BOOK REVIEWS

Agostino Abbate, Casimer M. DeCusatis, Pankaj K. Das, *Wavelets and Subbands. Fundamentals and Applications*, Birkhäuser, Boston Basel Berlin, 2002, ISBN 0-8176-4136-X.

The volume presents in a typical style some researches and noteworthy directions particularly aimed at providing stimulus and inspiration to workers interested in the broad areas of related to wavelets and their applications. The content of the book is divided into three parts.

In the first part (*Fundamentals*) the authors enlarge on a systematic study of wavelets and subbands concepts. It is written with a concern for simplicity and clarity. The section offers detailed and explanation of some concepts and methods accompanied by carefully selected worked examples. The aim of this part is to familiarize the reader with a lot of basic notions regarding wavelet and subband transforms, such as: Fourier transform as a wave transform, wavelet transform, time-frequency analysis, multiresolution analysis, wavelet frames, connection between wavelets and filters, analysis and synthesis filters, iterated filters for subbands, filter banks for subbands. Also, the link between discrete and continuous wavelets and subbands is explained.

The second part (*Wavelets and Subbands*) includes advanced topics and a more in-depth technical treatment of the subject matter. The information is structured in the following chapters: Time and Frequency Analysis of Signals; Discrete Wavelet Transform: from Frames to Fast Wavelet Transform; Theory of Subband Decomposition; Two-Dimensional Wavelet Transforms and Applications.
Within the scope of this section, the authors investigate the large body of work that has been done in applying wavelet and subband methods to image processing and compression.

The third part (*Applications*) contains some practical applications of wavelets and subbands. Divided in three chapters, these include image processing, image compression, pattern recognition, and signal-to-noise improvement. The communication application concentrates on spread spectrum systems which have applications to wireless communication, digital multitone, code division multiple access and excision.

At the end of the book are inserted four appendices: Fourier Transform, Discrete Fourier Transform, z-Transform and Orthogonal Representation of Signals.

We point out that for additional information, the reader is referred to the many excellent references in the literature which are listed at the end of each part. In the same time, in order to sustain the objectives of the book, a generous bibliography is listed over 20 pages.

In our opinion, the monograph is a valuable text for a broad audience including graduates, researches and professionals in signal processing.

Octavian Agratini

Erik M. Alfsen and Frederic W. Shultz, *Geometry of State Spaces of Operator Algebras,* Mathematics: Theory and Applications, Birkhäuser Verlag, Boston-Basel-Berlin 2003, xiii+467 pp., ISBN 0-8176-4319-2 and 3-7643-4319-2.

The aim of the present book is to give a complete geometric description of the state spaces of operator algebras, meaning to give axiomatic characterizations of those convex sets that are state spaces of C^*-algebras, von Neuman algebras, and of their nonassociative analogs - JB-algebras and JBW-algebras. A previous book by the same authors -*State spaces of operator algebras - basic theory, orientations and C^*-products,* published by Birkhäuser in 2001, contains the necessary prerequisites on C^*-algebras and von Neumann algebras but, for the convenience of the reader, these results are summarized in an appendix at the end of the present book with exact references to previous one for proofs.

The problem of the characterization of state spaces of operator algebras was raised in the early 1950s and was completely solved by the authors of the present book in Acta Mathematica **140** (1978), 155-190, and **144** (1980), 267-305 (the second paper has also H. Hanche-Olsen as co-author). Although the axioms for state spaces are essentially geometric, many of them have physical interpretations. The authors have included a series of remarks concerning these interpretations along with some historical notes.

The book is divided into three parts. Part I (containing Chapters 1 through 6) can serve as an introduction for novices to Jordan algebras and their states. Jordan algebras were originally introduced as mathematical model for quantum mechanics (in 1934 by P. Jordan, J. von Neumann and E. Wigner), starting from the remark that the set of observables is closed under Jordan multiplication, but not necessarily under associative multiplication. Part II (Chapters 7 and 8) develops the spectral theory for affine functions on convex sets. The functional calculus developed in this part reflects a key property of the subalgebra generated by a single element and, physically, it represents the application of a function to the outcome of an experiment. Part III (Chapters 9,10,11) gives the axiomatic characterization of operator algebra state spaces and explain how the algebras can be reconstructed from their state spaces.

This valuable book, together with the previous one on C^*-algebras, presents in a manner accessible to a large audience, the complete solution to a long standing problem, available previously only in research papers, whose understanding requires a solid background from the readers.

It is aimed to specialists in operator algebras, graduate students and mathematicians working in other areas (mathematical physics, foundation of quantum mechanics)

S. Cobzaş

Jan Andres and Lech Górniewicz, *Topological Fixed Point Principles for Boundary Value Problems,* Topological Fixed Point Theory and Its Applications 1, Kluwer Academic Publishers, Dordrecht-Boston-London, 2003, 761 + xvi pp., ISBN 1-4020-1380-9.

The monograph is devoted to the topological fixed point theory for single-valued and multivalued mappings in locally convex spaces and its applications to boundary value problems for ordinary differential equations and inclusions and to multivalued dynamical systems.

Chapter I, Theoretical background (126 pp.) gathers together several topological and analytical notions and results such as: locally convex spaces, absolute retracts (AR-spaces) and absolute neighborhood retracts (ANR-spaces), selections of multivalued mappings, admissible mappings, Lefschetz fixed point theorem, fixed point index in locally convex spaces, Nielson number etc.

In Chapter II, General principles (106 pp.), topological principles necessary for applications are presented, namely: Aronszajn-Browder-Gupta type results on the topological structure of fixed point sets, inverse limit method, topological dimension of fixed point sets, topological essentiality, relative theories of Lefschetz and Nielson, periodic point theorems, fixed point index for condensing maps, approximation methods in the fixed point theory of multivalued mappings, topological degree by means of approximation methods and continuation principles based on fixed point index and coincidence index.

Chapter III, Applications to differential equations and inclusions (366 pp.), is devoted to the applications of the general principles to boundary value problems for ordinary differential equations and inclusions on compact or non-compact intervals and to dynamical systems. The following problems are mainly considered: existence of solutions, topological structure of solution sets, topological dimension of solution sets, multiplicity results, periodic and almost periodic solutions and Wazewski type results.

Three Appendices concerning almost periodic and derivo-periodic functions and multivalued fractals are also included. A large and exhaustive list of References (58 pp.) and a subject Index are added.

The authors are known as experts in their field and most of presented results are their own. The book is self-contained and every chapter concludes by a section of Remarks and Comments giving to the reader historical information and suggestions for further studies.

Authors' intention has been to make deep results of algebraic topology and nonlinear analysis accessible to a wider auditorium and by this, to stimulate the interest of applied mathematicians (mathematical economists, population dynamics experts, theoretical physicists etc.) for such type of methods.

I believe that this monumental monograph will be extremely useful to post-graduate students and researchers in topological fixed point theory, nonlinear analysis, nonlinear differential equations and inclusions, dynamical systems, optimal control and chaos and fractals. This book should stimulate a great deal of interest and research in topological methods in general and in their applications in particular.

Radu Precup

Emmanuele DiBenedetto, *Real Analysis*, Birkhäuser Advanced Texts, Birkhäuser Verlag, Boston-Basel-Berlin 2002, xxiv+485 pp., ISBN 0-8175-4231-5.

The aim of this book is to present at graduate level the basic results in real analysis, needed for researchers in applied analysis - PDEs, calculus of variations, probability, and approximation theory. Assuming only the knowledge of the basic results about the topology of $\mathbb{R}^N$, series, advanced differential calculus and algebra of sets, the author develops the whole machinery of real analysis bringing the reader to the frontier of current research.

The emphasis is on measure and integration in $\mathbb{R}^N$, meaning Lebesgue and Lebesgue-Stieltjes measures, Radon measures, Hausdorff measure and dimension. The topological background, including Tihonov compactness theorem, Tietze and Urysohn theorems, is developed with full proofs. The specific of the book is done by the treatment of some more specialized topics than those usually included in introductory courses of real analysis. Between these topics I do mention a detailed presentation of covering theorems of Vitali and Besicovitch, the Marcinckiewicz integral, the Legendre transform, the Rademacher theorem on the a.e differentiability of Lipschitz functions. Fine topics, as a.e. differentiability of functions with bounded variation and of absolutely continuous functions and the relation with the integral, are worked out.

The spaces L^p are also presented in details in Chapter V - completeness, uniform convexity (via Hanner's inequalities), duality, weak convergence, compactness criteria. The next chapter of the book (Ch. VI) contains a brief introduction to abstract Banach and Hilbert spaces. Distributions, weak differentials and Sobolev spaces are presented in Chapter VII.

The last two chapters of the book, Chapters VIII and IX, contains more specialized topics as maximal functions and Fefferman-Stein theorem, the Calderón-Zygmund decomposition theorem, functions of bounded mean oscillation (BMO), Marcinkiewicz interpolation theorem, embedding theorems for Sobolev spaces, Poincaré inequality, Morrey spaces.

Each chapter is completed by a set of exercises and problems that add new features and shed new light on the results from the main text.

Bringing together, in a relatively small number of pages, important and difficult results in real analysis that are of current use in application to PDEs, Fourier and harmonic analysis and approximation, this valuable book is of great interest to researchers working in these areas, but it can be used for advanced graduate courses in real analysis as well.

Stefan Cobzaş

Stefan Czerwik, *Functional Equations and Inequalities in Several Variables*, World Scientific, New Jersey-London-Singapore-Hong Kong, 2002, ISBN 981-02-4837-7.

In the recent period (especially in the last three decades) functional equations became an important branch of mathematics. The book under review is intended to present a survey of classical results and more recent developments in the theory of functional equations in several variables. Particularly the results of the Polish school of functional equations are emphasized.

The book is divided into three parts. The first one is devoted to the study of additive and convex functions defined on linear spaces endowed with semilinear topologies. The classical results of Bernstein-Doetch, Picard, and Mehdi, concerning the relationship between the continuity and the local boundedness of a convex function, are presented in Chapter 4. Closely related to this problem are the so-called Kuczma-Ger set classes, studied in Chapter 5. The rest of the material included in the first part deals with familiar functional equations like Cauchy, D'Alembert, and quadratic equations.

Part II, entitled *Ulam-Hyers-Rassias Stability of Functional Equations*, is entirely concened with the examination of the stability problem. It has originally been posed by S. M. Ulam in 1940 with regard to the Cauchy functional equation. In 1941 D. H. Hyers gave a significant partial solution to this problem, but a substantial generalization of Hyers' result has been obtained by Th. M. Rassias in 1978. Rassias' paper has rekindled the interest of the mathematicians in the field of stability of functional equations. Since then a great number of articles have appeared in the literature. This second part of Czerwik's book brings together the stability results concerning several functional equations like Cauchy, Pexider, Jensen, D'Alembert, gamma, and quadratic, obtained by many authors.

Particularly valuable, Part III contains a systematic examination of set-valued functional equations, which has been lacking in the mathematical literature. Set-valued versions of the Cauchy, Jensen, Pexider, and quadratic equations are studied in this part. Finally, the author investigates some special kinds of set-valued functions like subadditive, superadditive, subquadratic, K-convex, and K-concave set-valued functions.

Twenty-one of the thirty-seven chapters contain valuable notes at the end, providing useful references to related material. The bibliography counts 216 references.

Written by an expert in domain, the book is an excellent tool for any reader interested to get an idea about the basic results and the latest research directions in the field of functional equations.

Lokenath Debnath, *Wavelet Transforms and Their Applications*, Birkhäuser, Boston Basel Berlin, 2002, ISBN 0-8176-4204-8.

The last two decades have produced tremendous developments in the mathematical theory of wavelets and their great variety of applications. Since wavelet analysis is a relatively new subject, this monograph is intended to be self-contained. The book is designed as a modern and authoritative guide to wavelets, wavelet transform, time-frequency signal analysis and related topics.

It is known that some research workers look wavelets upon as a new basis for representing functions, others consider them as a technique for time-frequency analysis and some others think of them as a new mathematical subject. All these approaches are gathered in this book, which presents an accessible, introductory survey of new wavelet analysis tools and the way they can be applied to fundamental analysis problems. We point out the clear, intuitive style of presentation and the numerous examples demonstrated thorough the book illustrate how methods work in a step by step manner.

This way, the book becomes ideal for a broad audience including advanced undergraduate students, graduate and professionals in signal processing. Also, the book provides the reader with a through mathematical background and the wide variety of applications cover the interdisciplinary collaborative research in applied mathematics. The information is spread over 565 pages and is structured in 9 chapters as follows:

1. Brief Historical Introduction
2. Hilbert Spaces and Orthonormal Systems
3. Fourier Transforms and Their Applications
4. The Gabor Transform and Time-Frequency Signal Analysis
5. The Wigner-Ville Distribution and Time-Frequency Signal Analysis
6. Wavelet Transforms and Basic Properties
7. Multiresolution Analysis and Construction of Wavelets
8. Newland's Harmonic Wavelets
9. Wavelet Transform Analysis of Turbulence.

At the end of the book a key and hints for selected exercises are included.

In order to stimulate further interest in future study and to sustain the present material, a generous bibliography is listed.

Octavian Agratini

Andrzej Granas and James Dugundji, *Fixed point theory,* Springer-Verlag, New York-Berlin-Heidelberg, 2003, 13 figs. xv + 690 pages, ISBN 0-387-00173-5.

Fixed point theory represents one of the most powerful tools for various problems from pure, applied and computational mathematics. The abstract theory, the computation of fixed points and various applications, mainly for proving the existence of solutions to several classes of nonlinear operator equations, occupies a central place in today's mathematics. Over 150 monographs and proceedings, as well as more then 10, 000 papers deal with this topic. Two very new journals are entirely dedicated to fixed point theory and its applications.

The new edition of Granas and Dugundji's book is, in my opinion, the most important and complete survey in the last years on fixed point theory and its applications. The book goes through almost all the basic results in fixed point theory, from elementary theorems to advanced topics, from ordered, metric or topological structures to algebraic topology. The main text is self-contained, the necessary background material being collected in an appendix at the end of the book. Each chapter ends with "Miscellaneous Results and Examples" and some very important "Notes and Comments". Several nice photographs of famous mathematicians in the field pigment the text.

The book is organized in six important parts, each of them containing several chapters, twenty on the whole.

Part I ("Elementary Fixed Point Theorems", 74 pages) includes basic results and applications in ordered and metric structures. The main topics of this part are: the Banach contraction principle, the continuation method for contractive maps, the Knaster-Tarski and Tarski-Kantorovitch theorems, the Bishop-Phelps result, Caristi's fixed point theorem, Nadler's extension of Banach contraction principle to set-valued operators, the KKM operator theory and the fixed point theory for nonexpansive operators.

Part II (Theorem of Borsuk and Topological Transversality, 112 pages) presents several fundamental results in the topological fixed point theory: the antipodal theorem of Borsuk (and as consequence, the Brouwer fixed point theorem), Schauder's fixed point principles, the infinite-dimensional version of Borsuk theorem, the theory on topological transversality based on the notion of essential map, the Leray-Schauder principle and the nonlinear alternative. As applications, the Fan coincidence theorem, the mini-max inequality and the Kakutani and Ryll-Nardzewski theorems are also presented.

Part III (Homology and Fixed Points, 50 pages) is dedicated to the Lefschetz-Hopf theorem for polyhedra.

Part IV (Leray-Schauder Degree and Fixed Point Index, 120 pages) presents the notions of topological degree and fixed point index. This part starts with the presentation of Brouwer's degree, defined for maps on the Euclidian spaces, and then the concept is extended for compact maps in normed linear spaces. Further on, the case of an arbitrary metric absolute neighborhood retracts is also considered. Bifurcation results in absolute neighborhood retracts and existence theorems for boundary value problems related to partial differential equations are nice applications of this theory.

Part V (The Lefschetz-Hopf Theory, 122 pages) is deals with the Lefschetz fixed point theorem and the Hopf index theorem. Several extensions of the Lefschetz theory to wider classes of maps and spaces are also included.

Part VI (Selected Topics, 97 pages) contains advanced topics of algebraic topology: Finite-Codimensional Čech Cohomology, Vietoris Fractions ans Coincidence Theory.

The Bibliography is organized as follows:

► General Reference Texts (Monographs, Lecture Notes, Surveys, Articles) with more than 700 titles

► Additional References with more than 400 titles.

An Appendix, a List of Symbols, an Index of Names and an Index of Terms are also included.

From the above considerations, it is more than obviously that this new edition of Granas-Dugundji's monograph is, in fact, a new book. New and interesting results and applications can be found all over the book. The style is alert and pleasant. The technical presentation of the book is exceptional.

The book **Fixed Point Theory, by Andrzej Granas and James Dugundji**, which appeared in the series **Springer Monograph in Mathematics**, is an inspired publication of Springer-Verlag Publishing House and I am sure that it will be a very useful work for anyone (postgraduate students, Ph.D. students, researchers, etc.) who is involved in fixed point theory in particular and nonlinear analysis in general.

Adrian Petruşel

Srdjan Stojanovic, *Computational financial mathematics using Mathematica: optimal trading stocks and options*, Birkhäuser Verlag, Boston - Basel - Berlin, 2003, XI+481 pages.

The book consists in 481 pages i.e. 8 chapters, a bibliography and an index and includes CD-ROM. Srdjan Stojanovic taught the course on Financial Mathematics at the University of Cincinnati since 1998 and at Purdue University during the academic year 2001-2002. This book is an expanded version of those courses, built with the help of the students during the time when Srdjan Stojanovic taught them computational financial mathematics and MATHEMATICA^R programming.

A very interesting and very actual book, because now, the computer make an integrand part of our life. The author, himself, underlines in the Introduction, that the book is addressed to students and professors of academic programs in financial mathematics (like computational finance and financial engineering). Anyway, the mathematical background would be Calculus, Differential Equations and Probability, but varies according to the objectives of the reader. The book is, as recommends the author, divided in some parts according to the required mathematical level as follows: the basics (for the Chapters 1-4), intermediate level (the Chapters 5 and 7), advanced level (for the Chapters 6 and 8).

In the Chapter 1, **Cash Account Evolution**, ordinary differential equations are solving with Mathematica^R, and symbolic and numerical solutions of ODEs are presented.

The Chapter 2, **Stock Price Evolution**, explains to the reader what are stocks and then presents the stock price modeling, i.e. some stochastic differential equations. An other aim of this chapter is to be acquainted with Itô calculus and with multivariable and symbolic Itô calculus. Also, some relationship between SDEs and PDEs are presented.

In the Chapter 3, **European Style Stock Options**, the first paragraph deals with the notion of stock option. Then, the Black and Scholes PDE and hedging are presented and the Black and Scholes PDE are symbolically solved. Also, the generalized Black and Scholes formulas are presented.

In the Chapter 4, **Stock Market Statistics**, the stock market data import and manipulation are presented. Then, the chapter deals with the volatility estimates, i.e. scalar case, and also deals with the appreciation rate estimates (the scalar case) and the statistical experiments (Bayesian and non-Bayesian). In the same chapter, the vector basic price model statistics and the dynamic statistics, like the filtering of conditional Gaussian processes, are treated.

In the Chapter 5, **Implied Volatility for European Options**, the option market data is presented. After that, the Black and Scholes theory is made obvious vs. market data (the implied volatility) and then, the numerical PDEs, the optimal control and the implied volatility are studied.

The Chapter 6, **American Style Stock Options**, deals with the american options, the obstacle problems and presents the general implied volatility for american options.

Very important, the Chapter 7, **Optimal Portfolio Rules,** presents the utility of wealth, the Merton's optimal portfolio rule derived and implemented, the portfolio rules under appreciation rate uncertainty, the portfolio optimization under equality constraints, the portfolio optimization under inequality constraints.

In the Chapter 8, **Advanced Trading Strategies,** the reduced Monge-Ampère PDEs of advanced portfolio hedging and the hypoelliptic obstacle problems in optimal momentum trading are presented.

As we have already said, the book is accompanied by a CD-ROM, but the book is not a software product. Informations about further developments might be available at the web site CFMLab.com. The reader may direct comments to the same address.

Diana Andrada Filip

Advances in Gabor Analysis, Hans G. Feichtinger and Thomas Strohmer - Editors, Applied and Numerical Harmonic Analysis, Birkhäuser Verlag, Boston-Basel-Berlin 2003, xviii+356 pp., ISBN 0-8176-4239-0 and 3-7643-4239-0.

In 1946 Dennis Gábor (Nobel prize for physics in 1971) had the idea to use linear combinations of a set of regularly spaced, discrete time and frequency translates of a single Gaussian function to expand arbitrary square-integrable functions. The idea turned out to be a very fruitful and far-reaching one, with spectacular applications to quantum mechanics and electrical engineering. The Heisenberg uncertainty principle, discussed at large in one of the included chapters, is the core of the time-frequency analysis and of Gabor analysis. Gabor analysis attracted many first rate mathematicians due to the highly non-trivial mathematics lying behind it. A strong impulse came from the development of frames in Hilbert space, leading to important problems of practical computation - rate of convergence, stability, density. In the last time, M.A. Rieffel, R.E. Howe and T.J. Steger found some unexpected connections with operator algebras.

The present book can be considered as a continuation of two previous ones: *Gabor Analysis and Algorithms: Theory and Applications,* H. G. Feichtinger and T. Strohmer - Editors, Birkhäuser 1998, and the book by K. Gröchenig, *Foundations of Time-frequency Analysis,* Birkhäuser 2001. It contains survey chapters, but new results that have been not published previously are also included. The introductory chapter of the book, written by H.G. Feichtinger and T. Strohmer, contains a clear outline of the contents as well as some comments on the future developments in Gabor analysis.

Beside this introductory chapter, the book contains other eleven chapters, written by different authors, and dealing with various questions in Gabor analysis and its applications: uncertainty principles, Zak transforms, Weil-Heisenberg frames, Gabor multipliers, Gabor analysis and operator algebras, approximation methods, localization properties, optimal stochastic encoding, applications to digital signal processing and to wireless communication.

Written by leading experts in the field, the volume appeals, by its interdisciplinary character, to a large audience, both novices and experts, theoretically inclined researchers and practitioners as well. It brilliantly illustrates how application areas and pure and applied mathematics can work together with profit for all.

S. Cobzaş

Enrico Giusti, *Direct Methods in the Calculus of Variations*, World Scientific, London-Singapore-Hong Kong, 2003, vii+403 pp., ISBN 981 238 043 4.

Let Ω be a domain in $\mathbb{R}^n$ and $F(x,u,z)$ a function from $\Omega\times\mathbb{R}^N\times\mathbb{R}^{n\times N}$ to $\mathbb{R}$. One denotes $x=(x_i)_{1\leq i\leq n}$, $u=(u^\alpha)_{1\leq\alpha\leq N}$, and $z=(z_i^\alpha)$, $1\leq i\leq n$, $1\leq\alpha\leq N$. The fundamental problem of the calculus of variations consists in finding a function $u:\Omega\to\mathbb{R}^N$ which minimizes the integral functional

$$\mathcal{F}(u,\Omega)=\int_\Omega F(x,u(x),Du(x))dx, \tag{1}$$

provided u satisfies some suitable conditions, the most frequent being a boundary condition, $u=U$ on $\partial\Omega$. Supposing F of class C^1, replacing u by $u+t\varphi$, where $\varphi=U$ on $\partial\Omega$, it follows that $g(t)=\mathcal{F}(u+t\varphi,\Omega)$ has a minimum at $t=O$, implying $g'(0)=0$. This condition leads to Euler (called sometimes Euler-Lagrange) equations

$$\frac{\partial}{\partial x_i}\left(\frac{\partial F}{\partial z_i^\alpha}(x,u(x),Du(x))\right)-\frac{\partial F}{\partial u^\alpha}(x,u(x),Du(x))=0, \tag{2}$$

for $\alpha=1,...,N$, that give a necessary condition of minimum. This approach is useful when the Euler equations can be explicitly integrated, particularly for $n=N=1$, leading to an explicit solution of the minimum problem, but with growing difficulties in higher dimensions.

The *direct method* in the calculus of variations, initiated by Riemann, consists in proving the existence of the minimum of $\mathcal{F}$ and discovering its properties, mainly regularity, without appealing to Euler equations. The usual assumptions, under which such an approach works, are lower semicontinuity of $\mathcal{F}$ and some convexity conditions (convexity, quasi-convexity, polyconvexity, rank-one convexity) on the function F, combined with some compactness hypotheses on the domain of $\mathcal{F}$, requiring compactness criteria in appropriate function spaces.

The present book follows this approach to the study of minima of the functional (1), an outline of its contents, along with some historical remarks, being given in the Introduction to the book. The book is divided into ten chapters: 1. *Semi-classical theory,* 2. *Measurable functions,* 3. *Sobolev spaces,* 4. *Convexity and semicontinuity,* 5. *Quasi-convex functionals,* 6. *Quasi-minima,* 7. *Hölder continuity,* 8. *First derivatives,* 9. *Partial regularity,* 10. *Higher derivatives.*

The prerequisites for the reading are basic properties of the Lebesgue integral and L^p spaces, and some elements of functional analysis, the more special topics being presented with full proofs in the second and the third chapters of the book. Compactness in L^p, Morrey-Campanato spaces, John-Nirenberg theorem on BMO functions, the interpolation theorems of Marcinkiewicz and Stampachia, and elements of Hausdorf measure in the second chapter, and a short introduction to Sobolev spaces (including embedding and trace theorems, Poincaré and Sobolev-Poincaré inequalities) in the third one.

The various aspects of the direct methods in the calculus of variations a treated in the rest of the book - semicontinuity (Chapters 4 and 5) and regularity (Chapters 6 to 10). A special emphasis is put on quasi-minima.

The field owes much to the Italian school of mathematics, starting with L. Tonelli, and continuing with the substantial contributions and ideas of E. De Giorgi, C. Miranda, M. Giaquinta, G. Modica, G. Anzelotti, L. Ambrosio and the author. The book reflects very well these contributions, along with those of other well known mathematicians as J. Moser, J. Nash, B. Dacorogna, S. Benstein, O. Ladyzenskaia, A. Ioffe, K. Uhlenbeck, J. Maly, H. Federer, L. Evans, R. Gariepy, et al.

It can be recommended for graduate courses or post-graduate courses in the calculus of variations, or as reference text.

J. Kolumbán

Israel Gohberg, Seymour Goldberg and Marinus A. Kaashoek, *Basic Classes of Linear Operators,* Birkhäuser Verlag, Basel-Boston-Berlin 2003, xvii+423 pp., ISBN 3-7643-6930-2.

The book provides an introduction to Hilbert and Banach spaces, with emphasis on operator theory, its aim being to stimulate the students to expand their knowledge of operator theory. It is designed for senior undergraduate and graduate students, the prerequisites being familiarity with linear algebra and Lebesgue integration (an appendix contains some results in this area with references). At the same time, the book is written in such a way that it can serve as an introduction to the two volume treatise by the same authors, *Classes of Linear Operators*, published by Birkhäuser, 1990 (Vol I), and 1993 (Vol. II).

The book is based on a previous one of the authors, *Basic Operator Theory,* Birkhäuser 1981, but the present one differs substantially from the previous one. The changes reflect the experience gained by the authors by using the old text in various courses, as well as the recent developments in operator theory. This affected the choice of the topics, proofs and exercises. They included more examples of concrete classes of linear operators as, for instance, Laurent, Toeplitz and singular integral operators, the theory of traces and determinants in an infinite dimensional setting and Fredholm theory. The theory of unbounded operators is expanded.

The material is presented in a way to make a natural transition from linear algebra and analysis to operator theory, keeping it at an elementary level. The main part of the book (Chapters I to X) deals with the Hilbert case. It starts with a chapter on the geometry of Hilbert spaces, and continues with the study of operators acting on them. This study comprises bounded linear operators, Laurent and Toeplitz operators on Hilbert space, unbounded operators, and spectral theory (including the operational calculus). As applications one considers the oscillations of an elastic string and iterative methods for solving linear equations in Hilbert space (relying on spectral theory).

The Banach space setting is treated in Chapters XI to XVI. These contain the basic principles of Banach spaces, linear operators, compact operators, Poincaré operators and their determinants and traces, Fredholm operators, Toeplitz and singular integral operators. The last chapter of the book, Chapter XVII, is concerned with some fix point theorems for non linear operators.

Each chapter ends with a set of exercises, chosen to expand reader's comprehension of the material or to add new results.

By the careful choice of the topics and by the numerous examples included, the book provides the reader with a firm foundation in operator theory, and demonstrates the power of the theory in applications. A list for further reading is presented at the end of the book.

S. Cobzaş

Nonlinear Analysis and its Applications to Differential Equations, M. R. Grossinho, M. Ramos, C. Rebelo and L. Sanchez, Editors, Progress in Nonlinear Differential Equations and Their Applications; Vol. 43, Birkhäuser, Boston-Basel-Berlin, 2001, 380 pp., ISBN 0-8176-4188-2.

This volume presents a significant part of the material given in the autumn school on "Nonlinear Analysis and Differential Equations" held at the CMAF (Centro de Matemática e Aplicações Fundamentais), University of Lisbon, in September-October 1998.

Part 1: Short courses (143 pp.), includes key articles offering a systematic approach to some classes of problems in ordinary differential equations and partial differential equations: C. De Coster and P. Habets, An overview of the method of lower and upper solutions for ODEs; E. Feireisl, On the long-time behaviour of solutions to the Navier-Stokes equations of compressible flow; J. Mawhin, Periodic solutions of systems with p-Laplacian-like operators; W.M. Oliva, Mechanics on Riemannian manifolds; R. Ortega, Twist mappings, invariant curves and periodic differential equations; and K. Schmitt, Variational inequalities, bifurcation and applications.

Part 2: Seminar papers, includes short articles representative of the recent research of participants: F. Alessio, M. Calanchi and E. Serra, Complex dynamics in a class of reversible equations; L. Almeida and Y. Ge, Symmetry and monotonicity results for solutions of certain elliptic PDEs on manifolds; J. Andres, Nielsen number and multiplicity results for multivalued boundary value problems; D. Arcoya and J.L. Gámez, Bifurcation theory and application to semilinear problems near the resonance parameter; P. Benevieri, Orientation and degree for Fredholm maps of index zero between Banach spaces; A. Cabada, E. Liz and R.L. Pouso, On the method of upper and lower solutions for first order BVPs; A. Cañada, J.L. Gámez and J.A. Montero, Nonlinear optimal control problems for diffusive elliptic equations of logistic type; A. Capietto, On the use of time-maps in nonlinear boundary value problems; P. Drábek, Some aspects of nonlinear spectral theory; C. Fabry and A. Fonda, Asymmetric nonlinear oscillators; T. Faria, Hopf bifurcation for a delayed predator-prey model and the effect of diffusion; M. Fečkan, Galerkin-averaging method in infinite-dimensional spaces for weakly nonlinear problems; D. Franco and J.J. Nieto, PBVPs for ordinary impulsive differential equations; M.R. Grossinho, F. Minhós and S. Tersian, Homoclinic and periodic solutions for some classes of second order differential equations; J. Jacobsen, Global bifurcation for Monge-Ampère operators; M. Kunze, Remarks on boundedness of semilinear oscillators; D. Lupo and K.R. Payne, The dual variational method in nonlocal semilinear Tricomi problems; F. Pacella, Symmetry properties of positive solutions of nonlinear differential equations involving the p-Laplace operator; A.M. Robles-Pérez, A maximum principle with applications to the forced Sine-Gordon equation; A.V. Sarychev and D.F.M. Torres, Lipschitzian regularity conditions for the minimizing trajectories of optimal control problems; and I. Schindler and K. Tintarev, Abstract concentration compactness and elliptic equations on unbounded domains.

We recommend this book to those mathematicians working in nonlinear analysis, ordinary differential equations, partial differential equations and related fields.

Radu Precup

Steven G. Krantz and Harold R. Parks, *The Implicit Function Theorem - History, Theory and Applications,* Birkhäuser, Boston-Basel-Berlin, 20002, ISBN 0-8176-4285-4 and 3-7643-4285-4.

The Implicit Function Theorem (IFT) and its closest relative - the Inverse Function Theorem - are two fundamental results of mathematical analysis with deep and far reaching applications to various domains of mathematics, as partial differential equations, differential geometry, geometric analysis, optimization. The aim of the present book is to present some fundamental implicit function theorems along with some nontrivial applications.

Some historical facts concerning the evolution of the ideas of function and implicit function, are presented in the second chapter of the book, *History.* It turns out that the origins of the notion of implicit function can be traced back to I. Newton (in 1669), G.W. Leibniz, who used implicit differentiation as early as 1684, and J.-L. Lagrange, who applied in 1670 the inverse function theorem for real analytic functions to some problems in celestial mechanics. The first explicit formulation of the implicit function theorem for holomorphic functions was done by A. Cauchy, and the first real variable formulation belongs to U. Dini in the academic year 1876/77 at the University of Pisa. In the third chapter, *Basic ideas,* the authors present two proofs of the IFT (the finite dimensional case) - one by induction and one via the inverse function theorem and Banach contraction principle.

Ch. 4, *Applications,* deals with existence results for differential equations (Picard's theorem), numerical homotopy methods and smoothness of the distance function to a smooth manifold.

Ch. 5, *Variations and generalizations,* is concerned with IFT for non-smooth functions and for function with degenerate Jacobian.

The highlight of the book is Ch. 6, *Advanced implicit function theorems,* presenting Hadamard's global inverse function theorem and the famous Nash-Moser implicit function theorem.

A Glossary of notions and a bibliography complete the book.

Collecting together disparate ideas in an important area of mathematical analysis and presenting them in an accessible but rigorous way, the book is of great interest to mathematicians, graduate or advanced undergraduate students, who want to learn or to apply the powerful tools supplied by implicit function theorems.

Tiberiu Trif

Sergiu Kleinerman and Francesco Nicolò, *The Evolution Problem in General Relativity,* Progress in Mathematical Physics, Vol. 25, Birkhäuser Verlag, Boston - Basel - Berlin 2003, xxii+385 pp., ISBN 3-7643-4254-4 and 0-8176-4254-4.

From the Preface: "The aim of the present book is to give a new self-contained proof of the global stability of the Minkowski space, given in D. Christodoulou and S. Kleinerman, *The global nonlinear stability of the Minkowski space,* Princeton Mathematical Series, Vol. 41. Princeton 1993 (Ch-Kl). We provide a new self-contained proof of the main part of that result, which concerns the full solution of the radiation problem in vacuum, for arbitrary asymptotically flat initial data sets. This can be also interpreted as a proof of the global stability of the external region of Scwarzschild spacetime.

The proof, which is a significant modification of the argument in Ch-Kl, is based on a *double null foliation* of spacetime instead of the *mixed null-maximal foliation* used in Ch-Kl. this approach is more naturally adapted to the radiation features of the Einstein equations and leads to important technical simplifications."

The book is fairly self-contained, the basic notions from differential geometry being reviewed in the first chapter. This chapter contains also a review of known results on Einstein equations and initial data value problems in general relativity, and the formulation of the main result.

The rest of the book is devoted to technical preparations for the proof, and to the proof of the main result. These chapters are headed as follows: 2. *Analytic methods in the study of the initial value problems,* 3. *Definitions and results,* 4. *Estimates for the connection coefficients,* 5. *Estimates for the Riemann curvature tensor,* 6. *The error estimates,* 7.*The initial hypersurface and the last slice,* and 8. *Conclusions.* This last chapter contains a rigorous derivation of the Bondi mass as well as of the connection between the Bondi mass and the ADM mass.

This important monograph, presenting the detailed proof of an important result in general relativity, is of great interest to researchers and graduate students in mathematics, mathematical physics, and physics in the area of general relativity.

Paul A. Blaga

JuliánLópez-Gómez, *Spectral theory and nonlinear functional analysis,* Research Notes in Mathematics, Vol. 426, Chapman & Hall/CRC, New York Washington 2001, xii+265 pp., ISBN 1-58488-249-2.

The general abstract problem this monograph deals with is the following one: For U and V real Banach spaces consider the operator

$$\mathfrak{F} : \mathbb{R} \times U \to V$$

of the form

$$\mathfrak{F}(\lambda, u) = \mathfrak{L}(\lambda)u + \mathfrak{N}(\lambda, u)$$

and the associated equation

$$(*) \qquad \mathfrak{F}(\lambda, u) = 0$$

where the following conditions are assumed to be satisfied:

The construction of the spectral theory is based on appropriate definitions of the notions of bifurcation point, nonlinear eigenvalue and algebraic eigenvalue. One of the principal goals of the monograph is characterizing the nonlinear eigenvalues of $\mathfrak{L}$ by means of a so called *generalized algebraic multiplicity* of $\mathfrak{L}$ at λ_0. As the author says "Our generalized algebraic multiplicity, subsequently denoted by $\chi[\mathfrak{L}(\lambda); \lambda_0]$ is a natural number that provides a finite order algorithm to calculate the change of the Leray-Schauder degree as λ crosses λ_0, thereby ascertaining and establishing the deep relationship between algebraic/analytic and topological invariants arising in nonlinear functional analysis."

The algebraic multiplicity can be defined if and only if λ_0 is an algebraic eigenvalue of $\mathfrak{L}$. The most crucial property of the algebraic multiplicity is established by Theorem 1.2.1: λ_0 is a nonlinear eigenvalue of $\mathfrak{L}$ if and only if $\chi[\mathfrak{L}(\lambda); \lambda_0]$ is odd.

If $V = U$ and $\mathfrak{L}(\lambda) = T - \lambda I_U$, where T is a continuous linear operator acting in U, and I_U stands for the identity operator of U, if λ_0 is an isolated eigenvalue of T and $T - \lambda_0 I_U$ is Fredholm of index zero, then the order ν of λ_0 is an algebraic eigenvalue of $\mathfrak{L}$. In this case $\chi[\mathfrak{L}(\lambda); \lambda_0] = \dim N[(T - \lambda_0 I_U)^\nu]$. Hence $\chi[\mathfrak{L}(\lambda); \lambda_0]$ equals the classical algebraic multiplicity of λ_0 as an eigenvalue of T.

If $U = V = \mathcal{R}^d, d \geq 1,$ and λ_0 is an algebraic eigenvalue of $\mathfrak{L}$, then $\chi[\mathfrak{L}(\lambda); \lambda_0]$ is odd if and only if the determinant of $\mathfrak{L}(\lambda)$ in any basis changes sign as λ crosses λ_0.

We have selected some basic properties of the algebraic multiplicity as they are listed in the Introduction of the monograph. This Introduction is in fact a well written detailed abstract of the book, with historical comments and, whenever possible, finite dimensional examples and counterexamples involved. As a general aspect of the approach, we remark the intention of the author to present the results in the most simple, but significant context. Hence, more sophisticated topological notions, variational aspects and monotonicity techniques appear only in the last two sections 6 and 7.

The book summarizes the authors new results in the nonlinear bifurcation theory some of which were subjects of the various lectures presented by him in the last decade. They extend and complete classical contributions in the field due to Ize, Fitzpatrick and Pejsachowicz, Rabinowitz, Rabier, Magnus, Ramm and others.

The book is addressed to researchers in nonlinear functional analysis and operator equations. It can be used also for advanced graduate or postgraduate courses.

A. B. Németh

Piotr Mikusiński and Michael D. Taylor, *An Introduction to Multivariable Analysis - From Vector to Manifold*, Birkhäuser Verlag, Basel-Boston-Berlin 2002, x+295 pp., ISBN 0-8176-4234-X and 3-7643-4234-X.

The aim of the present book is to provide a quick and smooth introduction to multivariable calculus, including differential calculus and Lebesgue integration in $\mathbb{R}^N$, and culminating with calculus on manifolds. The more geometric and intuitive approach, based on K-vectors and wedge product, allows the authors to overcome some of the difficulties encountered in the study of differential forms and, at a same time, to give full and rigorous coverage of the fundamental theorems, including Stokes generalized theorem.

The first two chapters of the book, Ch. 1, *Vectors and volumes*, and Ch. 2, *Metric spaces,* contain the algebraic and topological background needed for the development of multivariable analysis. As more specialized topics included in the first chapter we mention the Binet-Cauchy formula for determinants with applications to K-dimensional volumes of parallelipipeds in $\mathbb{R}^N$.

Differential calculus for mappings from open subsets of $\mathbb{R}^N$ to $\mathbb{R}^M$, including Taylor's formula and inverse and implicit function theorems, is developed in Ch. 3, *Differentiation.* As application one proves the Lagrange multiplier rule.

Lebesgue integration is developed in Ch. 4, *The Lebesgue integral,* following the approach proposed in the book by Jan and Piotr Mikusiński *Introduction to Analysis - from Number to Integral*, J. Wiley 1993, in the case of functions of one variable. The building starts from "bricks", which are intervals $[a, b) \subset \mathbb{R}^N$, and the integrals of step functions (= linear combinations of characteristic functions of bricks), and defining then the integrable functions f and their integrals by the conditions

$\int f = \sum_{n=1}^{\infty} \int f_n$, where (f_k) is a sequence of step functions with $\sum_{n=1}^{\infty} \int |f_n| < \infty$ and $f(x) = \sum_{n=1}^{\infty} f_n(x)$ whenever $\sum_{n=1}^{\infty} |f_n(x)| < \infty$. Measurable sets are defined later as sets whose characteristic functions are integrable.

Ch. 5, *Integral on manifolds*, starts with the proof of the change of variables formula for the integrals, the most complex proof in the book, and continues with the study of C^r manifolds embedded in $\mathbb{R}^N$ and the integrals of real-valued functions defined on manifolds.

Ch. 6, *$K-$vectors and wedge products*, develops the fundamental properties of $K-$ vectors in $\mathbb{R}^N$, the wedge and dot products, and the Hodge star operator. These are essential tools for the next and the last chapter of the book, Ch. 7, *Vector analysis on manifolds,* the highlight of the book. This chapter is dealing with integration of differential forms on oriented manifolds and the proofs of fundamental theorems of the calculus on manifolds: Stokes theorem and Poincaré lemma. The particular case of Green formula is emphasized.

The authors strongly motivate the abstract notions by a lot of intuitive examples and pictures. The exercises at the end at each section range from computational to theoretical.

The book is highly recommended for basic undergraduate or graduate courses in multivariable analysis for students in mathematics, physics, engineering or economics.

Ştefan Cobzaş

Laurenţiu Modan, *Calcul Diferenţial Real*, Editura CISON, Bucureşti, 2002.

This book is firstly destined to the students of Computer Science Faculties, but it is also recommended to the students of all other universities, where the Real Analysis is teaching.

Having as subject *Real Differential Calculus* the book looks for giving correct reasonings to the students, so that to permit them a high mathematical education becoming good specialists, endowed by the logic which insure them finding the best decision in the domain they will work.

The author chosed a walk between theoretical and practical knowledges supported by many excellent examples and exercises.

The book develops its all fundamental notions in 4 chapters: *Elements of topology*, *Elements of numerical and function sequences in* $\mathbb{R}$ *and* $\mathbb{R}^n$, *Numerical and function series, including Taylor and MacLaurin power series*, and finally *Functions of several variables*.

Of the end of the book in *Appendix* the author presents four sets of special problems of the great didactical interest.

This book is warmly recommended to the users, not only for its content and presentation, but also for its mathematical beauty.

Gh. Micula

Jürgen Moser, *Selected Chapters in the Calculus of Variations*, (Lectures notes by Oliver Knill), Lectures in Mathematics, ETH Zürich, Birkhäuser Verlag, Basel-Boston-Berlin 2003, xvi+132, ISBN 3-7643-2185-7.

This book is based on the lectures presented by J. Moser in the spring of 1998 at the Eidgenössische Technische Hochschule (ETH) Zürich. The course was attended by students in the 6th and 8th semesters, by some graduate students and visitors from ETH. The German version of the notes was typed in the summer of of 19988 and J. Moser carefully corrected it the same year in September. A translation was done in 2002 and figures were included, but the original text remained essentially unchanged.

The lectures are concerned with a new development in the calculus of variations - the so called Aubry-Mather Theory. It has its origins in the research of the theoretical physicist S. Aubry on the motion of electrons in two dimensional crystal, and in that of J. Mather on monotone twist maps, appearing as Poincaré maps in mechanics. They were studied by G. Birkhoff in 1920s, but it was J. Mather in 1982 who succeeded to make substantial progress proving the existence of a class of closed invariant subsets, called now Mather sets. The unifying topic of both Aubry and Mather approaches is that of some variational principles, a point that the book makes very clear.

The material is grouped in three chapters: 1. *One-dimensional variational problems;* 2. *Extremal fields and global minimals;* 3. *Discrete systems, Applications.*

The first chapter collects the basic results from the classical theory, the notion of extremal fields being a central one. In the second chapter the variational problems on the 2-dimensional torus are investigated, leading to the notion of Mather set. In the last chapter the connection with monotone twist maps is made, as a starting point of Mather's theory, and the discrete variational problems lying at the basis of Aubry's theory are presented.

The aim of the book is not to present the things in their greatest generality, but rather to emphasize the relations of the newer developments with classical notions.

The progress made in the area since 1998 is shortly presented in an Appendix along with some additional literature.

The book is ideal for advanced courses in the calculus of variations and its applications.

S. Cobzaş

Vladimir Müller, *Spectral Theory of Linear Operators (and spectral Systems in Banach Algebras),* Operator Theory: Advances and Applications, Vol. 139, Birkhäuser Verlag, Basel-Boston-Berlin 2003, x+381, ISBN 3-7643-6912-4.

The book is devoted to the basic results in spectral theory in Banach algebras for both single elements and for n-tuples of commuting elements, with emphasis on the spectral theory of operators on Banach and Hilbert spaces. The unifying idea, allowing to present in an axiomatic and elementary way various types of spectra - the approximate point spectrum, Taylor spectrum, local spectrum, essential spectrum, etc - is that of regularity in a Banach algebra. A regularity is a subset R of a unital Banach algebra $\mathcal{A}$ having some nice properties as : $ab \in R \iff a, b \in R$; $e \in R$; $\mathrm{Inv}(\mathcal{A}) \subset R$. The spectrum of an element $a \in \mathcal{A}$ with respect to a regularity R is defined by $\sigma_R(a) = \{\lambda \in \mathbb{C} : a - \lambda e \notin R\}$. For $R = \mathrm{Inv}(\mathcal{A})$ one obtains the usual spectrum $\sigma(a)$ of the element a. This notion, introduced and studied by the author and V. Kordula (Studia Math. **113** (1995), 127-139), is sufficiently general to cover many interesting cases of spectra but, at the same time, sufficiently strong to have non-trivial consequences as, e.g., the spectral mapping theorem.

The first chapter of the book, Ch. I, *Banach algebras*, presents the basic results on spectral theory in Banach algebras, including the axiomatic theory of spectrum via regularities. A special attention is paid to approximate point spectrum and its connection with removable and non-removable ideals. In the second chapter, Ch. II, *Operators,* these notions and results are specified to the very important case of operators on Banach and Hilbert spaces. Chapter III, *Essential spectrum,* is concerned with spectra in the Calkin algebra $\mathcal{B}(X)/\mathcal{K}(X)$, and with Fredholm and Browder operators. Although having a rather involved definition, the Taylor spectrum for commuting finite systems of operators seems to be the natural extension of ordinary spectrum for single operators, due mainly to the existence of functional calculus for functions analytic in a neighborhood of it. The presentation of Taylor functional calculus is done in the fourth chapter in an elementary way, without the use of sheaf theory and cohomological methods, following the ideas from a paper by the author of the book (Studia Math. **150** (2002), 79-97).

The last chapter of the book, Ch. IV, *Orbits and capacity*, is concerned with the study of orbits, meaning sequences $\{T^n x : n = 0, 1, ...\}$ in Banach or Hilbert spaces, a notion closely related to those of local spectral radius and capacity of an operator. Some Baire category results of the author on the boundedness of the orbit are included.

The book is clearly written and contains a lot of material, some of it appearing for the first time in book form. At the same time, the author tried successfully to keep the presentation at an elementary level, the prerequisites being basic functional analysis, topology and complex function theory (some needed results are collected in an Appendix at the end of the book).

The book, or parts of it, can be used for graduate or postgraduate courses, or as a reference text.

S. Cobzaş

Manfred Reimer, *Multivariate Polynomial Approximation*, International Series of Numerical Mathematics, Vol. 144, Birkhäuser Verlag, Basel-Boston-Berlin, 2003, pp. 358. ISBN 3-7643-1638-1.

This monograph brings a new breath over an old field of research - the approximation of functions by using multivariate polynomials. Besides surveying both classical and recent results in this field, the book also contains a certain amount of new material. The theory is characterized both by a large variety of polynomials which can be used and by a great richness of geometric situations which occur. Among these approached families of polynomials, we recall: Gegenbauer polynomials, the polynomial systems of Appell and Kampé de Feriét, the space $\mathbb{P}^r(S^{r-1})$, $r \in \mathbb{N} \setminus \{1\}$, of polynomial restriction, onto the unit sphere S^{r-1} and its subspaces, the most important of them being rotation-invariant subspaces.

The author investigates polynomial approximation to multivariate functions which are defined by linear operators. The reader will meet Bernstein polynomials, the Weierstrass theorem, the concept of best approximation and interpolatory projections in the space of the continuous real functions defined on a compact subset of $\mathbb{R}^r$, $r \in \mathbb{N}$, as well.

Distinct sections are devoted to quadratures. For example, the following are presented: Gauss quadratures, quadrature on the sphere, the geometry of nodes and weights in a positive quadrature, quadrature on the ball.

Hyperinterpolation represents another important concept treated by Manfred Reimer. It is a generalization of interpolation which shares with it the advantage of an easy evaluation but achieves simultaneously the growth order of the minimal projections. This new positive discrete polynomial approximation method is established on the sphere and then it is carried over to the balls of lower dimension.

By using summation methods such as Cesàro method or a method based on the Newman-Shapiro kernels, positive linear approximation operators are generated. A special consideration is given to the approximation on the unit ball B^r, $r \geq 2$. More precisely, orthogonal projections, Appell series and summation methods, interpolation on the ball are studied.

Among the book's outstanding features is the inclusion of some applications and a large variety of problems. As regards the applications, the author studies a recovery problem for real functions F belonging to a given space X and which are to be reconstructed from the values λF, where λ runs in a family of linear functionals on X. This way are presented both Radon transform, k-plane transform and reconstruction by approximation. As regards the problems, these are attached to help the reader to become familiar with the multivariate theory. All exercises are solved in a separate appendix.

Multivariate Polynomial Approximation includes the author's own research results developed over the last ten years, some of which build upon the results of others and some that introduce new research opportunities. His approach and proofs are straightforward constructive making the book accessible to graduate students in pure and applied mathematics and to researchers as well.

Octavian Agratini

Luigi Ambrosio and Paolo Tilli, *Selected Topics on "Analysis in Metric Spaces",* APPUNTI, Scuola Normale Superiore, Pisa 2000, 133 pp.

The aim of these notes, based on a course taught by the first author in the academic year 1988-89 at the Scuola Normale Superiore di Pisa, is to present the main mathematical prerequisites needed to study or to do research in the field of Analysis in Metric Spaces. This relatively new and rapidly expanding area of investigation has as target to transpose to the case of metric spaces as many as possible results from classical analysis. In order to obtain consistent results, one supposes the metric space (X, d) endowed with a Borel measure μ that is finite on bounded sets and doubling, meaning that it values on balls in X satisfy the inequality $\mu(B_{2r}(x) \leq C\mu(B_r(x))$. Important contributions to the subject have been done by the authors of this book, by their coworkers from SNS di Pisa, and by M. Gromov, J. Cheeger, P. Hajlasz, P. Koskela, J. Heinonen.

The book contains six chapters: 1. *Some preliminaries in measure theory*; 2. *Hausdorff measures and covering theorems in metric spaces*; 3. *Lipschitz functions in metric spaces*; 4. *Geodesic problem and Gromov-Hausdorff convergence*; 5. *Sobolev spaces in a metric framework*; 6. *A quick overview on the theory of integration.*

Although in some places the proofs are only sketched with the specification of a source, the book covers a lot of topics. The last chapter of the book presents De Giorgi's approach to the theory of integration based on Cavalieri's formula.

The bibliography at the end of the book contains the basic references in the field.

Written in a clear and pleasant style, the book is a good introductory text to this promising area of investigation - the Analysis on Metric Spaces.

S. Cobzaş

Lectures Notes on Analysis in Metric Spaces, a cura di Luigi Ambrosio and Francesco Serra Cassano, Scuola Normale Superiore, Pisa 2000, 121 pp.

The book contains the notes of an international Summer School on Analysis in Metric Spaces, organized by L. Ambrosio, N. Garofalo, P. Serapioni, and F. Serra Cassano in May of 1999 at the Scuola Normale Superiore di Pisa.

There are included five papers, representing the edited and a little expanded versions of lectures delivered at the school: 1. Thierry Coulhon, *Random walks and geometry on infinite graphs*, pp. 5-36; 2. Guy David *Uniform rectifiability and quasi-minimal sets,* pp. 37-54; 3. Pekka Koskela, *Upper gradients and Poincaré inequalities,* pp. 55-69; 4. Stephen Semmes, *Derivatives and difference quotients for Lipschitz or Sobolev functions on various spaces,* pp. 71-103; 5. Richard L. Wheeden, *Some weighted Poincaré estimates in spaces of homogeneous type,* pp. 105-121.

The main concern of Analysis in Metric Spaces is to see to what extent results from classical analysis extend to the more general framework of metric spaces. Among these results I do mention the introduction of Sobolev spaces via the methods of upper gradients and Poincaré inequalities, treated in several papers in the volume. The first paper discusses the discrete case of analysis on graphs, with special emphasis on Cayley graphs.

Surveying new results, some of them belonging to the authors of the contributions, in this rapidly growing field of investigation situated at the border between analysis, topology and geometry, the book is of great interest for researchers working in this area, as well as for people desiring to become acquainted with its powerful methods.

S. Cobzaş

Steven G. Krantz and Harold R. Parks, *A Primer of Real Analytic Functions*, Birkhäuser Advanced Texts, Birkhäuser Verlag, Boston-Basel-Berlin 2002, xii+205 pp., ISBN 0-8175-4264-1.

Complex analytic functions of one or several complex variables are presented in a lot of books, at introductory level and at advanced as well.

Their older and poorer relatives - the real analytic functions - having totally different features, found their first book treatment in the first edition of the present book, published by Birkhäuser in 1992. Real analytic functions are an essential tool in the study of embedding problem for real analytic manifolds. They have also applications in PDEs and in other areas of analysis.

With respect to the first edition, beside the revision of the presentation, some new material on topologies on spaces of real analytic functions and on the Weierstrass preparation theorem, has been added.

The basic results on real analytic functions are presented in the first two chapters: Ch. *Elementary properties,* and Ch. 2 *Multivariable calculus of real analytic functions*, including implicit and inverse function theorems, Cauchy-Kowalewski theorem.

Chapters 3, *Classical topics* and 4 ,*Some questions in hard analysis*, contain more advanced topics as Besicovitch's theorem, Whitney's extension and approximation theorems, quasi-analytic classes and Gevrey classes, Puiseux series.

Ch. 5, *Results motivated by PDEs*, is concerned with topics as division of distributions, the FBI transform (FBI comes here from the name of mathematical physicists Fourier, Bros and Iagnolitzer), and Paley-Wiener theorem.

The last chapter, Ch. 6, *Topics in geometry,* contains a discussion of some deep and difficult results as embedding of real analytic manifolds, sub- and semi-analytic sets, the structure theorem for real analytic varieties.

Bringing together results scattered in various journals or books and presenting them in a clear and systematic manner, the book is of interest first of all for analysts, but also for applied mathematicians and for researcher in real algebraic geometry.

Stefan Cobzaş

Steven G. Krantz, *Handbook of Logic and Proof Techniques for Computer Science*, Birkhäuser Boston, Inc., Boston, MA; Springer-Verlag, New York, 2002. xx+245pp., ISBN 0-8176-4220-X.

Logic plays a key role in modern mathematics and computer science. However, the vast number of topics, the unusual and sometimes difficult formalism and terminology, made most of the modern logic inaccessible to all but the experts.

The present book is a comprehensive overview of the most important topics in modern logic emphasizing ideas essential in Computer Science as axiomatics, completeness, consistency, decidability, independence, recursive functions, model theory, P/NP completeness. Some of these topics are: first-order logic, semantics and syntax, axiomatics and formalism in mathematics, the axioms of set theory, elementary set theory, recursive functions, the number systems, methods of mathematical proof, the axiom of choice, proof theory, category theory, complexity theory, Boolean algebra, the word problem.

The book was written to be accessible for non experts. It contains definitions, plenty of concrete examples and a clear presentation of the main ideas, on the other hand avoids complicated proofs, difficult notations, difficult formalisms. Self-contained, the book is designed for those mathematicians, engineers and especially computer scientists who need a quick understanding of some key ideas from logic.

The vast bibliography also makes this book an excellent modern logic resource for the working mathematician.

Csaba Szántó

Bhimsen K. Shivamoggi, *Perturbations Methods for Differential Equations,* Birkhäuser Verlag, Basel-Boston-Berlin 2003, xiv+354, ISBN 3-7643-4189-0 and 0-8176-4189-0.

The mathematical problems associated with nonlinear equations, generally, are very complex. So that, one practical approach is to seek the solutions of these nonlinear equations as the perturbations of known solutions of a linear equation. A perturbative solution of a nonlinear problem becomes viable if it is close to the solution of another problem we already know how to solve.

After a chapter containing the asymptotic series and expansions, this book presents the regular perturbation methods for differential and partial differential equations. Other methods, such as the strained coordinates method, the averaging method, the matched asymptotic expansion method, the multiple scales method, are also very detailed presented. Very important is the fact that each chapter contains certain important applications, especially to fluid dynamics, but also to solid mechanics and plasma physics. Moreover, each chapter contains a section of specific exercises, and an appendix with basic mathematical tools.

Many methods and procedures are very well described without technical proofs. It is obvious the intention of the author to convince the reader to understand the phenomena and to learn how to apply correctly the suitable presented method.

"Perturbation Methods for Differential Equations" can serve as a textbook for undergraduate students in applied mathematics, physics and engineering. Researchers in these areas will also find the book an excellent reference. A comprehensive bibliography and an index complete the book.

Gh. Micula

Alain Escassut, *Ultrametric Banach Algebras,*, World Scientific, London - Singapore - Hong Kong 2003, xiii+275 pp., ISBN 981-238-194-5.

The book is concerned with the spectral theory of ultrametric Banach algebras over an algebraically closed complete ultrametric field. As it is well known, in the classical case due to Gelfand's representation theory every commutative complex Banach algebra can be viewed as an algebra of functions on a compact space, whose points characterize all maximal ideals, which are all of codimension 1. Any such algebra admits a holomorphic functional calculus.

In the ultrametric case the situation is much more complicated, because an ultrametric Banach algebra can have maximal ideals of infinite codimension. It turns out that a key role in constructing an ultrametric spectral theory is played by the family of multiplicative semi-norms on an ultrametric Banach algebra, allowing to define a kind of Gelfand transform. There exists also a spectral semi-norm defined by $\|a\|_{si} = \lim_n \|a^n\|^{1/n}$, which is also equal to the supremum of all multiplicative semi-norms. It is also possible to construct a holomorphic functional calculus. The basic idea is to associate methods based on affinoid algebras (called also "Tate algebras") with methods based on holomorphic functional calculus involving very thin properties of analytic functions of one variable. The present book is the first that treats together both of these subjects. Concerning holomorphic mappings in the ultrametric case, references are given to another book by the same author, *Analytic Elements in p-adic Analysis*, World Scientific, Singapore 1995.

The author is well known specialist in the field and the book is largely based on his original results.

The book will be of interest to researchers in non-archimedean analysis (or ultrametric analysis), a field having its origins in the work of the Dutch mathematicians A. F. Monna and T. A. Springer, and which still is in the focus of attention of several research centers. Recently there have been found some applications of non-archimedean analysis to mathematical physics, see V. S. Vladimirov, I. V. Volovich and E. J. Zelenov, *p-Adic Analysis and Mathematical Physics,* World SCientific, Singapore 1994.

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Mathematics and War, Editors: Bernhelm Booß-Bavnek and Jens Høyrup, Birkhäuser Verlag, Basel-Boston-Berlin 2003, viii+416 pp., ISBN 3-7643-1634-9.

The volume contains some of the contributions delivered at the International Meeting on Mathematics and War, held in Karlskrona, Sweden, from 29 to 31 august, 2002, together with some invited papers. The idea was to bring together mathematicians, historians, philosophers and military, to discuss some of the interconnections between warfare and mathematics. As it is well known after the World War II, there has been a strong mathematization of warfare and of the concepts of modern war, which in its turn deeply influenced the development of some areas of mathematics.

The papers included in the volume deal with topics ranging from historical, philosophical, ethical aspects of the problem, to more technical aspects like the functioning of weapons, the actual planning of war and information warfare. The perspectives the authors approaches the treated theme also differ from one to other - some papers are written from a pacifist point of view (more or less explicitly), while others are not. Some of the papers are dealing with history, but focused on the last sixty years, e.g., WW II and the Kosovo war.

The volume is organized in four parts: I. *Perspectives from mathematics,* II. *Perspectives from the military,* III. *Ethical issues,* IV. *Enlightenment perspectives.*

The first part contains studies on military work in mathematics 1914-1945 (R. Siegmund-Schultz), on the Enigma code breaking (E. Rakus-Anderson), on the defence work of A. N. Kolmogorov (A. N. Shiryaev), on the discovery of maximum pricilple by Lev Pontryagin (R. V. Gamkrelidze), and on the mathematics and war in Japan (S. Fukutomi).

The second part is written by military and deals with topics as information warfare (U. Bernhard and I. Ruhmann), the exposure of civilians under the modern "safe" warfare (E. Schmägling), duels of systems and forces (H. Löfstedt).

The third part is concerned with N. Bohr's and A. Turing's involving in military research (I. Aaserud and A. Hodges, respectively), and K. Ogura and the "Great Asia War" (T. Makino).

The last part contains two studies - one on mathematical thinking and international law (I. M. Jarvard), and one on modeling the conflict and cooperation (J. Scheffran).

The aim of the volume is to draw the attention of scientists, military and philosophers on the dramatic consequences that the use of science, particularly of mathematics, for military purposes can have on the development of humanity, and to trace some possible way of preventing this disaster.

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