BOOK REVIEWS

Zdzislaw Denkowski, Stansislaw Migórski and Nikolas S. Papageorgiu, *An Introduction to Nonlinear Analysis,* Vol. I, *Theory,* 689 pp, ISBN 0-306-47392-5, Vol. II, Applications, 823 pp, ISBN 0-306-47456-5, Kluwer Academic Publishers, Boston-Dordrecht-London 2003.

The aim of this two volume treatise is to present some basic results in nonlinear analysis along with some applications. The methods and tools of nonlinear analysis rely heavily on results from other mathematical disciplines, first of all linear functional analysis (Banach space theory), topology and measure theory. In order to make a newcomer acquainted with the needed results from these areas and to prevent him to waste time and energy to browse over various specific books, the authors decided to gather in the first volume the basic results that are often used in nonlinear analysis. In the next we shall pass to a detailed analysis of the volumes.

Volume I: **Theory**

The volume is divided into five chapters corresponding to specific areas included: 1. *Elements of topology* (102 pp); 2. Elements of measure theory (150 pp); 3. *Banach spaces* (149 pp); 4. *Set-valued analysis* (112 pp); 5. *Nonsmooth analysis* (148 pp).

The first chapter contains the basic notions, constructions and results from topology – separation properties, nets and filters, connectedness, compactness, metrizability, continuity and uniform continuity, completeness, topologies on function spaces. Measure theory and integration is treated in the second chapter, and includes basic constructions in measure theory, Radon-Nikodym theorem, measures and measurable functions on topological spaces, Polish and Souslin spaces, Carathéodori functions (Scorza-Dragoni theorem). The third chapter is concerned with the fundamental properties of Banach and Hilbert spaces and of operators acting on them: Hahn-Banach theorem, fundamental principles, weak and weak* topologies, separation of convex sets. A special attention is paid to function spaces (including Sobolev spaces) and their duality, compactness and weak compactness criteria in such spaces.

Although the majority of the results are presented with full proofs, some difficult theorems (as, e.g., Tietze and Urysohn theorems, the paracompactness of metric spaces, Nikodym boundedness theorem, Lyapunov convexity theorem, some extension theorems for measures, James' criterium of weak compactness, Eberlein-Smulian theorem, Bishop-Phelps theorem) are only enounced with exact references to the sources where a proof can be found.

Chapter 4, *Set-valued analysis* (112 pp), presents the basic results of multivalued analysis – various convergence types for sets and multifunctions, continuity, measurability, set-valued measures and integration, measurable and continuous selections. A comprehensive treatment of these topics is given in another two volume treatise published also at Kluwer A. P. by S. Hu and N. S. Papageorgiu, Handbook on multivalued analysis, Vol. I (1997), Vol. II (2000).

The last chapter of this volume is concerned with nonsmooth analysis including differential calculus in Banach spaces, convex functions and their subdifferentials, generalized subdifferentials of locally Lipschitz functions, optimization and minimax theorems, tangent and normal cones.

Volume II, **Applications**

The first chapter of the second volume, *Nonlinear operators and fixed points* (168 pp), discusses nonlinear compact and Fredholm operators, measures of noncompactness and set-contractions, monotone operators, accretive operators and semigroups of nonlinear operators, Ekeland variational principle, fixed points.

After this somewhat transition chapter, making a bridge between the theory treated in the first chapter to the applications from the second one, one passes to more applied topics: 2. *Ordinary differential equations* (144 pp); 3. *Partial differential equations* (228); 4. *Optimal control and calculus of variations* (147 pp); 5. *Mathematical economics* (105 pp). Of course that it is impossible to give in one chapter a comprehensive treatment of the subject, the aim of the authors being rather to emphasize how the techniques developed so far work to give new insights.

For instance, in the second chapter, the approach to differential equations is done via critical point theory and minimax techniques (Mountain Pass, Saddle Point and Linking Theorems). Differential inclusions as well as Hamiltonian systems with emphasis on the existence of periodic trajectories, are also considered.

Partial differential equations, treated in the third chapter, are one of the main domain of applications of nonlinear analysis and, at the same time, a source for many problems and results. Here the main idea is to show that there are some unifying themes, lying underneath the huge amount of apparently unrelated techniques used to solve partial differential equations. The main topics are: eigenvalue problems and maximum principles, nonlinear elliptic problems, evolution equations, Γ-convergence for functions and G-convergence for operators.

Another important field of applications where the method of nonlinear analysis are essential is optimal control, treated in the fourth chapter. Again the treatment is restricted to topics that illustrate the techniques developed in the previous chapters, and they include: existence and relaxation, sensitivity analysis, the maximum principle, Hamilton-Jacobi-Belman equation, viscosity solutions, controllability and observability. The last section of this chapter is devoted to the calculus of variations, a field as old as the calculus itself, but still of great interest.

Finally, the last chapter of the book deals with some problems in mathematical economics, a domain that knew a remarkable progress in the last forty years, and allowed to some mathematicians to win a Nobel prize in economics. From this vast domain the authors selected some topics: Walras equilibria in competitive economies, growth models for both discrete time and continuous time cases, growth models under uncertainty, stochastic games.

Each chapter contains a set of exercises (around 50), followed by solutions, completing the main text. A section of remarks, containing historical comments, references to related results as well as indications for further reading, is also included in each chapter.

This fairly self-contained two volume book is a very good introductory text to a variety of topics in nonlinear analysis and its applications. It, or parts of it, can be used for graduate or post-graduate courses, or as a reference text.

S. Cobzaş

p-Adic Functional Analysis, Lecture Notes in Pure and Applied Mathematics: Vol. 222, A. K. Katsaras, W. H. Schikhof, L. Van Hamme - Editors, M. Dekker, New York 2001, viii+322 pp, ISBN 0-8247-0611-0.

These are the Proceedings of the Sixth International Conference on p-adic Functional Analysis held in 2000 at the University of Ioannina, Greece. Starting with Laredo, Spain 1990, each two years a conference on these topics was held in various countries, most of the proceedings being published by M. Dekker in the same series as the present one.

This conference was attended by about 40 mathematicians from various countries, reputed specialists who, in 30 minutes talks, reported on their latest results in p-adic or non-archimedean (n.a.) analysis. Among the participants were J. Aguayo, H. Ochsenius (Chile), J. Araujo, C. Perez-Garcia (Spain), K. Boussaf, A. Escassut (France), N. De Grande-De Kimpe (Belgium), J. Kakol (Poland), A.K. Katsaras, C.G. Petalas (Greece), A. Khrennikov, K.-O. Lindhal, M. Nilsson (Sweden), A.J. Lemin (Russia), P.N. Natarajan (India), W.H. Schikhof (The Netherlands), B. Dragovich (Yugoslavia), M. Berz (USA), H. Keller (Switzerland), et al.

The volume contains 26 research papers covering a large area of topics in p-adic analysis and its applications as – n.a. locally convex spaces (2 papers) and sequence spaces, n.a. vector measures and integral representations of linear operators, n.a probability measures, compact perturbations of linear operators, spctral radius of derivations, n.a. Banach-Stone theorem, p-adic analytic functions, p-adic differential equations, commutation relations for operators on non-classical Hilbert spaces, dynamical systems (3 papers), embedding n.a. metric spaces in classical L_p-spaces, ultrametric Hopf algebras, Levi-Civita fields, ergodicity of p-adic spheres, and more.

As the preceding ones, this volume will became an indispensable reference for those working in non-archimedean analysis and its applications.

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