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BOOK REVIEWS

Jürgen Appel, Espedito De Pascale and Alfonso Vignoli, ***Nonlinear Spectral Theory,*** De Gruyter Series in Nonlinear Analysis and Applications, Vol. 10, Walter de Gruyter, Berlin - New York 2004, xi + 408 pages, ISBN: 3-11-018143-6.

The spectral theory of bounded linear operators on Banach spaces is one of the most important branches of functional analysis and operator theory, with deep and far reaching applications to spectral theory of differential operators and to classical quantum mechanics.

It is expected that a reasonable definition of the spectrum of a continuous nonlinear operator F acting on a Banach space X should agree with the usual one when F is linear and, at a same time, to retain some of its essential properties, as nonemptiness, compactness, to contain the eigenvalues, etc. By a sequence of 8 simple examples given in the introduction the authors show some of the drawbacks of various natural definitions of the spectrum of a nonlinear operator, leading them to the conclusion that the main matter is not the intrinsic structure of the spectrum, but rather its usefulness in the study of nonlinear operator equations.

The book contains a systematic presentation of various spectra for nonlinear operators, along with some applications. Numerous examples and tables illustrate the relations between these spectra, as well as some shortcomings arising for each of them.

The first chapter contains an overview of the spectral theory of bounded linear operators, the second one is concerned with various metric and topological properties of nonlinear operators (the Lipschitz property, α-contractibility, etc), while the third one presents some results on the invertibility of nonlinear operators, a question closely related to the solvability of nonlinear equations - the main target of the nonlinear spectral theory.

Various kinds of spectra for nonlinear operators are presented in Chapters 4 through 9: 4. *The Rhodius and Neuberger spectra*, 5. *The Kachurovskij and Dörfner spectra*, 6. *The Furi-Martelli-Vignoli spectrum*, 7. *The Feng spectrum*, 8. *The Väth phantom*, 9. *Other spectra.*

Chapter 10 is concerned with the quite subtle notion of eigenvalue of a nonlinear operator. Again, a direct transpose of the definition to the nonlinear case does not fit best the needs of the theory, this being done by other equivalent definitions, apparently different from the familiar one. Chapter 11 emphasizes through an appropriate definition of the numerical range of a nonlinear operator, the influence of the geometry of the underlying Banach space in the study of nonlinear spectra. The last chapter of the book, Chapter 12, is devoted to applications to general solvability of nonlinear equations and to bifurcation theory. A nonlinear Fredholm theory is applied existence and perturbation results for p-Laplacian.

Each chapter ends with a section of bibliographical and historical notes and remarks. The book is fairly self-contained, the prerequisites being a modes background in nonlinear functional analysis and spectral theory.

As the authors point out in the introduction, the theory is far from being complete - in fact there is no a satisfactory definition of the spectrum in the nonlinear case. The book can be considered as a systematic introduction to this area, emphasizing the diversity of directions in which current research in nonlinear spectral is developing.

S. Cobzaş

Jon P. Davis, ***Methods of Applied Mathematics with a MATLAB overview***, Birkhäuser Verlag, 2004, XII, 721 p., ISBN: 0-8176-4331-1.

This book is devoted to the application of Fourier Analysis. The author mixed in a remarkable way theoretical results and application illustrating the results. Flexibility of presentation (increasing and decreasing level of rigor, accessibility) is a key feature.

The first chapter is an introductory one.

An introduction to Fourier series based mainly on inner product spaces is given in chapter 2.

The third chapter treats elementary boundary value problems. Besides applications of the Fourier series, it presents standard boundary value problem models and their discrete analogous problems.

Higher-dimensional, non rectangular problems is the topic of the fourth chapter. These includes Sturm-Liouville Theory, series solutions, Bessel equations and nonhomogeneous boundary value problems.

Chapter 5 is an introduction to functions of complex variable. Here ones discuss basic results and their applications to problems of fluid flow and transform inversion.

The sixth chapter introduces Laplace transform and their applications to ordinary differential equations, circuit analysis and input-output analysis of linear systems.

Continuous Fourier transform is the topic of seventh chapter. Also applications of Fourier transform to ordinary differential equations, integral equations, partial differential equations are included here.

Chapter eight is on discrete variable transforms. It treats discrete variable models, z-transform, discrete and fast Fourier transform and their properties. Computational aspects of fast Fourier transform are also pointed.

The last chapter "Additional Topics" introduces methods that are specialization of those treated previously such as two-sided and Walsh transform, wavelets analysis and integral transform.

The book contains extensive examples, presented in an intuitive way with high quality figure (some of them quite spectacular), useful MATLAB codes. MATLAB exercises and routines are well integrated within the text, and a concise introduction into MATLAB is given in an appendix. The emphasis is on program's numerical and graphical capabilities and its applications, not on its syntax. A large variety of problems graded from difficulty point of view. Applications are modern and up to date. Reach and comprehensive references are attached to each chapter.

Intended audience: especially students in pure and applied mathematics, physics and computer science, but also useful to applied mathematicians, engineers and computer scientists interested in applications of Fourier analysis.

Radu Trîmbiţaş

Donaldson, S.K., Eliashberg, Y., Gromov, M. (Eds.), ***Different Faces of Geometry***, Kluwer Academic / Plenum Press (International Mathematical Series), 2004, Hardback, 404 pp., ISBN 0-306-48657-1.

Everybody knows how difficult can be to give a proper definition. This is, particularly, true when it comes to geometry. I think it's quite impossible to give a definition of contemporary geometry. Definitely, the old ethymological definition doesn't do the job anymore. In fact, the editors (three of the most influential mathematicians of our times, who don't need any formal introduction) claim that "there is, perhaps, no branch of mathematics which cannot be considered a part of geometry, when approached in the right spirit". Their idea, therefore is that it is probably better to think of geometry as being rather a collection of subjects than a single field. To put it another way, the geometry has many "faces".

The aim of the editors of this book is to provide a readable description of some of these faces, by asking leading specialists to discuss the current state and prospects of their fields of expertise. These fields include (but are not restricted to): amoebas and tropical geometry, convex geometry, differential geometry of 4-manifolds, 3-dimensional contact geometry, Lagrangian and Special Lagrangian submanifolds, Floer homology. It is probably no accident that many of these topics are closely related to the research interests of the editors themselves.

While I didn't mentioned all the subjects touched in this book, I would like, nevertheless, to mention at least the authors of the contributions: G. Mikhalkin, V.D. Milman, A.A. Giannopoulos, C.LeBrun, Ko Honda, P. Ozsváth, Z. Szabó, C. Simpson, D. Joyce, P. Seidel and S. Bauer.

The books of this kind, providing a rapid access to reliable information on different fields of mathematics are of a great help for many people, from graduate students and researchers. Nowadays is quite difficult to find your way through a field which is not exactly your own and a hand lent by an expert is always very helpful. The book under review is no exception. The subjects chosen belong to the most active fields of research in the last period and the authors manage to describe them in an accessible way. I would gladly recommend it to anyone with an interest in geometry, even if he/she has no intention whatsoever to specialize in one of the fields described in the book. It's always nice to know what your neighbors are doing.

I'd like to finish this review by mentioning that this book (like any other in this series, edited by the Russian mathematician Tamara Rozhkovskaya) was simultaneously published in Russian.

Paul A. Blaga

Pei-Kee Lin, ***Köthe-Bochner Function Spaces,*** Birkhäuser Verlag, Boston-Basel-Berlin 2004, xii+370 pp, ISBN: 0-8176-3521-1.

Let (Ω, Σ, μ) be a complete measure space. A real Banach space E consisting of equivalence classes (modulo equality a.e.) of locally integrable real-valued functions is called a Köthe function space provided:

(i) if $h \in E$ and $g : \Omega \to \mathbb{R}$ is measurable and $g(\omega)| \leq |h(\omega)|$ a.e. on Ω, then $g \in E$ and $\|g\| \leq \|h\|$;

(ii) for every $A \in \Sigma$ with $\mu(A) < \infty$ the characteristic function 1_A of A belongs to E.

Every Köthe function space is a Banach lattice with respect to the pointwise order: $f \leq g \iff f(\omega) \leq g(\omega)$ a.e. on Ω. Köthe function spaces form an important class of Banach function spaces and Banach lattices as can be seen, for instance, from the second volume of the treatise J. Lindenstrauss and L. Tzafriri, *Classical Banach spaces,* Springer Verlag, Berlin 1979. If X is a Banach space and E is a Köthe function space over the complete measure space (Ω, μ), then the Köthe-Bochner function space $E(X)$ is formed by all strongly measurable functions $f : \Omega \to X$ such that the function $\omega \mapsto \|f(\omega)\|_X$ belongs to E. Equipped with the norm $\|\|f(\cdot)\|_X\|_E$, $E(X)$ is a Banach space.

The main questions the author of the present book addresses are: if both of the spaces E and X have a geometric property P, then does the space $E(X)$ have the same property, and conversely, if $E(X)$ has the property P, then must E and X have the property P ? For P one takes various rotundity and smoothness conditions (strict convexity, local uniform convexity, uniform convexity, smoothness, etc) or other properties of geometric or topological nature as Dunford-Pettis, Radon-Nikodým, Kadec-Klee properties. Chapters 5.I and 5.II, both headed *Stability properties,* are concerned with the following problem: if f is an extreme (smooth, exposed, etc) point of the unit ball of $E(X)$ then is $f(\omega)/\|f(\cdot)\|$ and extreme (smooth, exposed, etc) point of the unit ball of X for a.e. $\omega \in \text{supp} f$, and, conversely, is this property sufficient for f to be an extreme (smooth, exposed, etc) point of the unit ball of $E(X)$? These chapters contain also a discussion of the containment of c_0 and ℓ_1 in $E(X)$.

The basic properties of Köthe and Köthe-Bochner function spaces are treated in the third chapter *Köthe-Bochner function spaces.* Chapters 1, *Classical theorems* and 2, *Convexity and smoothness,* contain some basic results (most of them with complete proofs) on Banach spaces as strict convexity, uniform convexity, smoothness, Dunford-Pettis property, conditional expectations and martingales, tensor products. The last chapter of the book, Chapter 6, *Continuous function spaces,* is concerned with the Banach space $C(K, X)$.

Each chapter contains a set of exercises completing the main text, open questions for further study, remarks and historical notes, and bibliography.

The book is clearly written and succeeds to present in an accessible manner some deep and difficult results in the domain. It can be recommended for advanced graduate students and for researchers in functional analysis, probability theory, operator theory and related fields.

S. Cobzaş

Ole Christensen and Khadija L. Christensen, ***Approximation theory - From Taylor Polynomials to Wavelets***, Applied and Numerical Harmonic Analysis Series, Birkhäuser Verlag, Boston-Basel-Berlin 2004, xi+156 pp, ISBN:0-8176-3600-5.

This book contains an elementary introduction to approximation theory, in a way which naturally leads to the modern field of wavelets. One of the main goals of this presentation is to make it clear to the reader that the mathematics is a subject in a state of continuous evolution. The exposition demonstrates the dynamic nature of mathematics and how the classical disciplines influence many areas of modern mathematics and their applications.

The focus here is on ideas rather than on technical details. The book may be used in courses on infinite series and Fourier series, where ideas and motivation are more important than proofs. Some of the material from the two chapters on wavelets can be used as a guide towards more recent research. The wavelets are presented as a natural continuation of the material from the previous chapters.

The information is accessible to readers at several levels. Some basic material is placed at the beginning of each chapter preparing the reader for the more advanced concepts and topics in the latter part of that chapter. Only selected results are proved, while more technical proofs are included in an appendix.

The first chapter, dedicated to approximation by polynomials, contains elementary results. It also gives an idea about the content of the entire book. The next chapter presents the infinite series. It contains several classical entertaining examples and constructions. The Fourier analysis is treated in Chapter 3.

Wavelet analysis can be considered a modern supplement to classical Fourier analysis. Therefore, the chapters 4 and 5 are dedicated to this subject. Chapter 4 describes wavelets more in words rather than in symbols, but it gives the reader an understanding of the fundamental questions and concepts involved. It also tells the story of how the wavelets era began and discusses the applications in the signal processing.

In Chapter 5, which is slightly more technical, the multiscale representation associated to wavelets in the special case of the Haar wavelets is explained. It also presents the Gabor system. In this chapter the role of wavelets in digital signal processing and data compression is discussed, along with the FBI's manner of using wavelets to store fingerprints.

Each of the chapters contains more examples and ends with a few exercises. The book can be used as a good textbook or for self-study reference for students. Readers find the motivation and the background material pointing towards advanced literature and research topics in pure and applied harmonic analysis and related areas.

Radu Lupşa