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CLASSIFICATION OF NEAR EARTH ASTEROIDS WITH ARTIFICIAL NEURAL NETWORK

ZOLTÁN MAKÓ, FERENC SZENKOVITS, EDIT GARDA-MÁTYÁS, AND IHARKA CSILLIK

Abstract. Asteroids that can pass inside the orbit of Mars are said to be Near-Earth Asteroids (NEAs) or Earth-Approaching asteroids. The NEAs are subdivided into several groups based on their orbital characteristics. There are three important groups: Amor, Apollo and Aten. In this paper we show that fundamental characteristics of this classification, for which these groups are linear separable, are the focal distance and the semimajor axis. Starting from this property we construct a perceptron-type artificial neural network to classify automatically these objects into groups Amor, Apollo or Aten.

1. Introduction

Asteroids are rocky and metallic objects that orbit the Sun but are too small to be considered planets. They are also known as minor planets. Asteroids are divided into groups and families based on their orbital characteristics. Usually a group of asteroids is named after the first discovered member of the group. These groups are relatively loose dynamical associations.

Asteroids that can pass inside the orbit of Mars are known as Near-Earth Asteroids (NEAs) or Earth-Approaching asteroids. Rigorously NEAs are the asteroids with the perihelion distance $q < 1.3$ AU and the aphelion distance $Q > 0.983$ AU (see [2]). These asteroids probably came from the main asteroid belt, but were jolted from the belt by collisions or by interactions with the gravitational fields of other objects (primarily Jupiter). According to astronomers there are at least 1,000 NEAs

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whose diameter is greater than 1 kilometer and which could do catastrophic damage to the Earth (see for example [6], [1]). Even smaller NEAs could cause substantial destruction if they were to collide with the Earth.

From the point of view of the geometry of the orbit, there are four types of NEAs ([5], [4]):

1. The group of Aten asteroids (Atens) was named after 2062 Aten (discovered by E. F. Helin in 1976, with semimajor axis $a = 0.967$ AU, eccentricity $e = 0.183$ and inclination $i = 18^\circ.9$) . They have semimajor axes less than 1 AU and aphelion distance greater than or equal to 0.983 AU (the present perihelion distance of the Earth), namely

$$a(1+e) \geq 0.983 \text{ AU and } a < 1 \text{ AU}, \tag{1}$$

placing them inside the Earth's orbit.

2. The Apollos, named after 1862 Apollo (K. Reinmuth, 1932, $a = 1.471$ AU, $e = 0.560$, $i = 6^\circ.4$), have semimajor axes greater than or equal to 1 AU and perihelion distances less than or equal to 1.017 AU (the present aphelion distance of the Earth), namely

$$a \geq 1 \text{ AU and } a(1-e) \leq 1.017 \text{ AU}. \tag{2}$$

Some Apollos have eccentric orbits that cross the orbit of the Earth, making them a potential threat to our planet.

3. The Amors, named after the asteroid 1221 Amor (E.J. Delporte, 1932, $a = 1.920$ AU, $e = 0.435$ and $i = 11^\circ.9$), have perihelion distances between 1.017 AU and 1.3 AU (the present perihelion distance of the Mars), namely

$$1.017 \text{ AU} < a(1-e) < 1.3 \text{ AU}. \tag{3}$$

Amors often cross the orbit of Mars (if the orbit is eccentric enough), but they do not cross the orbit of Earth.

4. Inside of the orbit of the Earth, with perihelion distances less then 0.983 AU, orbit the Apoheles, for which

$$a(1+e) < 0.983 \text{ AU}. \tag{4}$$

"Apohele" is Hawaiian for "orbit". Other proposed names for this group are Inner-Earth Objects (IEOs) and Anons (as in "Anonymous"). Until May 2004 there are only two known Apoheles: 2003 CP20 and 2004 JG6.

In the presented classification (1–4) the semimajor axis (a) and the eccentricity (e) are used as fundamental parameters. In the plane of these parameters the separatrices between the different groups are mainly hyperbolas (see Figure 1). Generally is more convenient, if the separatrices of the groups are linear. Linear separatrices make possible for example the use of linear statistical methods in the different studies, and simple classification with a parallel computing artificial neural network.

In this paper we point out that if the semimajor axis and the focal distance, $c = ea$ are used as fundamental parameters, then the separatrices between the presented classical groups are all linear. Using this property we constructed a parallel computing artificial neural network to classify the NEAs.

2. **Linear separation of the NEA groups**

The different groups of asteroids (Atens, Apollos, Amos, Apoheles and other asteroids), defined in the above presented classification (1–4), are separated – in the plane of parameters (a, e) –, by curves of equation (see Figure 1)

$$f_i(a,e) = 0,\ i = 1,2,3,4, \tag{5}$$

where functions $f_i : (0,\infty) \times [0,1) \to \mathbb{R}$ are

$$\begin{aligned} f_1(a,e) &= a + ae - 0.983, \qquad f_2(a,e) = a - 1, \\ f_3(a,e) &= -a + ae + 1.017, \quad f_4(a,e) = a - ae - 1.3. \end{aligned}$$

The presence of the focal distance $c = ea$ in the hyperbolas $f_1 = 0, f_2 = 0$ and $f_4 = 0$ suggests us to transform the plane of parameters (a, e) in the plane (a, c). The corresponding function of transformation is $T : (0,\infty) \times [0,1) \to (0,\infty) \times [0,\infty)$, given by

$$T(a,e) = (a, ea)\,. \tag{6}$$

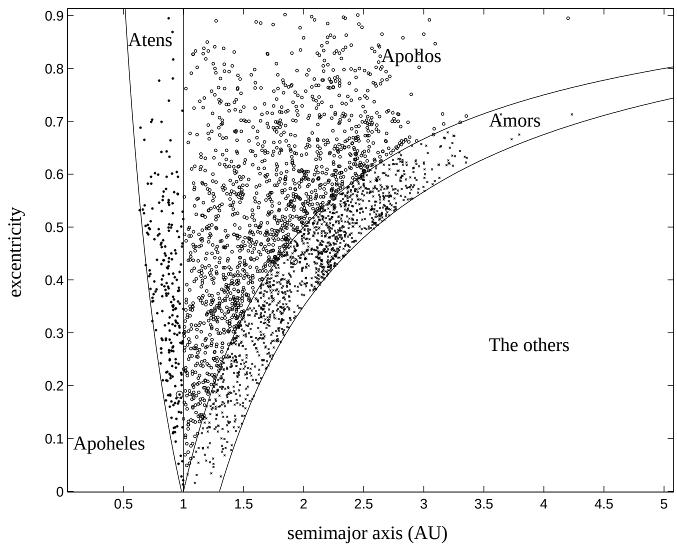

FIGURE 1. Distribution of NEAs in the (a, e) plane.

The four separatrices $f_i = 0$ are transformed by T in linear separatrices $g_i = 0$, $i = 1, 2, 3, 4$, where functions $g_i : (0, \infty) \times [0, \infty) \to \mathbb{R}$ are

$$\begin{aligned} g_1(a, c) &= a + c - 0.983, \qquad g_2(a, c) = a - 1, \\ g_3(a, c) &= -a + c + 1.017, \quad g_4(a, c) = a - c - 1.3. \end{aligned}$$

The distribution of the above defined groups of NEAs, in the plane of parameters (a, c), is illustrated in Figure 2.

In this plane the separatrices are all linear. The characterization of different groups by using the sign of the g_i separator functions is presented in Table 1 (here "+" means positive value or zero and "−" means negative value).

3. Classification of NEAs with artificial neural network

Artificial neural networks are composed of simple elements (*neurons*) operating in parallel. These elements are inspired by biological nervous systems. As in

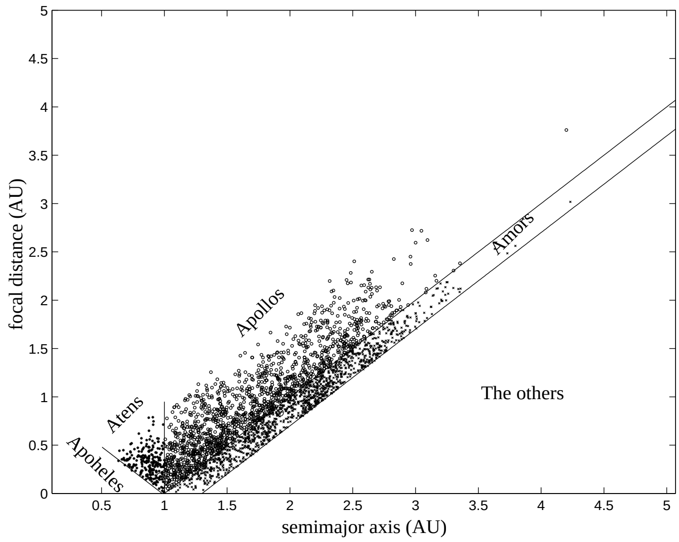

FIGURE 2. Distribution of NEAs in the (a, c) plane.

TABLE 1. The signs of the NEA groups in the plane (a, c).

Group	g_1	g_2	g_3	g_4
Apoheles	−	−	+	−
Atens	+	−	+	−
Apollos	+	+	+	−
Amors	+	+	−	−
others	+	+	−	+

nature, the network function is determined largely by the connections between elements. We can train a neural network to perform a classification by adjusting the values of the connections (*weights*) between elements ([3]).

A neuron with an input vector (a_1, a_2) and with bias scalar b appears on the Figure 3.

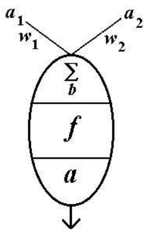

FIGURE 3. The neuron.

This neuron, denoted by a, transmits the input vector (a_1, a_2) to output scalar o, by using:

$$o = f(w_1 a_1 + w_2 a_2 + b), \tag{7}$$

where $f : \mathbb{R} \to \mathbb{R}$ is the *transfer function*, (w_1, w_2) is the *weights vector* and b is the *bias scalar*. Here f is a hardlim or a linear transfer function. Note that $\mathbf{w}$ and b are both adjustable parameters of the neuron.

The perceptron-type neural network, developed to classify exactly the NEAs in the above defined five groups is presented in Figure 4.

The transfer functions of our network is the hardlim function

$$H : \mathbb{R} \to \{-1, 1\}, \qquad H(x) = \begin{cases} 1, & \text{if } x \geq 0, \\ -1, & \text{if } x < 0, \end{cases}$$

in the neurons a_1, a_2, a_3 and a_4, and the identity function

$$I : \mathbb{R} \to \mathbb{R}, \qquad I(x) = x$$

in the neuron a_5 (Figure 4).

The input values of our network are the semimajor axis $a \in (0, \infty)$, and the focal distance $c = ea \in [0, \infty)$. The output value of this network is computed by formula

$$o = \frac{1}{2}\left[H(g_1(a,c)) + H(g_2(a,c)) - H(g_3(a,c)) + H(g_4(a,c))\right] + 3. \tag{8}$$

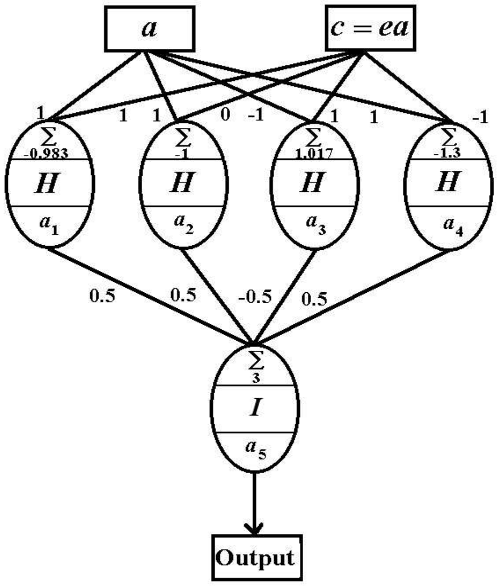

FIGURE 4. The neural network.

This value is: 1 for Apoheles, 2 for Atens, 3 for Apollos, 4 for Amors, and 5 for other asteroids.

4. Conclusions

In this study we proved that in the classification of the NEAs is more convenient to use as parameters the semimajor axis and the focal distance instead of the semimajor axis and eccentricity, because in this plane of parameters (a,c) the separatrices between different NEA groups are linear. This parameters make also possible the development of an artificial neural network to perform a parallel computing classification of NEAs. Another advantage of our linear classification is that it can be easily compared with other linear classifications.

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Department of Mathematics and Informatics, Sapientia University, 530104, Miercurea-Ciuc, Romania
E-mail address: makozoltan@sapientia.siculorum.ro

Department of Applied Mathematics, Babeş-Bolyai University, Str. M. Kogălniceanu 1, 400084 Cluj-Napoca, Romania
E-mail address: fszenko@math.ubbcluj.ro

Department of Mathematics and Informatics, Sapientia University, 530104, Miercurea-Ciuc, Romania

Astronomical Institute of Romanian Academy, Astronomical Observatory of Cluj-Napoca, Str. Cireşilor 19, 400487 Cluj-Napoca, Romania
E-mail address: iharka@math.ubbcluj.ro