BOOK REVIEWS

Jonathan M. Borwein and Qiji J. Zhu, ***Techniques of Variational Analysis,*** Canadian Mathematical Society (CMS) Books in Mathematics, Vol. 20, Springer 2005, vi+362 pp, ISBN 3-387-24298-8.

The term variational analysis concerns methods of proofs based on the fact that an appropriate auxiliary function attains a minimum, and has its roots in the physical principle of the least action. Probably that the first illustration of this method is Johann Bernoulli's solution to the Brachistocrone problem which led to the development of variational calculus.

A significant impact on variational analysis was done by the development of nonsmooth analysis, making possible the use of calculus of nonsmooth functions and enlarging substantially the area of applications. Other powerful tools are the decoupling method (a nonconvex substitute for Fenchel conjugacy and Hahn-Banach theorem from convex analysis), alongside with variational principles.

As it is well known, a lower semi-continuous (lsc) function attains its minimum on a compact set, a property that is not longer true in the absence of the compactness, even for bounded from below lsc functions. This drawback can be compensated by adding a small perturbation to the original function such that the perturbed function attains its minimum. The properties of the perturbation function depend on the geometric properties of the underlying space: the better these properties (smoothness) the nicer the perturbation function. This fact is well illustrated in the second chapter, *Variational Principles* - Ekeland variational principle holds in complete metric spaces, while the smooth Borwein-Preiss variational principle holds in Banach spaces with smooth norm. Another one, Stegall variational principle (proved in Chapter 6), holds in Banach spaces with the Radon-Nikodym property and ensures a continuous linear perturbation.

The aim of the book is to emphasize the strength of the variational techniques in various domains of analysis, optimization and approximation, dynamic systems, mathematical economics. These applications are arranged by chapters which are relatively independent and can be used for graduate topics courses.

The chapters are: 3. *Variational techniques in subdifferential theory* (Fréchet subdifferential and normal cone, sum rules, chain rules for Lyapunov functions, mean value theorems and inequalities, extremal principles); 4. *Variational techniques in convex analysis* (Fenchel conjugate, duality, entropy maximization); 5. *Variational techniques and multifunctions* (multifunctions, subdifferentials as multifunctions, distance functions, coderivatives of multifunctions, implicit multifunction theorems); 6. *Variational principles in nonlinear functional analysis* (subdifferential and Asplund spaces, nonconvex separation, Stegall variational principle, mountain pass theorem); 7. *Variational techniques in the presence of symmetry* (nonsmooth functions on smooth manifolds, manifolds of matrices and spectral functions, convex spectral functions).

The book contains a lot of exercises completing the main text, some of them, which are more difficult, being guided exercises with references.

Based mainly on developments and applications from the past several decades, the book is directed to graduate students in the field of variational analysis. The prerequisites for its reading are undergraduate analysis and basic functional analysis. Researchers who use variational techniques, or intend to do, will find the book very useful too.

S. Cobzaş

Dorin Bucur, Giuseppe Buttazzo, ***Variational Methods in Shape Optimization Problems***, Progress in Nonlinear Differential Equations and their Applications, Birkhäuser, 2005, ISBN 0-8176-4359-1.

Usually, problems of the calculus of variations concern optimization among an admissible class of functions. What is special about shape optimization problems is that the "competing objects" are shapes (domains of $\mathbf{R}^n$). Because of this, the existence of a solution is ensured only in certain cases, due to some geometrical restrictions on the admissible domains (shapes) or to a particular form of the cost function. In general, relaxed formulations of the problems have to be formulated.

The development of the field of shape optimization is due especially to the great number of applications in physics and engineering.

Several examples of shape optimization problems are presented in the first chapter of the book, in a detailed and clear manner: the isoperimetric problem, the Newton problem of minimal aerodynamical resistance, the optimal distribution of two different media in a fixed region, the optimal shape of a thin insulating layer.

The second chapter is about optimization problems over classes of convex domains and it deals with the case where an additional convexity constraint on the domains ensures the existence of an optimal shape (by providing some extra compactness). Some necessary conditions of optimality are given for the Newton problem.

Some shape optimization problems can be considered optimal control problems: the shape plays the role of the control and the state equation is usually a partial differential equation on the control domain. In Chapter 3, a topological framework for general optimization problems is given, together with the theory of relaxed controls and some examples of relaxed shape optimization problems.

Shape optimization problems with Dirichlet (Neumann) condition on the free boundary are treated in Chapters 4 (7, respectively). In both cases, is important to understand the stability of the solution to a PDE for nonsmooth perturbations of the geometric domain. This stability is related to the convergence in Mosco sense of the corresponding variational spaces. The relaxed form of a Dirichlet problem is given (in a case where the existence of an optimal solution does not occur), to understand the behavior of minimizing sequences. For Neumann boundary conditions, the problem of optimal cutting is treated completely.

Chapter 5 contains other particular cases where an unrelaxed optimal solution exists, in the family of classical admissible domains. The existence of solutions is ensured by some monotonicity properties of the cost functional or by some geometrical constraints on the domains.

Optimization problems for functions of eigenvalues are presented in Chapter 6. The case of the first two eigenvalues of the Laplace operator is studied, using the continuous Steiner symmetrization.

The book is addressed mainly to graduate students, applied mathematicians, engineers; it requires standard knowledge in the calculus of variations, differential equations and functional analysis.

The problems are treated from both the classical and modern perspectives, each chapter contains examples and illustrations and also several open problems for further research. A substantial bibliography is given, emphasizing the rapid development of the field.

Daniela Inoan

Stefaan Caenepeel and Freddy van Oystaeyen Editors, ***Hopf Algebras in Noncommutative Geometry and Physics***, Pure and Applied Mathematics; Vol. 239, Marcel Dekker, New York, 2005, 320 pp., ISBN 0-8247-5759-9.

The study of Hopf algebras and quantum groups has seen a great development during the last two decades. The present volume is devoted to these topics, and consists of high quality articles related to the lectures given at the meeting on "Hopf algebras and quantum groups" held at the Royal Academy in Brussels from May 28 to June 1, 2002. This volume contains refereed papers and surveys on different aspects of the subject, such as:

The list of contributors and their papers is as follows. *J. Abuhlail*, Morita contexts for corings and equivalences; *F. Aly and F. van Oystaeyen*, Hopf order module algebra orders; *G. Böhm*, An alternative notion of Hopf algebroid; *Ph. Bonneau and D. Sternheimer*, Topological Hopf algebras, quantum groups and deformation quantization; *T. Brzeziński, L. Kadison and R. Wisbauer*, On coseparable and biseparable corings; *D. Bulacu, S. Caenepeel and F. Panaite*, More properties of Yetter-Drinfeld modules over quasi-Hopf algebras; *S. Caenepeel, J. Vercruysse and S.H. Wang*, Rationality properties for Morita contexts associated to Corings; *L. El Kaoutit and J. Gómez-Torrecillas*, Morita duality for corings over quasi-Frobenius rings; *K.R. Goodearl and T.H. Lenagan*, Quantized coinvariants at transcendental q; *S. Majid*, Classification of differentials on quantum doubles and finite noncommutative geometry; *S. Majid*, Noncommutative differentials and Yang-Mills on permutation groups S_n; *C. Menini and G. Militaru*, The afineness criterion for Doi-Koppinen modules; *S. Montgomery*, Algebra properties invariant under twisting; *C. Ohn*, Quantum $\mathrm{SL}(3,\mathbb{C})$'s: the missing case; *A Paolucci*, Cuntz algebras and dynamical quantum group $SU(2)$; *B. Pareigis*, On symbolic computations in braided monomial categories; *P. Schauenburg*, Quotients of finite quasi-Hopf algebras; *K. Szlachányi*; Adjointable monoidal functors and quantum groupoids; *R. Wisbauer*, On Galois corings.

The book is highly recommended to researchers in algebraic geometry, number theory and mathematical physics, who will find here an excellent overview of the most significant areas of research in this field. Some of the new results are presented here for the first time. It is a valuable addition to the literature, and I warmly recommend it to algebraists and theoretical physicists.

Andrei Marcus

Leszek Gasiński and Nikolaos S. Papageorgiou, ***Nonlinear Analysis***, Series in Mathematical Analysis and Applications, Vol. 9, Chapman & Hall/CRC, Taylor & Francis Group, Boca Raton, London, New York, Singapore, 2006, xi +971 pp., ISBN 1-58488-484-3.

The aim of the present volume is to provide the reader with a solid background in several areas related to some modern topics in nonlinear analysis as critical point theory, nonlinear differential operators and related regularity and comparison principles.

The first chapter, *Hausdorff measures and capacity,* is concerned with topics as Vitali and Besicovitch covering theorems, Hausdorff measure and dimension, differentiability of Hausdorff measures and of Lipschitz functions (Rademacher theorem), the area, coarea and change of variables formulae for Lipschitz transforms.

The second chapter, *Lebesgue-Bochner and Sobolev spaces,* contains a brief introduction to integration of vector-functions (weak and strong measurability, Pettis, Gelfand and Bochner integrals), a treatment of Banach spaces of continuous vector-functions, of Lebesgue-Bochner spaces (completeness, duality, compactness), and of Sobolev spaces of vector-functions.

Chapter 3, *Nonlinear operators and Young measures,* discusses some classes of nonlinear operators (monotone, accretive) and semigroups of operators, exemplified on the case of Nemytskii composition operator. Some results on compact and on Fredholm linear operators on Banach and Hilbert spaces are also included, in order to emphasize the similarities and the differences between the linear and nonlinear case. The chapter ends with an introduction to Young measures.

The fourth chapter, *Smooth and nonsmooth variational principles,* contains an introduction to differential calculus on Banach spaces (Gâteaux and Fréchet derivatives) with applications to the differentiability of convex functions - Mazur and Asplund generic differentiability theorems. Christensen theorem on almost everywhere differentiability of locally Lipschitz functions on Banach spaces (the extension of Rademacher theorem) with respect to Haar null sets is also proved. Subdifferential calculus for convex functions, as well as Clarke generalized subdifferential calculus for locally Lipschitz functions are considered too. The chapter ends with the proof of Ekeland and Borwein-Preiss variational principles with applications.

Chapter 5, *Critical point theory,* is concerned with applications of the critical point theory to minimax, saddle point and mountain pass theorems. Lusternik-Schnirelman theory with applications to eigenvalue problems is the topic of the last section of this chapter.

In Chapter 6, *Eigenvalue problems and maximum principles,* the techniques and methods developed so far are applied to the study of linear and nonlinear elliptic PDEs.

Fixed point theorems (FPT) constitute the basic tool in the proofs of the existence of solutions to various kinds of equations and inclusions. The last chapter of the book, Chapter 7, *Fixed point theorems,* is devoted to the proofs of the main FPT of metrical nature (Banach contraction principle with extensions and applications, normal structure in Banach spaces and FPT for nonexpansive mappings), and of topological nature as well – the fixed point theorems of Brouwer, Schauder, Borsuk, and Sadovskii. A special attention is paid to FPT in ordered structure (Tarski, Bourbaki-Kneser, Amann) and in ordered Banach spaces – Krasnoselskii FPT with applications to positive eigenvalues and to fixed point index.

An appendix collects the essential results from topology, measure theory, functional analysis, calculus and nonlinear analysis, used throughout the book.

Together with the books *Nonsmooth Critical Point Theory and Nonlinear Boundary Value Problems,* CRC 2005, by the same authors, and *An Introduction to Nonlinear Analysis,* Vol. I. *Theory*, Vol. II, *Applications,* by Z. Denkowski, S. Migorski & N. Papageorgiou, the present one provides a comprehensive and fairly self-contained presentation of some important results in nonlinear analysis and applications.

It (or parts of it) can be used for graduate or post-graduate course, but also as reference text by specialists.

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