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A REMARKABLE STRUCTURE AND CONNECTIONS ON THE TANGENT BUNDLE

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Abstract. The present paper deals with the conformal almost symplectic structure on TM. Starting from the notion of conformal almost symplectic structure in the tangent bundle, we define the notion of general conformal almost symplectic d-linear connection and respective conformal almost symplectic d-linear connection with respect to a conformal almost symplectic structure $\hat{A}$, corresponding to the 1-forms ω and $\tilde{\omega}$ in TM. We determine the set of all general conformal almost symplectic d-linear connections on TM, in the case when the nonlinear connection is arbitrary and we find important particular cases.

1. Introduction

The geometry of the tangent bundle (TM, π, M) has been studied by R.Miron and M.Anastasiei in [6], by R.Miron and M.Hashiguchi in [7], by V.Oproiu in [8], by Gh.Atanasiu and I.Ghinea in [1], by R.Bowman in [2], by K.Yano and S.Ishihara in [10],etc.

Concerning the terminology and notations, we use those from [4].

Let M be a real C^∞-differentiable manifold with dimension n, (n=2n') and (TM, π, M) its tangent bundle.

If (x^i) is a local coordinates system on a domain U of a chart on M, the induced system of coordonates on $\pi^{-1}(U)$ is $(x^i, y^i), (i = 1, ..., n)$.

Let N be a nonlinear connection on TM, with the coefficients $N^j{}_i(x, y), (i, j = 1, ..., n)$.

We consider on TM an almost symplectic structure A defined by:

$$A(x, y) = \frac{1}{2}a_{ij}(x, y)dx^i \wedge dx^j + \frac{1}{2}\tilde{a}_{ij}(x, y)\delta y^i \wedge \delta y^j, \tag{1}$$

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where $(dx^i, \delta y^i), (i = 1, ..., n)$ is the dual basis of $\left(\frac{\delta}{\delta x^i}, \frac{\partial}{\partial y^i}\right)$, and $(a_{ij}(x, y), \tilde{a}_{ij}(x, y))$ is a pair of given d-tensor fields on TM, of the type (0,2), each of them alternate and nondegenerate.

We asociate to the lift A the Obata's operators:

$$\begin{cases} \Phi^{ir}_{sj} = \frac{1}{2}(\delta^i_s \delta^r_j - a_{sj} a^{ir}), \ \Phi^{*ir}_{sj} = \frac{1}{2}(\delta^i_s \delta^r_j + a_{sj} a^{ir}), \\ \tilde{\Phi}^{ir}_{sj} = \frac{1}{2}(\delta^i_s \delta^r_j - \tilde{a}_{sj} \tilde{a}^{ir}), \ \tilde{\Phi}^{*ir}_{sj} = \frac{1}{2}(\delta^i_s \delta^r_j + \tilde{a}_{sj} \tilde{a}^{ir}). \end{cases} \tag{2}$$

Obata's operators have the same properties as the ones associated with a Finsler space [7].

Let $\mathcal{A}_2(TM)$ be the set of all alternate d-tensor fields, of the type (0,2) on TM. As is easily shown, the relations on $\mathcal{A}_2(TM)$ defined by (3):

$$\begin{cases} (a_{ij} \sim b_{ij}) \Leftrightarrow \left((\exists)\, \lambda(x, y) \in \mathcal{F}(TM), a_{ij}(x, y) = e^{2\lambda(x,y)} b_{ij}(x, y)\right), \\ \left(\tilde{a}_{ij} \sim \tilde{b}_{ij}\right) \Leftrightarrow \left((\exists)\, \mu(x, y) \in \mathcal{F}(TM), \tilde{a}_{ij}(x, y) = e^{2\mu(x,y)} \tilde{b}_{ij}(x, y)\right), \end{cases} \tag{3}$$

is an equivalence relation on $\mathcal{A}_2(TM)$.

Definition 1.1. *The equivalent class: $\hat{A}$ of $\mathcal{A}_2(TM)/_\sim$ to which the almost symplectic tensor field A belongs, is called conformal almost symplectic structure on TM.*

Thus:

$$\hat{A} = \{A' | A'_{ij}(x, y) = e^{2\lambda(x,y)} a_{ij}(x, y) \text{ and } \tilde{A}'_{ij}(x, y) = e^{2\mu(x,y)} \tilde{a}_{ij}(x, y)\}. \tag{4}$$

2. General conformal almost symplectic d-linear connections on TM.

Definition 2.1. *A d-linear connection, D, on TM, with local coefficients $D\Gamma(N) = (L^i{}_{jk}, \tilde{L}^i{}_{jk}, \tilde{C}^i{}_{jk}, C^i{}_{jk})$, is called general conformal almost symplectic d-linear connection on TM if:*

$$a_{ij|k} = K_{ijk}, \ a_{ij}|_k = Q_{ijk}, \ \tilde{a}_{ij|k} = \tilde{K}_{ijk}, \ \tilde{a}_{ij}|_k = \tilde{Q}_{ijk}, \tag{5}$$

where $K_{ijk}, Q_{ijk}, \tilde{K}_{ijk}, \tilde{Q}_{ijk}$ are arbitrary tensor fields, of the type $(0, 3)$ on TM, with the properties:

$$K_{ijk} = -K_{jik}, \ Q_{ijk} = -Q_{jik}, \ \tilde{K}_{ijk} = -\tilde{K}_{jik}, \ \tilde{Q}_{ijk} = -\tilde{Q}_{jik} \tag{6}$$

and $|, |$ denote the h-and respective v-covariant derivatives with respect to D.

Particularly, we have:

Definition 2.2. *A d-linear connection, D, on TM, with local coefficients $D\Gamma(N) = (L^i{}_{jk}, \tilde{L}^i{}_{jk}, \tilde{C}^i{}_{jk}, C^i{}_{jk})$, for which there exists the 1-forms ω and $\tilde{\omega}$ in TM, $\omega = \omega_i dx^i + \dot{\omega}_i \delta y^i$, $\tilde{\omega} = \tilde{\omega}_i dx^i + \dot{\tilde{\omega}}_i \delta y^i$ such that:*

$$\begin{cases} a_{ij|k} = 2\omega_k a_{ij}, \quad a_{ij}|_k = 2\dot{\omega}_k a_{ij}, \\ \\ \tilde{a}_{ij|k} = 2\tilde{\omega}_k \tilde{a}_{ij}, \quad \tilde{a}_{ij}|_k = 2\, \dot{\tilde{\omega}}_k\, \tilde{a}_{ij}, \end{cases} \tag{7}$$

where $|$ *and* $\big|$ *denote the h-and v-covariant derivatives with respect to* D,*is called conformal almost symplectic d-linear connection on* TM, *with respect to the conformal almost symplectic structure* $\hat{A}$, *corresponding to the 1-forms* $\omega, \tilde{\omega}$ *and is denoted by:* $D\Gamma(N, \omega, \tilde{\omega})$.

We shall determine the set of all general conformal almost symplectic d-linear connections, with respect to $\hat{A}$.

Let $\overset{0}{D}\Gamma(\overset{0}{N}) = (\overset{0}{L^i}_{jk}, \overset{0}{\tilde{L}^i}_{jk}, \overset{0}{\tilde{C}^i}_{jk}, \overset{0}{C^i}_{jk})$ be the local coefficients of a fixed d-linear connection $\overset{0}{D}$ on TM. Then any d-linear connection, D, on TM, with local coefficients: $D\Gamma(N) = (L^i{}_{jk}, \tilde{L}^i{}_{jk}, \tilde{C}^i{}_{jk}, C^i{}_{jk})$,can be expresed in the form:

$$\begin{cases} N^i{}_j = \overset{0}{N^i}_j - A^i{}_j, \\ L^i{}_{jk} = \overset{0}{L^i}_{jk} + A^l{}_k \overset{0}{\tilde{C}^i}_{jl} - B^i{}_{jk}, \\ \tilde{L}^i{}_{jk} = \overset{0}{\tilde{L}^i}_{jk} + A^l{}_k \overset{0}{C^i}_{jl} - \tilde{B}^i{}_{jk}, \\ \tilde{C}^i{}_{jk} = \overset{0}{\tilde{C}^i}_{jk} - \tilde{D}^i{}_{jk}, \\ C^i{}_{jk} = \overset{0}{C^i}_{jk} - D^i{}_{jk}, \\ A^l{}_{j\overset{0}{|}k} = 0, \end{cases} \tag{8}$$

where $(A^i{}_j, B^i{}_{jk}, \tilde{B}^i{}_{jk}, \tilde{D}^i{}_{jk}, D^i{}_{jk})$ are components of the difference tensor fields of $D\Gamma(N)$ from $\overset{0}{D}\ \Gamma(\overset{0}{N})$, [4] and $\overset{0}{|}$, $\overset{0}{\big|}$ denotes the h-and v-covariant derivatives with respect to $\overset{0}{D}$.

Theorem 2.1. *Let* $\overset{0}{D}$ *be a given d-linear connection on* TM, *with local coefficients* $\overset{0}{D}\Gamma(\overset{0}{N}) = (\overset{0}{L^i}_{jk}, \overset{0}{\tilde{L}^i}_{jk}, \overset{0}{\tilde{C}^i}_{jk}, \overset{0}{C^i}_{jk})$. *The set of all general conformal almost symplectic d-linear connections on* TM, *with local coefficients* $D\Gamma(N) = (L^i{}_{jk}, \tilde{L}^i{}_{jk}, \tilde{C}^i{}_{jk}, C^i{}_{jk})$ *is given by:*

$$\begin{cases} N^i_j = \overset{0}{N^i_j} - X^i_j, \\ L^i{}_{jk} = \overset{0}{L^i}_{jk} + \overset{0}{\tilde{C}^i}_{jm} X^m{}_k + \frac{1}{2}a^{is}(a_{sj\overset{0}{|}k} + a_{sj} \overset{0}{|}_m X^m{}_k - K_{sjk}) + \Phi^{ir}_{hj} X^h{}_{rk}, \\ \tilde{L}^i{}_{jk} = \overset{0}{\tilde{L}^i}_{jk} + \overset{0}{C^i}_{jm} X^m{}_k + \frac{1}{2}\tilde{a}^{is}(\tilde{a}_{sj\overset{0}{|}k} + \tilde{a}_{sj} \overset{0}{|}_m X^m{}_k - \tilde{K}_{sjk}) + \tilde{\Phi}^{ir}_{hj} \tilde{X}^h{}_{rk}, \\ \tilde{C}^i{}_{jk} = \overset{0}{\tilde{C}^i}_{jk} + \frac{1}{2}a^{is}(a_{sj} \overset{0}{|}_k - Q_{sjk}) + \Phi^{ir}_{hj} \tilde{Y}^h{}_{rk}, \\ C^i{}_{jk} = \overset{0}{C^i}_{jk} + \frac{1}{2}\tilde{a}^{is}(\tilde{a}_{sj} \overset{0}{|}_k - \tilde{Q}_{sjk}) + \tilde{\Phi}^{ir}_{hj} Y^h{}_{rk}, \ X^i{}_{\overset{0}{j|k}} = 0, \end{cases} \tag{9}$$

where $X^i{}_j,\ X^i{}_{jk},\ \tilde{X}^i{}_{jk},\ \tilde{Y}^i_{jk},\ Y^i_{jk}$ are arbitrary tensor fields on TM, $\overset{0}{|},\ \overset{0}{|}$ denote the h-and respective v-covariant derivatives with respect to $\overset{0}{D}$ and $K_{ijk}, Q_{ijk}, \tilde{K}_{ijk}, \tilde{Q}_{ijk}$ are arbitrary tensor fields of the type (0,3) on TM with the properties (6).

Particular cases:

1. If $X^i{}_j = X^i{}_{jk} = \tilde{X}^i{}_{jk} = \tilde{Y}^i{}_{jk} = Y^i{}_{jk} = 0$ in Theorem 2.1. we have:

Theorem 2.2. *Let $\overset{0}{D}$ be a given d-linear connection on TM, with local coefficients $\overset{0}{D}\Gamma(\overset{0}{N}) = (\overset{0}{L^i}_{jk}, \overset{0}{\tilde{L}^i}_{jk}, \overset{0}{\tilde{C}^i}_{jk}, \overset{0}{C^i}_{jk})$. Then the following d-linear connection D, with local coefficients $D\Gamma(\overset{0}{N}) = (L^i{}_{jk}, \tilde{L}^i{}_{jk}, \tilde{C}^i{}_{jk}, C^i{}_{jk})$ given by (10) is a general conformal almost symplectic d-linear connection with respect to $\hat{A}$:*

$$\begin{cases} L^i{}_{jk} = \overset{0}{L^i}_{jk} + \frac{1}{2}a^{is}(a_{sj\overset{0}{|}k} - K_{sjk}), \\ \tilde{L}^i{}_{jk} = \overset{0}{\tilde{L}^i}_{jk} + \frac{1}{2}\tilde{a}^{is}(\tilde{a}_{sj\overset{0}{|}k} - \tilde{K}_{sjk}), \\ \tilde{C}^i{}_{jk} = \overset{0}{\tilde{C}^i}_{jk} + \frac{1}{2}a^{is}(a_{sj} \overset{0}{|}_k - Q_{sjk}), \\ C^i{}_{jk} = \overset{0}{C^i}_{jk} + \frac{1}{2}\tilde{a}^{is}(\tilde{a}_{sj} \overset{0}{|}_k - \tilde{Q}_{sjk}), \end{cases} \tag{10}$$

where $\overset{0}{|},\ \overset{0}{|}$ denote the h-and respective v-covariant derivatives with respect to the given d-linear connection $\overset{0}{D}$ and $K_{ijk}, Q_{ijk}, \tilde{K}_{ijk}, \tilde{Q}_{ijk}$ are arbitrary tensor fields of the type (0,3) on TM with the properties (6).

2. If $K_{ijk} = \tilde{K}_{ijk} = \tilde{Q}_{ijk} = Q_{ijk} = 0$ in Theorem 2.1 we have :

Theorem 2.3. *Let $\overset{0}{D}$ be a given d-linear connection on TM, with local coefficients $\overset{0}{D}\Gamma(\overset{0}{N}) = (\overset{0}{L^i}_{jk}, \overset{0}{\tilde{L}^i}_{jk}, \overset{0}{\tilde{C}^i}_{jk}, \overset{0}{C^i}_{jk})$. The set of all almost symplectic d-linear connections on TM, with local coefficients $D\Gamma(N) = (L^i{}_{jk}, \tilde{L}^i{}_{jk}, \tilde{C}^i{}_{jk}, C^i{}_{jk})$ is given by:*

$$\begin{cases} N^i_{\ j} = \overset{0}{N^i_{\ j}} - X^i_{\ j}, \\ L^i_{\ jk} = \overset{0}{L^i_{\ jk}} + \overset{0}{\tilde{C}^i_{\ jm}} X^m_{\ k} + \frac{1}{2} a^{is} (a_{\underset{sj|k}{0}} + a_{sj} \overset{0}{|}_m X^m_{\ k}) + \Phi^{ir}_{hj} X^h_{\ rk}, \\ \tilde{L}^i_{\ jk} = \overset{0}{\tilde{L}^i_{\ jk}} + \overset{0}{C^i_{\ jm}} X^m_{\ k} + \frac{1}{2} \tilde{a}^{is} (\tilde{a}_{\underset{sj|k}{0}} + \tilde{a}_{sj} \overset{0}{|}_m X^m_{\ k}) + \tilde{\Phi}^{ir}_{hj} \tilde{X}^h_{\ rk}, \\ \tilde{C}^i_{\ jk} = \overset{0}{\tilde{C}^i_{\ jk}} + \frac{1}{2} a^{is} a_{sj} \overset{0}{|}_k + \Phi^{ir}_{hj} \tilde{Y}^h_{\ rk}, \\ C^i_{\ jk} = \overset{0}{C^i_{\ jk}} + \frac{1}{2} \tilde{a}^{is} \tilde{a}_{sj} \overset{0}{|}_k + \tilde{\Phi}^{ir}_{hj} Y^h_{\ rk}, \ X^i_{\underset{j|k}{0}} = 0, \end{cases} \tag{11}$$

where $X^i_{\ j},\ X^i_{\ jk},\ \tilde{X}^i_{\ jk},\ \tilde{Y}^i_{\ jk},\ Y^i_{\ jk}$ are arbitrary tensor fields on TM and $\overset{0}{|}$, $\overset{0}{\Big|}$ denote the h-and respective v-covariant derivatives with respect to $\overset{0}{D}$.

3. If $K_{ijk} = 2a_{ij}\omega_k, \tilde{K}_{ijk} = 2\tilde{a}_{ij}\tilde{\omega}_k, \tilde{Q}_{ijk} = 2\tilde{a}_{ij}\dot{\tilde{\omega}}_k, Q_{ijk} = 2a_{ij}\dot{\omega}_k$, such that $\omega = \omega_i dx^i + \dot{\omega}_i \delta y^i$ and respective $\tilde{\omega} = \tilde{\omega}_i dx_i + \dot{\tilde{\omega}}_i \delta y^i$ are two 1-forms in TM, then from (9) we have the set of all conformal almost symplectic d-linear connections on TM:

Theorem 2.4. *Let $\overset{0}{D}$ be a given d-linear connection on TM, with local coefficients $\overset{0}{D}\Gamma(\overset{0}{N}) = (\overset{0}{L^i_{\ jk}}, \overset{0}{\tilde{L}^i_{\ jk}}, \overset{0}{\tilde{C}^i_{\ jk}}, \overset{0}{C^i_{\ jk}})$. Then set of all conformal almost symplectic d-linear connections on TM, with respect to $\hat{A}$, corresponding to the 1-forms ω and $\tilde{\omega}$, with local coefficients $D\Gamma(N, \omega, \tilde{\omega}) = (L^i_{\ jk}, \tilde{L}^i_{\ jk}, \tilde{C}^i_{\ jk}, C^i_{\ jk})$ is given by:*

$$\begin{cases} N^i_{\ j} = \overset{0}{N^i_{\ j}} - X^i_{\ j}, \\ L^i_{\ jk} = \overset{0}{L^i_{\ jk}} + \overset{0}{\tilde{C}^i_{\ jm}} X^m_{\ k} + \frac{1}{2} a^{is} (a_{\underset{sj|k}{0}} + a_{sj} \overset{0}{|}_m X^m_{\ k}) - \delta^i_{\ j}\omega_k + \Phi^{ir}_{hj} X^h_{\ rk}, \\ \tilde{L}^i_{\ jk} = \overset{0}{\tilde{L}^i_{\ jk}} + \overset{0}{C^i_{\ jm}} X^m_{\ k} + \frac{1}{2} \tilde{a}^{is} (\tilde{a}_{\underset{sj|k}{0}} + \tilde{a}_{sj} \overset{0}{|}_m X^m_{\ k}) - \delta^i_{\ j}\tilde{\omega}_k + \tilde{\Phi}^{ir}_{hj} \tilde{X}^h_{\ rk}, \\ \tilde{C}^i_{\ jk} = \overset{0}{\tilde{C}^i_{\ jk}} + \frac{1}{2} a^{is} a_{sj} \overset{0}{|}_k - \delta^i_{\ j}\dot{\omega}_k + \Phi^{ir}_{hj} \tilde{Y}^h_{\ rk}, \\ C^i_{\ jk} = \overset{0}{C^i_{\ jk}} + \frac{1}{2} \tilde{a}^{is} \tilde{a}_{sj} \overset{0}{|}_k - \delta^i_{\ j}\dot{\tilde{\omega}}_k + \tilde{\Phi}^{ir}_{hj} Y^h_{\ rk}, \\ \\ X^i_{\underset{j|k}{0}} = 0, \end{cases} \tag{12}$$

where $X^i_{\ j},\ X^i_{\ jk},\ \tilde{X}^i_{\ jk},\ \tilde{Y}^i_{\ jk},\ Y^i_{\ jk}$ are arbitrary tensor fields on TM, $\omega = \omega_i dx^i + \dot{\omega}_i \delta y^i$ and respective $\tilde{\omega} = \tilde{\omega}_i dx_i + \dot{\tilde{\omega}}_i \delta y^i$ are arbitrary 1-forms in TM and $\overset{0}{|}$, $\overset{0}{\Big|}$ denote the h-and respective v-covariant derivatives with respect to $\overset{0}{D}$.

4. If $X^i_j = X^i_{jk} = \tilde{X}^i_{jk} = \tilde{Y}^i_{jk} = Y^i_{jk} = 0$ in Theorem 2.4. we have:

Theorem 2.5. *Let $\overset{0}{D}$ be a given d-linear connection on TM, with local coefficients $\overset{0}{D}\Gamma(\overset{0}{N}) = (\overset{0}{L^i}_{jk}, \overset{0}{\tilde{L}^i}_{jk}, \overset{0}{\tilde{C}^i}_{jk}, \overset{0}{C^i}_{jk})$. Then the following d-linear connection D, with local coefficients $D\Gamma(\overset{0}{N}, \omega, \tilde{\omega}) = (L^i_{jk}, \tilde{L}^i_{jk}, \tilde{C}^i_{jk}, C^i_{jk})$ given by (13) is a conformal almost symplectic d-linear connection with respect to $\hat{A}$, corresponding to the 1-forms ω and $\tilde{\omega}$:*

$$\begin{cases} L^i_{jk} = \overset{0}{L^i}_{jk} + \frac{1}{2} a^{is} a_{sj \overset{0}{|} k} - \delta^i_j \omega_k, \\ \tilde{L}^i_{jk} = \overset{0}{\tilde{L}^i}_{jk} + \frac{1}{2} \tilde{a}^{is} \tilde{a}_{sj \overset{0}{|} k} - \delta^i_j \tilde{\omega}_k, \\ \tilde{C}^i_{jk} = \overset{0}{\tilde{C}^i}_{jk} + \frac{1}{2} a^{is} a_{sj} \overset{0}{|}_k - \delta^i_j \dot{\omega}_k, \\ C^i_{jk} = \overset{0}{C^i}_{jk} + \frac{1}{2} \tilde{a}^{is} \tilde{a}_{sj} \overset{0}{|}_k - \delta^i_j \dot{\tilde{\omega}}_k, \end{cases} \tag{13}$$

where $\overset{0}{|}$, $\overset{0}{\Big|}$ denote the h-and respective v-covariant derivatives with respect to the given d-linear connection $\overset{0}{D}$ and $\omega = \omega_i dx^i + \dot{\omega}_i \delta y^i$ and respective $\tilde{\omega} = \tilde{\omega}_i dx_i + \dot{\tilde{\omega}}_i \delta y^i$ are two given 1-forms in TM.

5. If we take an almost symplectic d-linear connection as $\overset{0}{D}$ in Theorem 2.5, then (13) becomes:

$$\begin{cases} L^i_{jk} = \overset{0}{L^i}_{jk} - \delta^i_j \omega_k, \\ \tilde{L}^i_{jk} = \overset{0}{\tilde{L}^i}_{jk} - \delta^i_j \tilde{\omega}_k, \\ \tilde{C}^i_{jk} = \overset{0}{\tilde{C}^i}_{jk} - \delta^i_j \dot{\omega}_k, \\ C^i_{jk} = \overset{0}{C^i}_{jk} - \delta^i_j \dot{\tilde{\omega}}_k. \end{cases} \tag{14}$$

6. If we take a conformal almost symplectic d-linear connection with respect to $\hat{A}$ as $\overset{0}{D}$ in Theorem 2.4, we have

Theorem 2.6. *Let $\overset{0}{D}$ be a given conformal almost symplectic d-linear connection on TM, with local coefficients: $\overset{0}{D}\Gamma(\overset{0}{N}, \omega, \tilde{\omega}) = (\overset{0}{L^i}_{jk}, \overset{0}{\tilde{L}^i}_{jk}, \overset{0}{\tilde{C}^i}_{jk}, \overset{0}{C^i}_{jk})$. The set of all conformal almost symplectic d-linear connections on TM, with respect to $\hat{A}$, corresponding to the 1-forms ω and $\tilde{\omega}$, with local coefficients $D\Gamma(N, \omega, \tilde{\omega}) = (L^i_{jk}, \tilde{L}^i_{jk}, \tilde{C}^i_{jk}, C^i_{jk})$ is given by:*

$$\begin{cases} N^i_j = \overset{0}{N^i_j} - X^i_j, \\ L^i{}_{jk} = \overset{0}{L^i}{}_{jk} + (\overset{0}{\tilde{C}^i}{}_{jm} + \delta^i_j \dot{\omega}_m) X^m{}_k + + \Phi^{ir}_{hj} X^h{}_{rk}, \\ \tilde{L}^i{}_{jk} = \overset{0}{\tilde{L}^i}{}_{jk} + (\overset{0}{C^i}{}_{jm} + \delta^i_j \dot{\tilde{\omega}}_m) X^m{}_k + \tilde{\Phi}^{ir}_{hj} \tilde{X}^h{}_{rk}, \\ \tilde{C}^i{}_{jk} = \overset{0}{\tilde{C}^i}{}_{jk} + \Phi^{ir}_{hj} \tilde{Y}^h{}_{rk}, \\ C^i{}_{jk} = \overset{0}{C^i}{}_{jk} + \tilde{\Phi}^{ir}_{hj} Y^h{}_{rk}, \\ X^i{}_{j\overset{0}{|}k} = 0, \end{cases} \tag{15}$$

where $X^i_j,\ X^i{}_{jk},\ \tilde{X}^i{}_{jk},\ \tilde{Y}^i_{jk},\ Y^i_{jk}$ are arbitrary tensor fields on TM, $\omega = \omega_i dx^i + \dot{\omega}_i \delta y^i$ and respective $\tilde{\omega} = \tilde{\omega}_i dx_i + \dot{\tilde{\omega}}_i \delta y^i$ are two arbitrary 1-forms in TM and $\overset{0}{|}$, $\overset{0}{\big|}$ denote h-and respective v-covariant derivatives with respect to $\overset{0}{D}$.

7. If we take $X^i_j = 0$ in Theorem 2.6 we obtain:

Theorem 2.7. *Let $\overset{0}{D}$ be a given conformal almost symplectic d-linear connection on TM, with local coefficients: $\overset{0}{D}\Gamma(\overset{0}{N}, \omega, \tilde{\omega}) = (\overset{0}{L^i}{}_{jk}, \overset{0}{\tilde{L}^i}{}_{jk}, \overset{0}{\tilde{C}^i}{}_{jk}, \overset{0}{C^i}{}_{jk})$. The set of all conformal almost symplectic d-linear connections on TM, with respect to $\hat{A}$, which preserve the nonlinear connection $\overset{0}{N}$, corresponding to the 1-forms ω and $\tilde{\omega}$, with local coefficients $D\Gamma(\overset{0}{N}, \omega, \tilde{\omega}) = (L^i{}_{jk}, \tilde{L}^i{}_{jk}, \tilde{C}^i{}_{jk}, C^i{}_{jk})$ is given by:*

$$\begin{cases} L^i{}_{jk} = \overset{0}{L^i}{}_{jk} + \Phi^{ir}_{hj} X^h{}_{rk}, \\ \tilde{L}^i{}_{jk} = \overset{0}{\tilde{L}^i}{}_{jk} + \tilde{\Phi}^{ir}_{hj} \tilde{X}^h{}_{rk}, \\ \tilde{C}^i{}_{jk} = \overset{0}{\tilde{C}^i}{}_{jk} + \Phi^{ir}_{hj} \tilde{Y}^h{}_{rk}, \\ C^i{}_{jk} = \overset{0}{C^i}{}_{jk} + \tilde{\Phi}^{ir}_{hj} Y^h{}_{rk}, \end{cases} \tag{16}$$

where $X^i_j,\ X^i{}_{jk},\ \tilde{X}^i{}_{jk},\ \tilde{Y}^i_{jk},\ Y^i_{jk}$ are arbitrary tensor fields on TM.

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