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FREE CONVECTION BOUNDARY LAYER OVER A VERTICAL PERMEABLE CONE EMBEDDED IN A FLUID SATURATED POROUS MEDIUM WITH INTERNAL HEAT GENERATION

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1. **Introduction**

Heat transfer through porous media has important practical applications such as oil extraction, thermal insulation, geophysical flows, water waste disposal, etc. Recent monographs by Ingham and Pop (1998, 2002), Nield and Bejan (1999), Vafai (2000) and Pop and Ingham (2001) give excellent summary of the work on the subject.

The phenomenon of internal heat generation is present in many situations, especially in the field of nuclear energy and composite superconductors (see Horvat et al., 2001; Malinowski, 1993). Studies in natural convection driven by internal heat generation has been done by Roberts (1967), Jahn and Reinke (1974), Hardee and Nilson (1977), Stewart and Dona (1988), Crepeau and Clarksean (1997), etc.

The present paper studies the free convection from a vertical permeable cone embedded in a fluid saturated porous medium the effects of internal heat generation being present. The case of a variable temperature at the cone surface is considered, see Fig. 1.

2. **Basic equation**

Under Boussinesq and boundary layer approximation the governing equations can be written as:

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial y} = 0 \tag{1}$$

$$u = \frac{g\cos\gamma K\beta}{\nu}(T - T_\infty) \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_m \frac{\partial^2 T}{\partial y^2} + \frac{q^{'''}}{\rho C_p} \tag{3}$$

where $r = x\cos\gamma$ is the cone radius, ν is the kinematic viscosity, K the permeability,α_m is the thermal diffusivity, $q^{'''}$ is the internal heat, ρ the density and C_p is the specific heat at constant pressure. Indexes w and ∞ refer to the cone surface and ambient conditions.

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Further we introduce the stream function ψ given by

$$ru = \frac{\partial \psi}{\partial x}, \;\; rv = \frac{\partial \psi}{\partial y} \tag{4}$$

so that the equations (1)-(3) become:

$$\frac{1}{r}\frac{\partial \psi}{\partial y} = \frac{g\cos\gamma K\beta}{\nu}(T - T_\infty) \tag{5}$$

$$\frac{1}{r}\left(\frac{\partial \psi}{\partial y}\frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial T}{\partial y}\right) = \alpha_m \frac{\partial^2 T}{\partial y^2} + \frac{q^{'''}}{\rho C_p} \tag{6}$$

subject to the boundary conditions:

$$-\frac{\partial \psi}{\partial x} = ax^n, \;\; T_w = T_\infty + Ax^\lambda \;\; for \;\; y = 0 \tag{7}$$

$$\frac{\partial \psi}{\partial y} \to 0, \;\; T \to T_\infty \;\; for \;\; y \to 0 \tag{8}$$

where A is a positive constant and a, n, λ are constants with $a > 0$ for injection and $a < 0$ for suction.

In order to solve equations (5)-(6)we introduce the following similarity variables:

$$\psi = \alpha_m r Ra_x^{1/2} f(\eta), \;\; \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \;\; \eta = Ra_x^{1/2}(y/x) \tag{9}$$

where Ra is the local Rayleigh number defined as by:

$$Ra_x = \frac{g\beta K\cos\gamma(T - T_\infty)x}{\nu\alpha_m} = \frac{g\beta K\cos\gamma A x^{\lambda+1}}{\nu\alpha_m} \tag{10}$$

In order that the similarity solution of equations (5)-(6) exist, we assume following Crepeau and Clarksean (1997), that the internal heat generation q''' is given by:

$$q^{'''} = \frac{k_m(T_w - T_\infty)}{x^2} Ra_x e^{-\eta} \tag{11}$$

On using (9) and (11) in (5) and (6) we obtain the following ordinary differential equations of the motion:

$$f' = \theta \tag{12}$$

$$\theta^{''} + \frac{\lambda+3}{2} f\theta' - f\lambda f'\theta + e^{-\eta} = 0 \tag{13}$$

Combining (12) and (13) we get

$$f^{'''} + \frac{\lambda+3}{2} f\theta' - \lambda f'^2 + e^{-\eta} = 0 \tag{14}$$

along with the boundary conditions

$$f(0) = -f_w, \;\; f'(0) = 1, \;\; f \to 0 \;\; for \;\; \eta \to \infty \tag{15}$$

where f_w is the mass flux parameter given by

$$f_w = -\frac{2a}{\lambda+3}\left(\frac{\alpha_m \nu}{g\beta K\cos\gamma A}\right)^{1/2} \tag{16}$$

For the above similar equations we considered that

$$n = \frac{\lambda - 1}{2} \tag{17}$$

as it was found in Postelnicu et al. (2000), for the corresponding flat plate case.

In this case, the local Nusselt number is given by:

$$Nu_x / Ra_x^{1/2} = -f''(\lambda, 0) \tag{18}$$

3. **Results and discussions**

Equation (14) with the boundary condition (17) has been solved numerically using a shooting method (see, Chakraborty 1998) for $\lambda = 0, 1/3$ and $1/2$ and $f_w = -2, -1, -0.5, 0, 0.5, 1$ and 2. Results obtained for the Nusselt number were compared in Table 1 with the results previously obtained by Cheng et al.(1985), and we can see that the results are in a very good agreement.

This table shows also that the presence of internal heat generation leads to the decrease of the local heat transfer. Figures 2-4 present the non-dimensional temperature profiles in the absence of internal heat generation and figures 5-7 the same profiles when internal heat generation is present. It can be seen from these figures that the thickness of boundary layer increases with the increase of the mass flux parameter, f_w. This phenomenon is more significant for law values of λ.

Figure 8 shows the variation of Nusselt number, $f''(\lambda, 0)$, with the mass flux parameter f_w. It is noticed that in the both cases with and without internal heat generation, the heat transfer is more significant for higher values of the mass flux parameter.

Table 1. Values of the local Nusselt number, $-f''(\lambda, 0)$

λ	Without internal heat generation		With internal heat generation
	Cheng et al. (1985)	present results	present results
0	0.769	0.7687	0.1963
1/3	0.921	0.9210	0.3937
1/2	0.992	0.9900	0.4799

Figure 1. Physical model

Figure 2. Velocity profiles for $\lambda = 0$ and some values of the mass flux parameter f_w when the effect of internal heat generation is not present

Figure 3. Velocity profiles for $\lambda = 1/3$ and some values of the mass flux parameter f_w when the effect of internal heat generation is not present

Figure 4. Velocity profiles for $\lambda = 1/2$ and some values of the mass flux parameter f_w when the effect of internal heat generation is not present

Figure 5. Velocity profiles for $\lambda = 0$ and some values of the mass flux parameter f_w when the effect of internal heat generation is present

Figure 6. Velocity profiles for $\lambda = 1/3$ and some values of the mass flux parameter f_w when the effect of internal heat generation is present

Figure 7. Velocity profiles for $\lambda = 1/2$ and some values of the mass flux parameter f_w when the effect of internal heat generation is present

Figure 8. Variation of the local Nusselt number, $-f''(0)$, with the mass flux parameter f_w when the effect of internal heat generation is not present (........) and when the effect of internal heat generation is present (----)

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