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RADIATION EFFECTS ON FREE CONVECTION FROM A VERTICAL CONE EMBEDDED IN A FLUID SATURATED POROUS MEDIUM

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Abstract. The radiation effects on the steady free convection boundary layer over a vertical cone embedded in a fluid saturated porous medium are studied. We adopt for the radiative model the well-known Rosseland model. It has been found that similarity solutions exist and the ordinary differential equations were solved using a combined Runge-Kutta method and shooting technique.

1. Introduction

Heat transfer in porous media is involved in many practical applications in geophysics, energy related problems, environment problems, etc. The monographs: Ingham and Pop (1998, 2002), Vafai (2002), Pop and Ingham (2001) and Ingham et al. (2004) give an excellent summary of the work on this subject.

If the heat transfer process take place at high temperature radiative effects can't be neglected (Modest, 2003; Siegel and Howell, 1992). The radiative models used for fluids are not always appropriate for porous media. A synthesis of radiative models in porous media is given by Kaviany and Singh (1993). Using Rosseland approximation (see Rosseland, 1936), Hossain and Takhar (1996), Raptis (1998), Hossain and Pop (2001) and Bakier (2001) studied the free mixed convection from vertical surfaces placed in porous media. Chamka (1997), Chamka et al. (2001, 2002) considered the solar radiation case or, in addition, the mass transfer.

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In this paper we will study the radiation effect in the free convection from a vertical cone embedded in a fluid saturated porous medium using the Rosseland radiative model.

2. **Basic equations**

We consider a vertical cone having a constant surface temperature, T_w, while the cone is placed in an opaque fluid saturated porous medium having the temperature T_∞ (see Fig. 1). Under boundary layer Boussinesq approximations and using the Rosseland radiative model the governing equations are given by:

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial y} = 0 \tag{1}$$

$$u = \frac{g\cos\gamma K\beta}{v}(T - T_\infty) \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_m \frac{\partial^2 T}{\partial y^2} - \frac{1}{(\rho_\infty c)_f}\frac{\partial q^r}{\partial y} \tag{3}$$

where $r = x\sin\gamma$ is the cone's radius, v is the kinematic viscosity, K is the permeability, α_m is the thermal diffusivity, ρ is the density and c is the specific heat. The subscripts w and ∞ are related to the surface and to the ambient medium, respectively. The radiative heat flux, q^r, has the form:

$$q^r = -\left(\frac{4\sigma}{3\chi}\right)\frac{\partial T^4}{\partial y} \tag{4}$$

where σ is the Stefan-Boltzman's constant and χ is the mean absorption coefficient in the Rosseland approximation.

The boundary conditions for eqs. (1)-(3) are:

$$\begin{aligned} &v = 0, \quad T = T_w \quad \text{for} \quad y = 0 \\ &u \to 0, \quad T \to T_\infty \quad \text{for} \quad y \to \infty \end{aligned} \tag{5}$$

In order to obtain similar solutions the following transformations are introduced:

$$\psi = \alpha_m r Ra_x^{1/2} f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \eta = Ra_x^{1/2}(y/x) \tag{6}$$

where ψ is the stream function, $ru = \dfrac{\partial\psi}{\partial x}$, $rv = -\dfrac{\partial\psi}{\partial y}$, η is the similar variable and Ra_x is the local Rayleigh number defined as:

$$Ra_x = \frac{g\beta K \cos\gamma (T_w - T_\infty) x}{v\alpha_m} \tag{7}$$

Using (6) in eqs. (1)-(3) and in boundary conditions (5) the governing equations became:

$$f' = \theta \tag{8}$$

$$\left\{\left[1 + \frac{4}{3}N[1 + (\theta_w - 1)\theta]^3\right]\theta'\right\}' + \frac{3}{2}f\theta' = 0 \tag{9}$$

$$f(0) = 0, \quad \theta(0) = 1, \quad f'(\infty) = 0 \tag{10}$$

where the radiative and temperature parameters N and θ_w, respectively, have the form:

$$N = \frac{4\sigma T_\infty^3}{k\chi}, \quad \theta_w = \frac{T_w}{T_\infty} \tag{11}$$

From the energetic balance on the cone's surface it is possible to deduce the convective heat transfer coefficient, h:

$$-k\frac{\partial T}{\partial y}\bigg|_{y=0} + q^r = h(T_w - T_\infty) \tag{12}$$

and thus the local Nusselt number is given by:

$$Nu_x = -\theta'(0) Ra_x^{1/2}\left[1 + \frac{4}{3}N\theta_w^3\right] \tag{13}$$

We must mention that in the absence of the radiation effect ($N = 0$), eqs. (8)-(10) reduce to those obtained by Cheng et al. (1985).

3. Results and discussions

Eqs. (8)-(10) have been solved numerically using a combined Runge-Kutta and shooting method for the following values of the radiative parameter $N = 0,\ 1,\ 5$ and 10 for the temperature parameter $\theta_w = 1.1,\ 1.5$ and 2. In the case $N = 0$ (i.e. radiation effects are negligible), the calculated value for the local Nusselt number, $-\theta_w$, is in very good agreement with that obtained by Cheng et al. (1985). The values for the Nusselt number are given in Table 1 for different values of parameters N and θ_w. Figs. 2-4 present the dimensionless temperature's profiles variation with

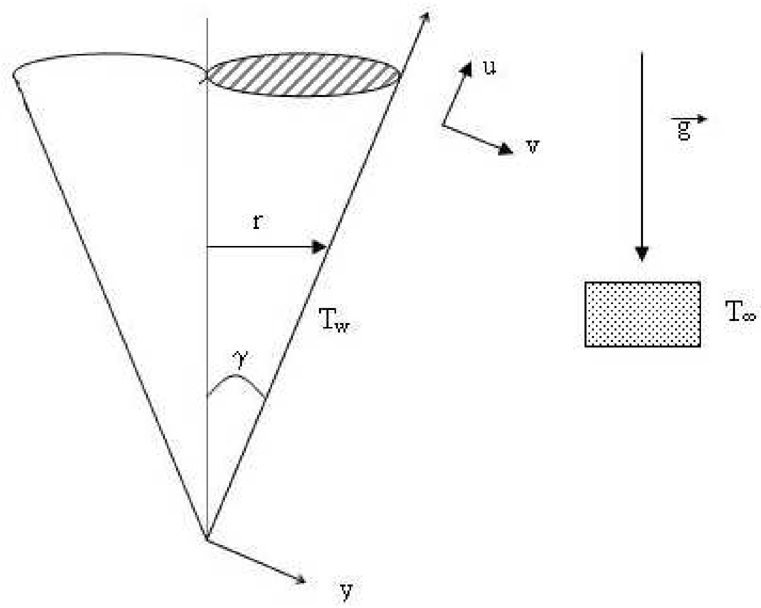

FIGURE 1. Geometry of the problem

the variation of the radiation parameter N. It is observed that the thickness of the boundary layer increase with the increasing of the parameter N. It is also observed in Figs. 5-7 that the dimensionless temperature increase with the increasing of the parameter θ_w.

N	$-\theta'(0)$		
	$\theta_w = 1.1$	$\theta_w = 1.5$	$\theta_w = 2.0$
0	0.768596(*0.769)	0.768596	0.768596
1	0.250263	0.212828	0.187691
5	0.107103	0.094502	0.085511
10	0.074887	0.066666	0.060616

Table 1. Values of the local Nusselt number, $\theta'(0)$

*Result obtained by Cheng et al. (1985)

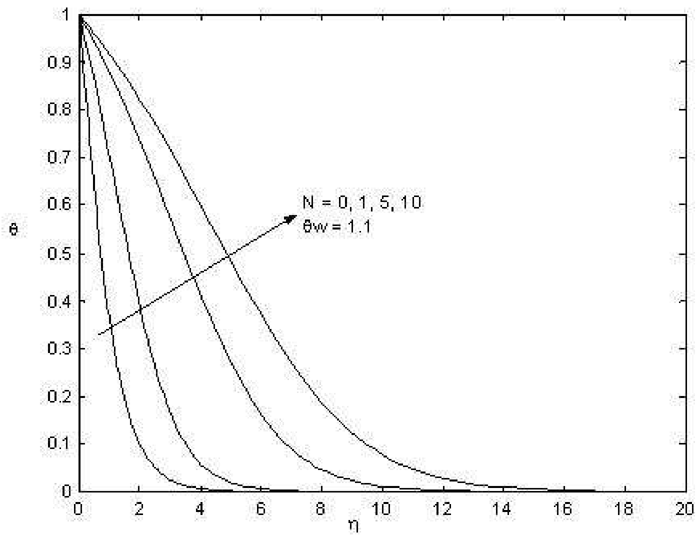

FIGURE 2. Dimensionless temperature profiles for $N = 0, 1, 5, 10$ and $\theta_w = 1.1$

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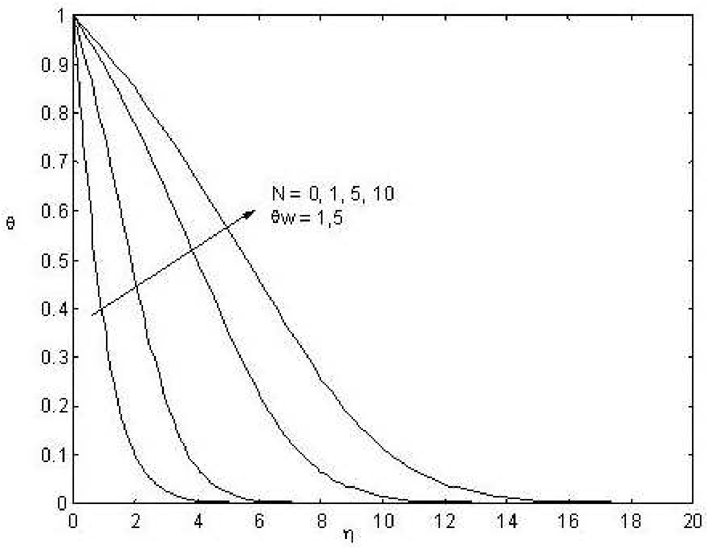

FIGURE 3. Dimensionless temperature profiles for $N = 0, 1, 5, 10$ and $\theta_w = 1.5$

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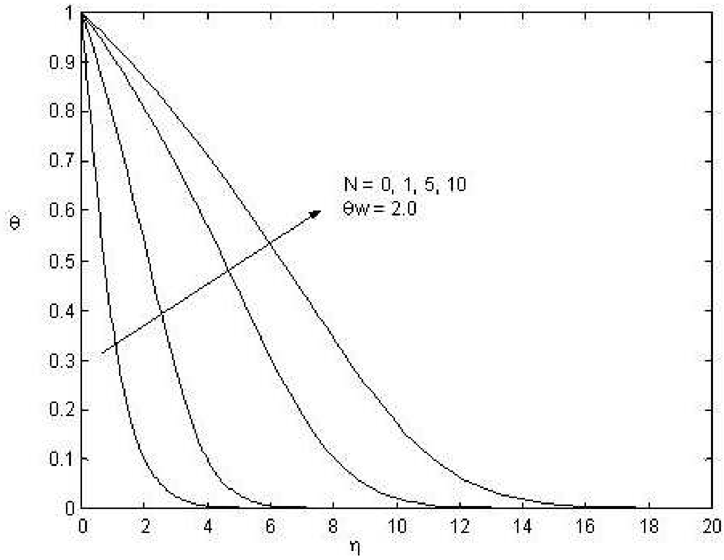

FIGURE 4. Dimensionless temperature profiles for $N = 0, 1, 5, 10$ and $\theta_w = 2$

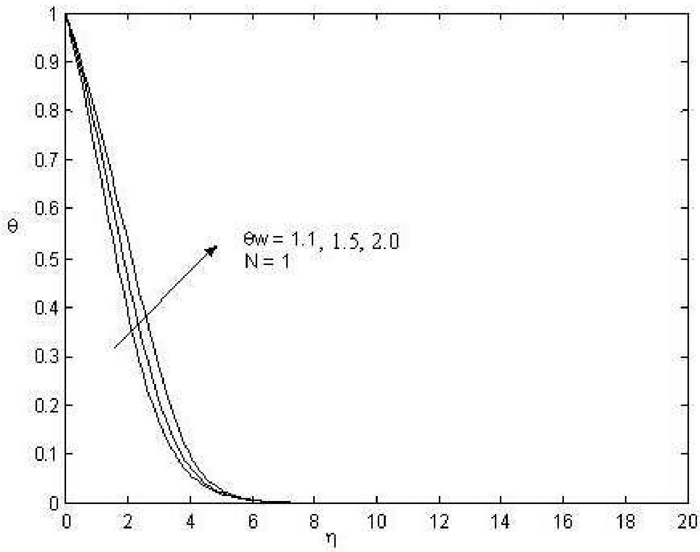

FIGURE 5. Dimensionless temperature profiles for $N = 1$ and $\theta_w = 1.1, 1.5, 2$

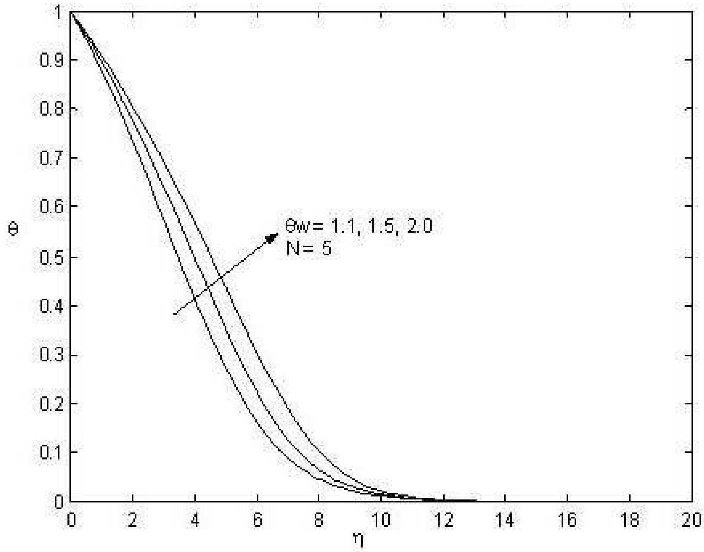

FIGURE 6. Dimensionless temperature profiles for $N = 5$ and $\theta_w = 1.1, 1.5, 2$

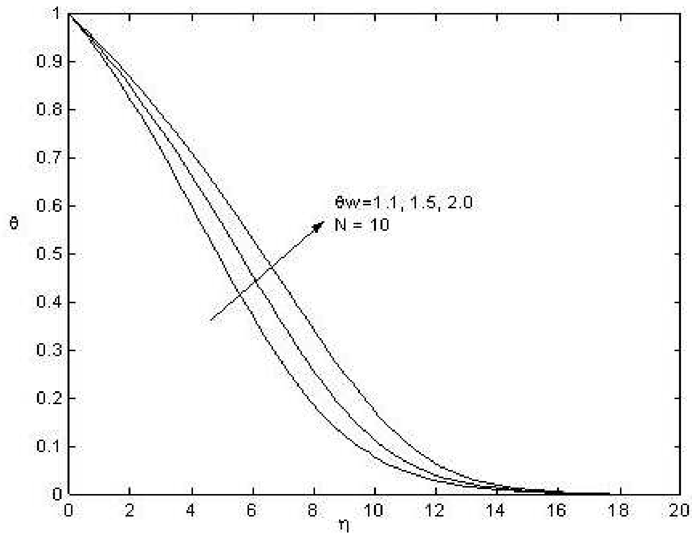

FIGURE 7. Dimensionless temperature profiles for $N = 10$ and $\theta_w = 1.1, 1.5, 2$

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