot{z} = (\dot{x}, \dot{y}) = A(\theta, z(t)) \text{ for all } t \in [0.T] \tag{39}$$

$$z(0) = z_0 \tag{40}$$

In order to study the problem (39)-(40), let us remark that, by lemma 3.2, A is a continuous operator and

$$|A(t, z_1) - A(t, z_2)|_Z \leq C|z_1 - z_2|_Z, \text{ for all } t \in [0.T] \text{ and } z_1, z_2 \in Z.$$

Moreover, by (14), (38), $\tilde{u} \in C^1(0, T, H_1)$ and $\tilde{\sigma} \in C^1(0, T, \mathcal{H}_1)$

We get z_0 belongs to Z and by lemma 3.2 and the classical Cauchy-Lipschitz theorem we have that $z \in C^1(0, T, Z)$ and the proof of theorem 3.1 is complete.

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