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BOOK REVIEWS

Titu Andreescu and Bogdan Enescu, ***Mathematical Olympiad Treasures***, Birkhäuser Verlag, Boston-Basel-Berlin, 2004, 234 pp., ISBN 0-8176-4305-2.

This excellent book deals with some important topics of elementary mathematics necessarily in the process of training students for various contests and olympiads. One of the main intention of the authors is to build a bridge between ordinary high school exercises and more sophisticated, intricate and abstract concepts and problems in undergraduate mathematics. Mathematical Olympiad Treasures reflects the depth of experience of two seasoned professors and coaches from the USA and Romanian Olympiad teams. The book is organized into three chapters each of them containing eight sections. Each sections contains some suggestive completely solved problems and some proposed problems which are solved in the second part of the book. In what follows we will briefly present each of them.

Chapter 1 is entitled *Algebra* and contains some basic notions and results concerning the following topics: An algebraic identity, Cauchy-Schwartz revisited, Easy ways through absolute values, Parameters, Take the conjugate, Inequalities with convex functions, Induction at work, Roots and coefficients.

Chapter 2, *Geometry and Trigonometry*, contains eight sections dealing with Geometric inequalities, An interesting locus, Cyclic quads, Equiangular polygons, More on equilateral triangles, The carpets theorem, Quadrilaterals with an inscribed circle, Dr. Trig learns complex numbers.

In Chapter 3, *Number Theory and Combinatorics*, the authors present some fundamental ideas and interesting problems concerning Arrays of numbers, Functions defined on sets of points, Count twice, Sequences of integers, Equations with infinitely many solutions, Equations with no solutions, Powers of 2, Progressions.

The book ends with a useful Glossary, an Index of notation and an Index containing the sources of the problems. The book is written in a very clear and rigorous manner and it is recommended for students, graduated students and their teachers and for anyone interested in mathematical contests and olympiads.

Dorin Andrica

Titu Andreescu and Zuming Feng, ***A Path to Combinatorics for Undergraduates. Counting Strategies***, Birkhäuser Verlag, Boston-Basel-Berlin, 2004, 228 pp., ISBN 0-8176-4288-9.

This is a unique approach to combinatorics centered around challenging examples, fully-worked solutions, and hundreds of problems many from mathematical contests and olympiads. This excellent book deals with some important topics of combinatorics which are very useful in the process of training students for various contests and olympiads and it reflects the depth of experience of two seasoned professors and coaches from the USA Olympiad team. The book is organized into nine chapters containing the basic notions and results in the following topics : Addition or multiplication, Combinations, Properties of binomial coefficients, Bijections, Recursions, Inclusion and exclusion, Calculating in two ways: Fubinis principle, generating functions, review exercises. The book ends with a useful Glossary, an Index of notions and a suggestive list of references. The book is written in a very clear and rigorous manner and it is recommended for students, graduated students and their teachers and for anyone interested in challenging mathematics. It can be used as a solid stepping stone for other advanced mathematical readings.

Dorin Andrica

Titu Andreescu and Zuming Feng, ***103 Trigonometry Problems From the Training of the USA IMO Team***, Birkhäuser Verlag, Boston-Basel-Berlin, 2005, 214 pp., ISBN 0-8176-4334-6.

This excellent book contains 103 highly selected problems used in the training and testing of the USA IMO (International Mathematical Olympiad) team. From the authors preface it follows that "It is not a collection of very difficult, impenetrable questions. Instead, the book gradually builds students trigonometric skills and techniques". The book contains five chapters. The first chapter provides a comprehensive introduction to trigonometric functions, their relations and functional properties, and their applications into plane and solid geometry. Chapters two and three contain 52 introductory and 51 advanced proposed problems, respectively. In Chapters four and five the authors present the solutions and some comments to the proposed problems. The book also contains a Glossary and a rich list of references.

This work aims to broaden students view of mathematics and better prepare them for possible participation in various mathematical contests and olympiads. The book further stimulates interest for the future study of mathematics.

Dorin Andrica

Vasile Berinde, ***Exploring, Investigating and Discovering in Mathematics***, Birkhäuser Verlag, Basel-Boston-Berlin, 2004, 246 +xix pp., ISBN 3-7643-7019-x.

This book represent the English version of the Romanian edition (V.Berinde, Explorare, investigare si descoperire in matematică, Editura Efemeride, Baia Mare, 2001). The author writes in his Preface to the book: "The book is addressed mainly to students, young mathematicians, and teachers, involved or/and actively working in mathematics competitions and training gifted people. It collects many valuable techniques for solving various classes of difficult problems and, simultaneously, offers a comprehensive introduction to creating new problems. The book should also be of

interest to anybody who is in any way connected to mathematics or interested in the creative process and in mathematics as a art". Indeed the author has been greatly successful. The reader can find here ideas and problems which combine a number of classical topics from various fields of mathematics.

The book is organized into 24 chapters, most of them independent, involving the following topics: Chase problems, Sequences of integers simultaneously prime, A geometric construction using ruler and compass, Solving a class of nonlinear systems, A class of homogeneous inequalities, The first decimal of some irrational numbers, Polynomial approximation of continuous functions, On an interesting divisibility problem, Determinants with alternate entries, Solving some cyclic systems, On a property of recurrent affine sequences, Binomial characterizations of arithmetic progressions, Using duality in studying homographic recurrences, Exponential equations having exactly two solutions, A class of functional equations, An extension of Leibniz-Newton formula, A measurement problem, A class of discontinuous functions admitting primitives, On two classes of inequalities, Another problem of geometric construction, How can we discover new problems by means of the computer, On the convergence of some sequences of real numbers, An applications of the integral mean, Difference and differential equations.

Each chapter ends with a suggestive and useful bibliography concerning the topic. Some basic and general principles regarding creativity in solving problems are discussed in an Addendum at the end of the book. The book is strongly recommended to all students and teachers but also to everyone who has a special love for mathematical problems that are stated and also solved in a simple and in an ingenious way.

Dorin Andrica

Advanced Courses in Mathematical Analysis, I. A. Aizpuru-Tomas and F. Leon-Saavedra (Editors), World Scientific Publishers, London-Singapore 2004, vii+155 pp., ISBN 981-256-060-2.

The volume contains the written versions of the lectures delivered at the First International Course of Mathematical Analysis in Andalucia, organized by the University of Cadiz from 23 to 27 September, 2005. The aim of the course was to bring together different research groups working in mathematical analysis and to provide the young researchers of these groups with access to the most advanced lines of research. A second course took place in September 2004 in Granada.

There are included five survey papers: 1. Y. Benyamini, *Introduction to uniform classification of Banach spaces* ; 2. M. Gonzáles, *An introduction to local duality for Banach spaces* ; 3. V. Müller, *Orbits of operators* ; 4. E. Matouškova, S. Reich and A. J. Zaslavski, *Genericity in nonexpansive mapping theory* ; 5. A. R. Palacios, *Absolute-valued algebras, and absolute-valuable Banach spaces.*

The first paper is the only updated survey on the classification of Banach spaces under uniformly continuous mappings. Its aim is to introduce the reader to this area and to present some results and open questions, a complete presentation of these problems and of other related ones being given in the recent treatise of Y. Beniaminy and J. Lindenstrauss, *Nonlinear Geometric Functional Analysis,* I., AMS, 2000.

The local duality for Banach spaces is a tool recently developed by the author of the second paper and some co-workers, which turned to be very useful in the study Banach spaces, mainly in the case when the dual of a Banach space is too large.

V. Müller emphasizes in the third paper the relevance of the orbit method and of Scott Brown's technique in the study of invariant subspaces. A more comprehensive treatment is given in his recent book ????.

It is known that nonexpansive mappings could note have fixed points, but, as it was shown by F. S. De Blasi and J. Myjak in 1976, most of them (in the sense of Baire category) do have. The fourth paper surveys various category and porosity

results concerning the well-posedness of the fixed point problem for nonexpansive mappings, most of them being obtained recently by the authors.

An absolute-valued algebra is a normed algebra A such that $\|xy\| = \|x\|\|y\|$, for all $x, y \in A$. As it is well-known, if A is associative and commutative then it agrees with $\mathbb{R}$ or $\mathbb{C}$, and with the quaternion field $\mathbb{H}$ if A is only associative. Therefore, the interesting case is that of non-associative absolute-valued algebras, presented in the last paper of the book. The results are presented from historical perspective to the frontier of current research in the field.

The book contains surveys of some topics of interest in the current research in functional analysis, written by leading experts in the area. It can be used as an introductory material for young researchers, as a guide to more advanced books or research papers.

S. Cobzaş

Michael Růžička, ***Nicthlineare Funktionalanalysis – Einr Einführung***, Springer Verlag, Berlin-Heidelberg-New York 2004, xii+208 pp., ISBN 3-540-20066-5.

The book is based on a one-semester course (the 6th semester) taught by the author at the Universities of Bonn (1999) and Freiburg (2002 and 2003). Its aim is to provide the reader with the basic results and techniques in the field, which can form a basis for the reading of more advanced books, as, e.g, the monumental four volume treatise of E. Zeidler, *Nonlinear Functional Analysis and Applications*, Springer Verlag, 1985-1990.

The topics covered by the present volume are best illustrated by the headings of the chapters: 1. *Fixed point theorems*; 2. *Integration and differentiation in Banach spaces*; 3. *The theory of monotone operators*; 4. *The topological degree theory.*

The first chapter of the book contains the basic fixed point theorems: Banach's contraction principle, Brouwer and Schauder fixed point theorems. The proof

of Brouwer fixed point theorem uses some techniques from variational calculus. Applications are given to existence results for ordinary differential equations (Picard's iterative method).

The second chapter is concerned with Bochner integration, L^p-spaces of Banach valued functions and differential calculus in Banach spaces (Gâteaux and Fréchet derivatives).

The third chapter contains an introduction to monotone operators and includes results of Browder, Minty and Brezis. Maximal monotone operators, subdifferentials of convex functions, and the duality mapping are also included. Applications are given to quasilinear parabolic and elliptic partial differential equations.

The last chapter of the book is devoted to a presentation of Brouwer and Leray-Schauder topological degree theories wit applications to Brouwer fixed point theorem and to quasilinear elliptic equations.

The prerequisites from topology, measure theory and linear functional analysis, needed for the reading of the book are included in an Appendix (37 pp.). There are no problems and exercises in the book.

The book is clearly written, with complete and carefully written proofs and illuminating examples. It can serve as a base text for introductory courses in nonlinear functional analysis.

S. Cobzaş

Kehe Zhu, ***Spaces of Holomorphic Functions in the Unit Ball***, Graduate Texts in Mathematics, Vol. 226, Springer, Berlin-Heidelberg-New York, 2005, x+271 pp., ISBN 0-387-22036-4.

The book is concerned with the basic properties of the most well-known and widely used spaces of holomorphic functions in the open unit ball $\mathbb{B}_n$ of $\mathbb{C}^n$. The restriction to the unit ball of $\mathbb{C}^n$ allows the author to present direct proofs of most of the results by straightforward formulas. The central idea of these proofs is to use

integral representations for holomorphic functions and elementary properties of the Bergman kernel, the Bergman metric, and the automorphism group. In this way, although few of the results are new, most of the proofs are new and simpler than the existing ones. On the other hand, this reduces the prerequisites to a minimum: only familiarity with single variable complex analysis, no prior knowledge of several complex variables theory being required.

The first chapter of the book, Ch.1, *Preliminaries*, has an introductory character and contains some results on holomorphic functions, the authomorphism group of $\mathbb{B}_n$, Lebesgue spaces, Bergman metric, subharmonic functions, complex interpolation.

Each of the remaining chapters is devoted to a class of spaces of holomorphic functions in the unit ball: Ch. 2. *Bergman spaces*; Ch. 3. *The Bloch space*; Ch. 4. *Hardy spaces*; Ch. 5. *Functions of bounded mean oscillation* (the study of BMOA spaces); Ch. 6. *Besov spaces*; Ch. 7. *Lipschitz spaces*. For each class of spaces, the author discusses integral representations, characterizations in terms of various derivatives (radial derivatives, holomorphic gradients, fractional derivatives), atomic decompositions, complex interpolation and duality.

All these spaces are intimately related as it is emphasized in the book: the Bloch space $\mathcal{B}$ can be thought as a limit case of Bergman space A^p_α as $p \to \infty$. In particular, $\mathcal{B}$ can be naturally identified with the dual of the Hardy space H^1. The Besov space B_p is the image of the Bergman space A^p_α under a suitable fractional integral operator, and B_∞ is just the Bloch space. In their turn Lipschitz spaces Λ_α are images of Bloch spaces under some fractional integral operator. In fact, as the author points out in the Preface to the book, all these spaces are special cases of a more general family of holomorphic Sobolev spaces, but the direct treatment of these particular cases is far more interesting and appealing than a cumbersome presentation of an exhaustive class of functions containing all of them.

Each chapter ends with a set of exercises of varying difficulty. For the difficult results, completing the main text, exact references are given. The Notes sections contain bibliographical mentions and references to related results.

Of course that the title of the book automatically directs us to the classic book of W. Rudin, *Function Theory in the Unit Ball of* $\mathbb{C}^n$ (Springer 1980). Although there are some inevitable overlaps, they are not substantial and the two books complement each other.

Professor Zhu is an authoritative personality in the area and the author of the books: *Operator Theory in Function Spaces* (M. Dekker 1990), *Theory of Bergman Spaces*, with H. Hedenmalm and B. Korenblum (Springer 2000), and *An Introduction to Operator Algebras* (CRC Press 1993).

The book is well written and can be used as a textbook for advanced graduate courses in complex analysis and spaces of holomorphic functions.

Mirela Kohr