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BOOK REVIEWS

John J. Benedetto, *Harmonic Analysis and Applications*, Studies in Advanced Mathematics, CRC Press, Boca Raton-New York-London-Tokyo 1997, xix+336 pp., ISBN 0-8493-7879-6.

The present book is a textbook and an essay, the author goal being "to present harmonic analysis at level that exhibits its vitality, intricacy and simplicity, power, elegance, and usefulness" (from the Preface). The author restricts to classical harmonic analysis, the fundamental components being the the trigonometric functions, with emphasis on *analysis,* meaning determination of harmonics or components of a given function, and *synthesis*, meaning the reconstruction of this function in terms of its components. The methods are primarily those of real analysis with very little complex analysis, the development being done within the framework of spaces L^1 and L^2. The prerequisites for the reading of the book are a basic course in real analysis as, e.g., J. Benedetto, "Real Variable and Integration", B.G. Teubner, Stuttgart 1976. Although abstract harmonic analysis (invariant measures on locally compact groups, Banach algebras, representation theory) are not considered, the treatment has a Banach algebra flavor, and is a substantial part of the harmonic analysis on a commutative locally compact group.

A selection of the book (the corresponding numbers of definitions and propositions are listed in Prologue I) was used by the author as material for upper undergraduate courses, taught for many years to students in engineering, physics, computer science, and mathematics. The exercises at the end of each chapter range from elementary to difficult and from theoretical to computational and/or computed oriented (using MATLAB programs). The first 30 exercises of each chapter are appropriate for Course I.

The book contains many examples from engineering and physics and very interesting historical comments on the evolution of the ideas in this very fertile areas of mathematics, which shaped the development of mathematics in the 20th century (measure theory, topology, set theory, functional analysis).

The book is, in essence, on classical harmonic analysis, including careful proofs of the basic theorems, but the exposition is done in a way to provide perspectives of many topics, some of them (e.g. Wiener's Generalized Harmonic Analysis) being extensively treated. Due to these perspectives, of lengthy historical comments and exercises, the book can serve also as a textbook for more advanced courses than Course I. Also, the limitation to classical harmonic analysis is compensated to some extent by a serious bibliography, referenced at appropriate junctures in the text.

Written by a leading specialist in harmonic analysis, with over than 100 published papers (including 9 books), the present book is a very good text on harmonic analysis, its applications and evolution, and can be used as a textbook as well as an essay for students and as general reference for engineers, mathematicians, physicists, and other people using harmonic analysis.

S. Cobzaş

Joseph A. Cima and William T. Ross, *The Backward Shift on the Hardy Space*, Mathematical Surveys and Monographs Vol. 79, xi+ 199 pp., American Mathematical Society 2000, ISBN: 0-8218-2083-4.

The book is devoted to the study of invariant subspaces of the backward shift operator on the Hardy space H^p of analytic functions on the open unit disc $\mathbb{D} = \{|z| < 1\}$. The backward shift operator B is defined by

$$Bf = \frac{f - f(0)}{z} = a_1 + a_2 z + a_3 z^2 + ...$$

for $f = a_0 + a_1 z + a_2 z^2 + ... \in H^p$.

As the backward shift operator on H^2 (the Hilbert case) is presented in detail in Nikolskiĭ's book *Treatise on the Shift Operator*, Springer Verlag, Berlin-New York 1986, the authors of the present book focus on the Banach case ($p \in [1, \infty)$) and the Fréchet case ($p \in (0, 1)$). The characterization of the invariant subspaces of the backward operator on H^p for $1 \leq p < \infty$ was settled down by R. Douglas, H. S. Shapiro and A. Shields, Annale Institut Fourier (Grenoble) **20** (1970), 37-76. The case

$p \in (0,1)$ was solved by A.B. Aleksandrov, Investigations on Linear Operators and the Theory of Functions IX, Zap. Nauchn. Sem. Leningrad Otdel. Mat. Inst. Steklov (LOMI), **92** (1979), 7-29, in a paper which was never translated from its original Russian and using a quite complicated technique–distribution theory and Coiffman's atomic decomposition for the Hardy space. The authors gather up these results together with the necessary background material which is surveyed in appropriate places. The reader is supposed to be acquainted with the basic of functional analysis (at the level of Rudin's book), complex function theory and H^p spaces (Duren's and Garnett's books), and harmonic analysis (Stein's book).

The main results and the technique used for their proofs are briefly, but in a very clear manner, explained in the first chapter of the book entitled *Overview*.

A good idea on the organization of the book is given by the headings of the rest of its chapters: 2. *Classical boundary value results*; 3. *The Hardy space on the disk*; 4. *The Hardy spaces on the upper-half plane*; 5. *The backward shift on* H^p *for* $p \in [1, \infty)$; 6. *The backward shift on* H^p *for* $p \in (0,1)$.

Written by two eminent specialists and combining techniques from functional analysis, operator theory, harmonic analysis, real and complex analysis, this beautiful book appeals to a large audience, meaning people interested in the topics listed above. It can be used also as a textbook for advanced graduate or post-graduate courses.

Stefan Cobzaş

David L. Jagerman, *Difference Equations with Applications to Queues*, Pure and Applied Mathematics Series, Vol. 233, M. Dekker, Inc., Basel - New York 2000, xi+241 pages, ISBN: 0-8247-9007-3.

This monograph presents a theory of difference and functional equations with continuous argument, based on a generalization of the Riemann integral introduced by N.E. Nörlund in his famous monograph published in 1924. This approach permits greater flexibility in constructing solutions and approximate solving nonlinear first order equations by a variety of of methods, including an adaptation of the Lie-Gröbner theory.

Ch. 1, *Operators and Functions*, is a general overview of the operators and functions which are important in the difference calculus. Ch. 2, *Generalities on Difference Equations*, considers the genesis of difference and gives a number of exercises. Casorati's determinant is introduced, and Heyman's theorem and a theorem of of Milne-Thompson on the asymptotic behavior of the linear independence of solutions are proved.

Chapters 3 and 4, *Nörlund Sums: Part one* and *Part two*, respectively, contain the basic properties of Nörlund sums as well as representations obtained by means of Euler-Maclaurin expansions. Fourier expansions and the extension to the complex plane of the Euler-Maclaurin representation are also studied. Some examples are included.

Ch. 5, *The First Order Difference Equation*, as the title shows, deal with first order difference equations, both linear and nonlinear. The method of Truesdell for differential-difference equations is discussed and applied to a queuing model. Simultaneous first-order nonlinear equations are solved approximately.

In Ch. 6, *The Linear Equation with Constant Coefficients*, beside the study of linear equations with constant coefficients, some methods of solving partial difference equations are also included. Application is made to the probability $P(t)$ that an $M/M/1$ queue be empty, given that it is initially empty. An asymptotic development for $P(t)$ is obtained for large t and a practical approximation is constructed.

The final chapter, Ch. 7,*Linear Difference Equations with Polynomial Coefficients*, describes the linear difference equations with polynomial coefficients. The method of depression of the order, the Casorati's determinant and Heyman's theorem, are some of the tools used in this chapter. However, the main technique for solution is based on the π, ρ operator method of Boole and Milne-Thompson, which constructs the solution in terms of factorial series. Application is made to the last-come-first-served queue with exponential reneging; in particular, the Laplace transform is obtained for the waiting time distribution.

J. Sándor

Kenneth L. Kuttler, *Modern Analysis,* Studies in Advanced Mathematics, CRC Press, Boca Raton-New York-London-Tokyo 1998, 572 pp., ISBN 0-8493-7166-X.

This is an advanced course on real and abstract analysis, meaning topology, functional analysis, measure theory and integration, and applications.

The first two chapters, 1. *Set theory and topology* and 2. *Compactness and continuous functions,* contain the basic of general topology including Urysohn's lemma, Stone-Weierstrass theorem and Arzela-Ascoli compactness criterium. Tichonoff's theorem on the compactness of the product is proved in the chapter on locally convex spaces (Chapter 6). Functional analysis is developed in three chapters: 3.*Banach spaces,* 4. *Hilbert spaces,* and 6. *Locally convex topological vector spaces* (separation theorems, weak and weak topologies, Tychonoff's fixed point theorem). Brouwer fixed point theorem is proved in Appendix 4 at the end of the book. There is also a chapter, 5. *Calculus in Banach space,* exposing the basic results on Fréchet differentiability, including the inverse function theorem and applications to ordinary differential equations.

A good part of the book is devoted to measure theory and integration, with emphasis on Lebesgue measure and integral, and on Radon measures. This is done in the chapters: 7. *Measures and measurable functions* (monotone classes and algebras, Egoroff's convergence theorem), 8. *The abstract Lebesgue integral,* 9. *The construction of measures* (outer measures and Caratheodory's definition of measurable sets, Radon measures and Riesz representation theorem for positive functionals on $C_c(\Omega)$), 10. *Lebesgue measure* (Lebesgue measure in $\mathbb{R}^n$, change of variables by linear transformations, polar coordinates), 11. *Product measures* (Fubini and Tonelli theorems, completion of a product measure), 12. *The L^p spaces* (completeness, density of simple functions, continuity of translation operator, separability, convolution, mollifiers, and density of smooth functions), 13. *Representation theorems* (Radon-Nikodym theorem, Clarkson inequality, the duals of L^p, $1 \leq p < \infty$, and $C(T)$, for T compact), 14. *Fundamental theorem of calculus* (Vitali covering theorem, differentiation with respect to Lebesgue measure, the change of variables for multiple integrals), 15. *General Radon measures* (Besicovitch covering theorem, differentiation with respect to Radon measures, Young measures). A chapter, 23. *Integration of vector valued functions*, presents the Bochner integral and Riesz representation theorem for the dual of $L^p(\Omega, X)$, X a Banach space).

Three chapters, 19. *Hausdorff measures,* 20. *The area formula,* and 21 *The coarea formula,* deal with Hausdorff measures and very general change of variable formulas for surface integrals in $\mathbb{R}^n$.

Among the applications, we mention Chapter 17. *Probability,* containing a short but thorough exposition of basic results in probability theory. Other applications are to Fourier analysis and distribution theory given in chapters 16. *Fourier transforms* (based on Schwartz class of rapidly decreasing smooth functions and on tempered distributions), 18 *Weak derivatives* (Morrey's inequality and Rademacher theorem on a.e. differentiability of Lipschitz functions), 22. *Fourier analysis in* $\mathbb{R}^n$ (Marcinckiewicz interpolation theorem, Calderon-Zygmund decomposition, Michlin's generalization to L^p of Plancherel theorem, Calderon-Zygmund theory of singular integrals).

The last chapter of the book, 24. *Convex functions,* presents some of the most important results on convex functions, culminating with a proof of Alexandrov's theorem on a.e. twice differentiability of convex functions.

Three appendices: 1. *The Hausdorff maximal theorem,* 2. *Stone's theorem and partitions of unity,* 3. *Taylor series and analytic functions,* and 4. *The Brouwer fixed point theorem*, complete the main text. There are also a set of well chosen exercises at the end of each chapter, some of them routine, others containing more advanced topics and results which were not included in the main body of the book.

The book is an ideal text for graduate-level real analysis courses and basic courses on measure theory, using a modern approach. Its specific feature is the presentation, with complete proofs and in an accessible but rigorous way, of some deep results in modern analysis, available only in more specialized texts and needing a lot of technicalities for their understanding.

We warmly recommend the book to all people desiring to teach or to learn some fundamental results in modern analysis, in a reasonable period of time.

S. Cobzaş

Rafael H. Villarreal, *Monomial Algebras*, Pure and Applied Mathematics 238, Marcel Dekker 2010, ix+455pp, ISBN 0-8247-0524-6.

The volume under review presents methods which can be used to study monomial algebras and their presentation ideals including computational methods.

The book is divided in 11 chapters. Chapter 1 contains the basic facts and methods on commutative algebra and homological algebra. In order to present the basic properties of monomial algebras, the author presents in Chapter 2 the affine and graded algebras and in Chapter 3 he exhibits the importance of Rees algebras and associated graded algebras. Chapters 4 and 5 present the Hilbert series of graduates modules and Stanley-Reisner rings which are used in the Sanley's proof of the upper bound conjecture for simplicial spheres. In Chapters 6, 8 and 9 the connections between monomial algebras, graph theory and polyhedral theory are presented. The author presents in Chapter 7, 9 and 10 some features of toric ideals the monomial curves, the affine toric varieties and their toric ideals.

The book contains 280 exercises and numerous examples and graphs. Therefore, graduate students and researchers interested in commutative algebra and in its connections with computational issues in algebraic geometry and combinatorics will find this volume very useful.

S. Breaz

Sorin Dăscălescu, Constantin Năstăsescu, Şerban Raianu, *Hopf Algebras. An Introduction.* Monographs and textbooks in pure and applied mathematics 235,

Marcel Dekker, New York-Basel, 2001, ix+401 pp., Hardcover, ISBN 0-8247-0481-9.

The volume under review is aimed to introduce the reader to modern results on Hopf algebras. The material, presented from a ring theoretical point of view, has grown out of courses given over several years by the authors at the University of Bucharest.

The book is divided into 7 chapters. Chapter 1 presents basic facts on algebras and coalgebras, while Chapter 2 studies categories of comodules over a coalgebra. Chapter 3 examines in some depth cosemisimple, semiperfect and co-Frobenius coalgebras. Chapter 4 introduces bialgebras, Hopf algebras and Hopf modules, and Chapter 5 is devoted to integrals, the case of Hopf algebras obtained by Ore extensions being thoroughly treated. Chapter 5 discusses actions and coactions of Hopf algebras on algebras, and Hopf-Galois extensions. The last chapter presents various results on finite dimensional Hopf algebras, such as the order of the antipode, the Nichols-Zoeller theorem, character theory, the Taft-Wilson theorem, pointed Hopf algebras of dimension p^n. Appendices on the language of category theory and on C-groups and C-cogroups are also included. Each section contains many exercises accompanied by detailed solutions.

The authors are among the most important contributors to the field, and the above choice of topics reflects their interests. The presentation is very clear and reasonably self contained for a graduate student. The book is one of the best choices for a graduate course on Hopf algebras, and it will definitely be a valuable investment for any student and researcher interested in algebra.

Andrei Marcus

Martin Väth, *Volterra and Integral Equations of Vector Functions,* Pure and Applied Mathematics, Vol. 224, M. Dekker, Inc., Basel - New York 2000, vi+349 pages, ISBN: 0-8247-0342-1.

The book is dealing with Volterra-type integral equations of the form

$$x(t) = \int_0^t f(t, s, x(s))ds + g(t) \quad (1)$$

where f, g are given functions and x is the unknown function, all taking values in a Banach space (usually infinite dimensional). In fact the author considers a more general situation of some operator equations assumed to satisfy some "Volterra-typical" conditions. The emphasis is on the well-posedness of the problems, meaning existence, uniqueness and continuous dependence on the data, however the main part of the book is concerned with the existence of the solutions. A specific feature of the book is the extensive use of methods based on measures on noncompactnes, on fixed point theorems of Darbo type, and on quasinormed preideal spaces of vector functions.

The first chapter of the book, Ch. 1, *Preliminaries,* is concerned with fixed point theorems (mainly for operators which are condensing with respect to a measure of noncompactness), Bochner measurable functions and integrals, Lebesgue-Bochner function spaces, and ideal spaces. The framework is that of functions with values in a pseudometric space (more general than a Banach space), which is more appropriate for the subsequent development.

Ch. 2, *General Existence Results*, based mainly on author's original results, deals with existence results for abstract Volterra operators satisfying some boundedness and compactness conditions, containing as particular cases many types of Volterra operators.

The main tool used in the third chapter, *Integral Operators in Banach Spaces*, for defining integral operators and studying their properties (boundedness and compactness) is that of Carathéodori functions. To prove compactness results for integral operators, the author uses merely equimeasurability conditions rather than equicontinuity ones, leading to the notion of strict Carathéodori function. The general framework for the study of integral operators is that of ideal spaces. In fact, the author have written a book on this topic - "Ideal Spaces", Lect. Notes in Math. Vol. 1664, Springer Verlag, Berlin 1997.

The last chapter of the book, Ch. 4, *Dependence on Parameters*, is concerned with continuous dependence on the data, the averaging principle in nonlinear mechanics and Bogoljubov type theorems.

Developing general principles and results for Volterra type integral equations, most based on author's original results, and specifying them to particular equations arising in models from physics, mechanics and biology, the book will be of great interest for researchers in applied functional analysis, differential and integral equations, and their applications in other areas of human knowledge.

S. Cobzaş