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THERMAL RADIATION EFFECT ON FULLY DEVELOPED FREE CONVECTION IN A VERTICAL RECTANGULAR DUCT

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Dedicated to Professor Gheorghe Coman at his 70^{th} *anniversary*

Abstract. The effect of radiation on the steady free convection flow, i.e. the case of purely buoyancy-driven flow, in a vertical rectangular duct is investigated for laminar and fully developed regime. The Rosseland approximation is considered and temperatures of the walls are assumed constants. The governing equations are expressed in non-dimensional form and are solved both analytically and numerically. It was found that the governing parameters have a significant effect on the velocity and temperature profiles.

1. Introduction

Heat transfer in free and mixed convection in vertical channels occurs in many industrial processes and natural phenomena. It has therefore been the subject of many detailed, mostly numerical studies for different flow configurations. The fluid flow and heat transfer has been the subject of many recent books, such as, for example Bejan [1], Pop and Ingham [2], Kohr and Pop [3], etc. Most of the interest in this subject is due to its applications, for instance, in the design of cooling systems for electronic devices and in the field of solar energy collection. Some of the published papers on this topic, such as Aung [4], Aung et al.[5], Aung and Worku [6,7], Barletta [8,9], and Boulama and Galanis [10], deal with the evaluation of the temperature and velocity profiles for the vertical parallel-flow fully developed regime. As is well

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known, heat exchangers technology involves convective flows in vertical channels. In most cases, these flows imply conditions of uniform heating of a channel, which can be modelled either by uniform wall temperature (UWT) or uniform heat flux (UHF) thermal boundary conditions.

All the above quoted analyses of free and mixed convection flow in vertical channels are based on the hypothesis that the thermal radiation effect within the fluid is negligible. However, effects of conduction-radiation on convective flows are very important in the context of space technology and processes involving high temperatures. The inclusion of conduction-radiation effects in the energy equation however leads to a highly nonlinear partial or ordinary differential equations. The aim of the present paper is therefore to analyse the effects of thermal radiation on the steady fully developed free convection in a vertical channel such that the walls of the channels are subjected to uniform but different wall temperatures (UWT) using the Rosseland approximation model which leads to ordinary differential equations for the free convection flow of an optically dense viscous incompressible fluid that flows through the channel. The ordinary differential equations are solved both analytically and numerically using the Runge-Kutta method. Flow and heat transfer results for a range of values of the pertinent parameters have been reported. Effects of pertinent parameters, such as the radiation parameter, Rd , and the thermal parameter θ_R velocity and temperature profiles are shown graphically.

2. **Basic equations**

Consider a viscous and incompressible fluid, which steadily flows between two infinite vertical and parallel plane walls. The distance between the walls, i.e., the channel width, is L . A coordinate system is chosen such that the $x-$ axis is parallel to the gravitational acceleration vector $\mathbf{g}$, but with the opposite direction. The $y-$ axis is orthogonal to the channel walls, and the origin of the axes is such that the positions of the channel walls are $-L/2$ and $L/2$, respectively. A sketch of the system and of the coordinate axes is reported in Figure 1. The wall at $y = -L/2$ is at the given uniform temperature T_1, while the wall at $y = L/2$ is subjected to a uniform temperature T_2, where $T_2 > T_1$. The fluid velocity $\mathbf{v}(u, v)$ is assumed to be parallel to the $x-$ axis, so that only the $x-$ component u of the velocity vector does not vanish. The Boussinesq and Rosseland approximations are employed. Fluid rises in the duct driven by buoyancy forces. Hence the flow is due to difference in temperature and the convection sets in instantaneously. Moreover the gradient of $T_2 - T_1$ is perpendicular to the gravity which we call it as Oberbeck convection and therefore there will be no pressure gradient in the basic equation. All the fluid properties except density in the buoyancy term are considered as constant. The flow being fully developed the following relations apply here

$$v = 0, \frac{\partial v}{\partial y} = 0, \frac{\partial p}{\partial x} = \frac{\partial p}{\partial y} = 0 \tag{1}$$

where p is the fluid pressure. Therefore, the continuity equation gives $\partial u/\partial x = 0$. One can thus conclude that u does not depend on x, i.e. $u = u(y)$. Under these assumptions the momentum and energy equations for the flow and heat transfer are

$$\mu \frac{\partial^2 u}{\partial y^2} + \rho_0 g \beta \left(T - T_0\right) = 0 \tag{2}$$

$$k \frac{\partial^2 T}{\partial y^2} - \frac{\partial q^r}{\partial y} = 0 \tag{3}$$

where T is the fluid temperature, g is the acceleration due to gravity, k is the thermal conductivity, β is the thermal expansion coefficient, μ is the dynamic viscosity, ρ_0 is the characteristic density, q^r is the radiative heat flux and T_0 is the characteristic temperature. We assume that q^r under the Rosseland approximation has the form

$$q^r = -\left(\frac{4\sigma}{3\chi}\right) \frac{\partial T^4}{\partial y} \tag{4}$$

where σ is the Stefan-Boltzman's constant and χ is the mean absorption coefficient. We also assume that . Equations (2) and (3) have to be solved subject to the boundary conditions

$$u\left(\mp L/2\right) = 0, T\left(-L/2\right) = T_1, T\left(L/2\right) = T_2 \tag{5}$$

In order to solve Eqs. (2) and (3), we introduce the following non-dimensional variables

$$Y = \frac{y}{L}, U(Y) = \frac{u}{U_0}, \theta(Y) = \frac{T - T_0}{T_2 - T_1} \tag{6}$$

where $U_0 = g\beta(T_2 - T_1)$ is the characteristic velocity. Substituting (6) into Eq. (2) and (3), we get the following ordinary differential equations

$$\frac{d^2U}{dY^2} + \theta = 0 \tag{7}$$

$$\frac{d}{dY}\left\{\left[1 + \frac{4}{3}Rd(1 + \theta_R\theta)^3\right]\frac{d\theta}{dY}\right\} = 0 \tag{8}$$

subject to the boundary conditions (5) which become

$$U\left(\mp\frac{1}{2}\right) = 0, \theta\left(-\frac{1}{2}\right) = -\frac{1}{2}, \theta\left(\frac{1}{2}\right) = \frac{1}{2} \tag{9}$$

where the radiation parameter Rd and the temperature parameter θ_R are given by

$$Rd = \frac{4\sigma T_0^3}{k\chi}, \theta_R = 2\frac{T_2 - T_1}{T_2 + T_1} \tag{10}$$

We notice that in the case when the radiation effect is absent ($Rd = 0$), Eqs. (7) and (8) reduce to those obtained by Aung [4]. The analytical solution of Eq. (7) and (8) can be expressed as

$$U = -\int_0^Y \int_0^s \theta ds dY + C_1 Y + C_2 \tag{11}$$

$$\theta + \frac{Rd}{4\theta_R}(1 + \theta_R\theta)^4 = C_3 Y + C_4 \tag{12}$$

where C_1, C_2, C_3 and C_4 are constants of integration. When $Rd = 0$ (radiation effect is absent), we get

$$\theta = Y, U = \frac{Y}{6}\left(\frac{1}{4} - Y\right) \tag{13}$$

3. **Results and discussion**

Equations (7) and (8), subject to the boundary conditions (9) were solved numerically using the finite-difference method for different values of the parameters $Rd = 0, 0.1, 1, 5, 10$ and $\theta_R = 1.1, 1.5, 2.0$.The velocity U and temperature θ profiles are shown on Figures 2 to 9. When the radiation is absent($Rd = 0$) one can see from Figures 4 to 9 that the numerical results are in very good agreement with the analytical solution. It means that we are confident that the present results are correct.

We notice that a reversed flow exist for small values of the radiation and temperature parameters, which is similar with the case studied by Aung [4] when the radiation is absent. The reversed flow dissapears for large values of the radiation and temperature parameters(see Figures 2, 4, 6 and 8). Further, we can see that the velocity profiles increase with the increasing of the temperature parameter θ_R (see Figure 2) and also with the increasing of the radiation parameter Rd(see Figures 4, 6 and 8).

The temperature profiles are shown in Figures 3, 5, 7 and 9. We can seen that the temperature profiles increase with the increasing of the parameters θ_R and Rd. The effect of the temperature parameter is more significant for larger values of the parameter Rd. We also notice that the radiation effects modify the simmetry of the temperature profiles, the temperature gradients are larger near the cold wall(left wall of the channel).

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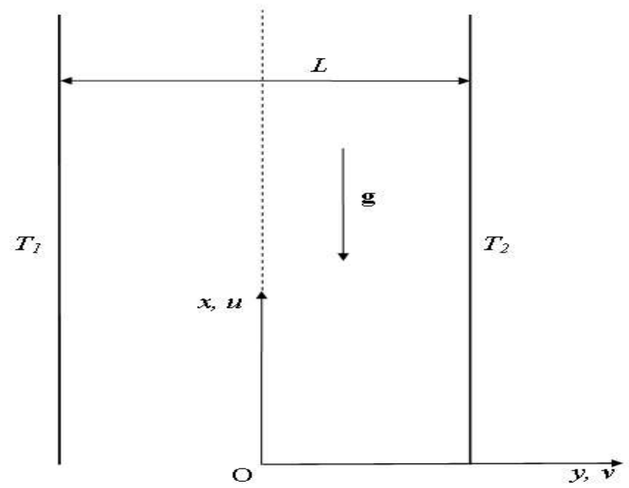

FIGURE 1. Phisycal model and co-ordinate system

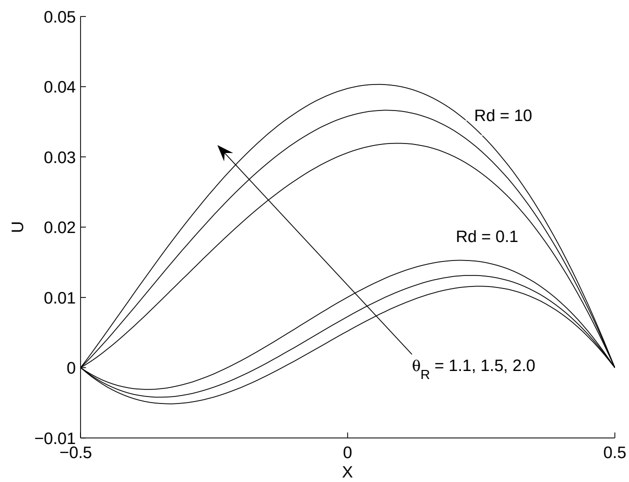

FIGURE 2. Dimensionless velocity profiles U for $Rd = 0.1$ and 10 and $\theta_R = 1.1, 1.5, 2.0$

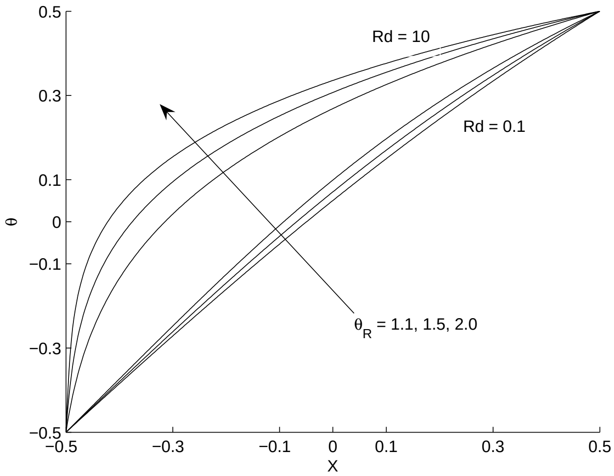

FIGURE 3. Dimensionless temperature profiles θ for $Rd = 0.1$ and 10 and $\theta_R = 1.1, 1.5, 2.0$

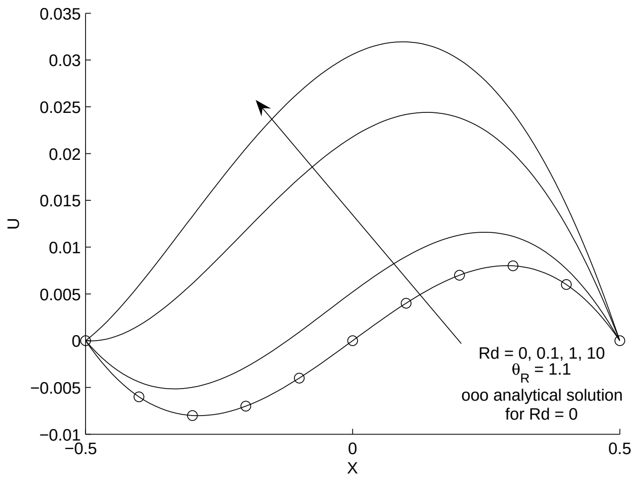

FIGURE 4. Dimensionless velocity profiles U for $Rd = 0, 0.1, 1, 10$ and $\theta_R = 1.1$

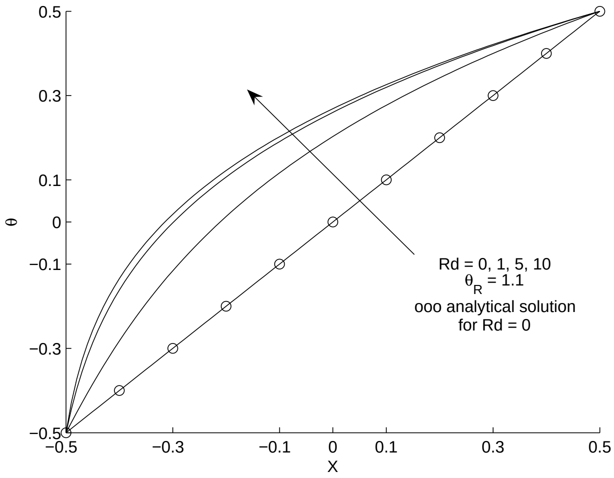

FIGURE 5. Dimensionless temperature profiles θ for $Rd = 0, 1, 5, 10$ and $\theta_R = 1.1$

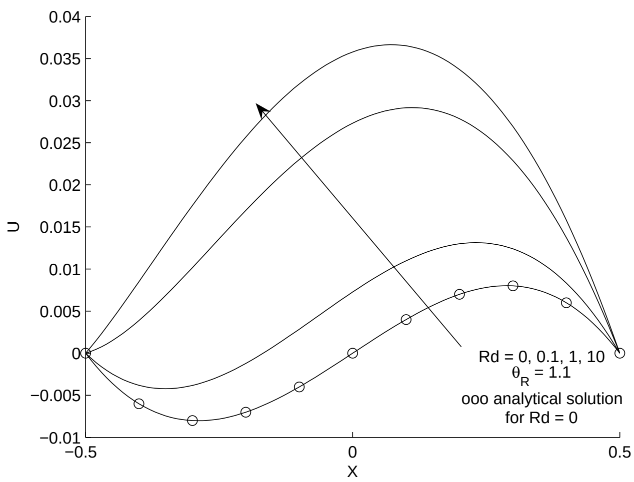

FIGURE 6. Dimensionless velocity profiles U for $Rd = 0, 0.1, 1, 10$ and $\theta_R = 1.5$

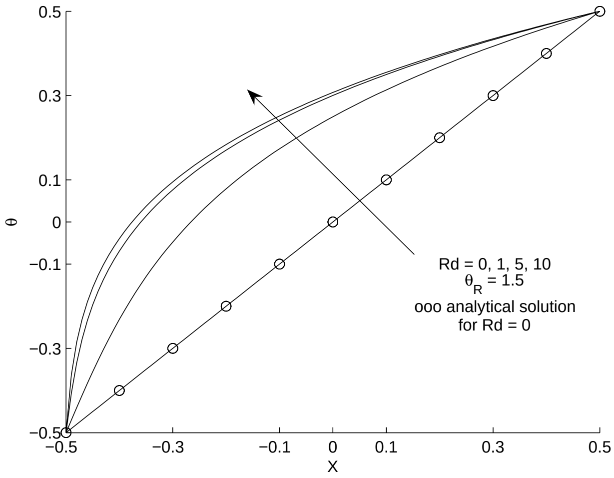

FIGURE 7. Dimensionless temperature profiles θ for $Rd = 0, 1, 5, 10$ and $\theta_R = 1.5$

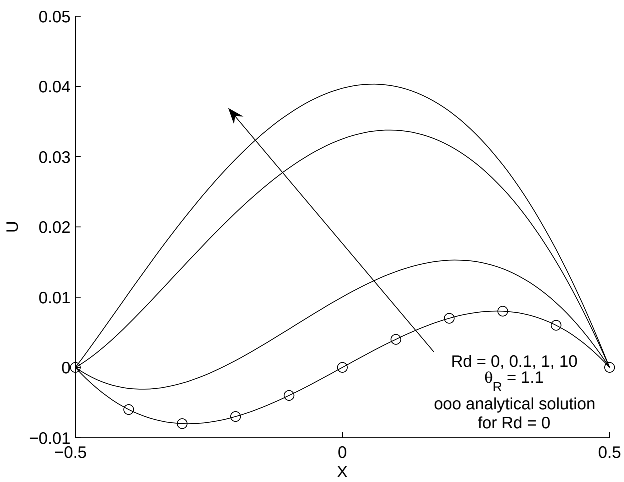

FIGURE 8. Dimensionless velocity profiles U for $Rd = 0, 0.1, 1, 10$ and $\theta_R = 2.0$

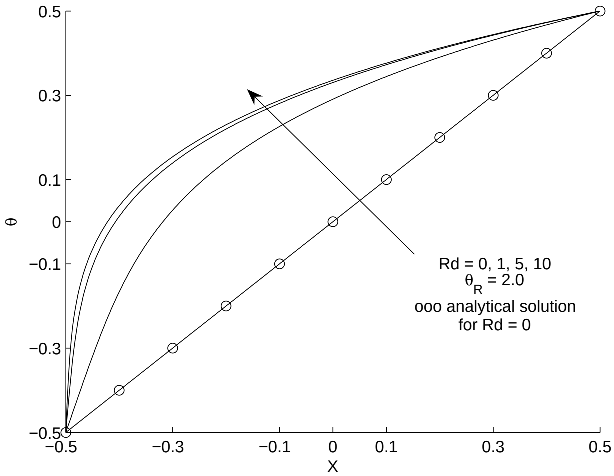

FIGURE 9. Dimensionless temperature profiles θ for $Rd = 0, 1, 5, 10$ and $\theta_R = 2.0$

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