STUDIA UNIV. "BABEŞ–BOLYAI", MATHEMATICA, Volume **XLVI**, Number 1, March 2001

THE ROLE OF APPLIED MATHEMATICS

WOLFGANG L. WENDLAND

Magnificus, Spectabiles, Professors, dear friends, ladies and gentlemen!

This is a very touching moment for my dear wife and me. The honorous doctorate of the Babeş-Bolyai University is a very great honour for me and I want to express my sincere gratitude to my dear colleagues, in particular to Professor Micula - thank you so much - to Professor Rus and to Professor Ţâmbulea. Dear Prorector Szilágyi, thank you so much for your kindness and for your flattering laudatio which makes me rather uncomfortable but also very happy. And Magnificus Simon, thank you much for this great honour! I will try to be a good scholar and ambassador.

Now, let me say a few words on my profession.

Mathematics is one of the oldest cultural achievements of mankind, as early as literature, art and music. "Mathematics must be understood as a human activity, a social phenomenon, part of human culture, historically evolved, intelligible only in a social context" [1].

In fact, mathematics has always been developed or discovered (?) in combination with other fields. Here are some historical examples:

Pythagoras (582-505 b. Chr.) discovered irrational numbers from the profane problem of partitioning a given surface, which triggered his studies of geometry and numbers. He introduced the concepts of axioms, theorems, proofs and logical reasoning. As we know, because of troubles with tyrant Polykrates of Samos, he became a political refugee immigrating to Italy.

Archimedes (287-212 b. Chr.) was regarded by his king Hieron of Syracuse as top engineer because of his numerous inventions and discoveries such as fundamental laws of hydrostatics, the spiral pump, sets of pulleys, catapults and war machines,

Given at the University Babeş-Bolyai in Cluj-Napoca, Romania, on April 7th 1999, on the occasion of his nomination as "Doctor Honoris Causa".

the concave mirror. King Hieron was very angry at Archimedes' absurd mathematical activities like approximation methods for computing roots and π, developing the concept of the Riemann integral and of differentiation. We all know about the tragic end of Archimedes and of Greek sophistication due to the brutal Roman conquest of Syracuse.

I. Newton (1643-1727) was a famous physicist who developed differential and integral calculus for deriving Kepler's laws from the fundamental law of gravity, and his theory of light arose in connection with his work on optical instruments. However, he himself rated his mathematical work much higher than his achievements in physics and astronomy.

G. Leibniz (1646-1716) was an allround genius, a famous lawyer, historian and philosopher who enjoyed mathematics almost like a hobby. He developed differential and integral calculus, theory of determinants, power series, successive iteration, division of polynomials and two mechanical calculators; one of them based on dual numbers. Modern analysis owes him the elegant and simple rules of calculus.

C.F. Gauss (1777-1855) started his career with number theory and the proof of the fundamental theorem of algebra as a student - he was 22 years old when getting his PhD in absentia at Helmstedt - but professionally he was hired as an astronomer, continued as the director of surveying engineers, then developed the theory of electromagnetism and with Weber constructed the telegraph. But mathematics fascinated him most: He pioneered complex function theory, differential geometry and geodesy, least squares methods and the solution of linear equations, theory of algebraic equations and potential theory - he even proved the existence of solutions to boundary integral equations (my hobby). He was a close friend of Farkas Bolyai, who worked here in Cluj and published a great two-volume book on mathematics. His son Janos Bolyai from Cluj invented the non-Euclidean geometry independently of Gauss which became a necessary prerequisite for Einstein's theory of relativity. This university is named after F. and J. Bolyai!

S. Kowalewskaya (1850-1891) corrected a conjecture of Weierstrass and gave a constructive proof for the convergence of Cauchy's algorithm providing the most general existence proof for systems of partial differential equations. She then made significant contributions to complex function theory, potential theory, Abelian integrals and algebraic geometry. But she achieved her greatest success in the theory of the gyroscope apart from contributions to optics, celestial mechanics and astrophysics.

V. von Neumann (1903-1957) began with complex function theory and was then working with D. Hilbert on the foundation of mathematics, continuing with topology, functional analysis, measure theory, quantum field theory and stochastics, treating problems in meteorology, hydro- and aerodynamics, spherical shock waves, game theory (for the Navy) and the foundation of computer science. He developed the ENIAC in 1944 and the MANIAC in 1951 which was the basic computational tool for America's hydrogen bomb.

Mathematics as a profession by itself, however, is not older than 150 years. And the distinction between pure and applied mathematics is even younger - to me this distinction seems rather artificial. The greatest mathematicians do both: pure and applied mathematics; each side fertilizes the other.

But what is pure and what is applied mathematics? Mathematical theories in their complex and abstract structures grow at their frontiers through continuous interaction between mathematicians and all kinds of researchers. The advance is fed by information and desire from all areas of real and intellectual life. In order to make predictions about future possible states, mathematical models provide the corresponding tools. But "no real phenomenon is perfectly described by any mathematical model. There's usually a choice among several incomplete models, each more or less suitable" [1]. And H.M. Enzensberger says [2]: "The unforeseen utility of mathematical models is somewhat puzzling. It is no means clear why highly precise mental productions, devised entirely in isolation from empirical reality... should be so capable of explaining and manipulating the real world around us. Many have marveled at "the unreasonable effectiveness of mathematics". ... One explanation that presents itself - though not especially popular among the guardians of tradition - might be that one and the same evolutionary process has produced the universe at large and our brain, so that a weak anthropic principle determines that we observe the same operating rules in physical reality and in our own thought processes."

So, applied mathematics can be seen as that part of mathematics that deals with models of real life problems. This kind of work is done by many people, not only professional mathematicians. Among the Nobel Prize winners since 1970, I found at least 14 in physics, 3 each in chemistry, medicine and economy who used and developed deep mathematics for their outstanding achievements. Examples:

1979: A.M. Cormack, Sir G.N. Hounsfield: Computer assisted tomography.

1979: S.L. Glashow, A. Salan, S. Weinberg: Formulation of the standard model unifying weak and electromagnetic interaction.

1982: K.G. Wilson: Theory of critical phenomena in connection with phase transitions.

1990: J.I. Friedman, H.W. Kendall, R.E. Taylor: Scattering of electrons, protons, bound neutrons and the development of the quark model.

1990: H.M. Markowitz, M.M. Miller, W.F. Sharpe: Theory of financial economics.

1998: J.A. Pole: Computational methods in quantum chemistry.

Mathematics, however, is explicitly excluded from the Nobel prize; probably since Mittag-Leffler, who was at the same time a famous mathematician at the Royal Academy and the University of Stockholm disliked the playboyish A. Nobel. These feelings were probably mutual.

During the first half of this century, the terminus applied mathematics was used for teaching mathematics to engineering students. But in the second half, "the most striking development in engineering... has been the increasing use of mathematics in the analysis of engineering problems. No longer is skill in the use of a slide rule sufficient mathematical equipment for a practising engineer. For instance, control engineers use sophisticated, and very often abstract, mathematical concepts, some electrical engineers have to be acquainted with quantum mechanics, others with transform theory, and civil and mechanical engineers reading research papers on continuum mechanics encounter a bewilderingly wide range of mathematical techniques;" writes I. Sneddon in [5]. Nowadays exist engineering sciences.

Sneddon's Encyclopaedic Dictionary [5] and also the Mathematical Handbook [6] by G.A. and T.M. Korn covers mathematical analysis of very high level, I doubt that I am familiar with more than 20% of it. Thus, our engineering colleagues do applied mathematics to a great extent, too.

In [5] and [6] one can also find references to the work of two famous mathematicians and professors of this university: To L. Fejer (1880-1959) who taught in Cluj from 1905 till 1911 and whom we owe fundamental results in harmonic analysis, complex function theory and interpolation, and to F. Riesz (1880-1956) who worked here 1911-1920 and who has made decisive inventions in the functional analysis of Hilbert and Banach spaces, harmonic analysis and approximation theory.

In all engineering fields, the modern tools of electronic computers led to the new branch of scientific computing and simulation in applied mathematics [3] which now is indispensable in computer tomography, geometric design, reconstruction and visualization, direct and inverse scattering of electromagnetic and acoustic fields, heat transfer and radiation, stress and damage analysis in solid mechanics, all branches of fluid mechanics from aircraft design to sedimentation and groundwater pollution, signal processing, network analysis and planning, chemical processing, etc., etc. - and the combination of several field models to multifield problems resulting in new technologies such as the intelligent wing, nondestructive thermography, noise reduced helicopters or most effective sex-segregated baby diapers.

Nowadays industry uses mathematics often directly. This resulted in the fashionable new field of industrial mathematics [4]. The E & E Chief of Exxon, E.E. David in a report for the NSF of the US [7]: "Apparently, too few people recognize that the high technology that is so celebrated today is essentially mathematical technology... Mathematics is, or should be playing an integral role in America's industry's approach to its challenges, at home and abroad."

Indeed, mathematics seems to be the key technology of our future (see [8]) and also the "language of engineers".

The enormous demand for applied mathematics creates completely new mathematical concepts such as qualitative reliability and corresponding tools for critical evaluation of physical and constutive models; cost and time efficient computational algorithms, new solution methods due to new computer technologies (e.g. parallelization), attractive presentation of computational results, adequate learning and teaching of the new mathematical technologies.

These challenges will also influence mathematics in general since "applied mathematics is not illegitimate or marginal. Advances in mathematics for science and technology often are inseparable from advances in pure mathematics" [1].

As it seems, we are living in the high time of mathematics!

However, professional mathematicians are somehow excluded from corresponding benefits. After a stimulating and optimistic start into a seemingly new mathematical era about 20 years ago public opinion has changed more recently. Of course, mathematicians with their persistent obsession of truth and typical slowness in acceptance of empirical relations or new - often false - algorithms of pushing economists and engineers, are then seen as blocking obstacles of progress. One of the leading engineers in finite element analysis claims that mathematicians are rubber stampers but not inventors - they "prove" 10 years behind nothing but that engineers are doing right. Mathematicians think in counterexamples whereas engineers think in examples; and a mathematician always hesitates to publish anything that is not yet completely and rigorously justified within a mathematical theory. However, there are several examples where mathematical cautiousness would have saved a lot of money and even human lives. I recall the destruction of cooling towers at Ferrybridge in 1966, the desastrous sinking of a new Norwegian oil platform in 1993, or less spectacular, the elk test affair of the new Mercedes compact cars, convergence of finite element approximations towards wrong solutions because of negligence of distributional derivatives in modern analysis. In all these cases, design engineers blindly trusted professional software. So, on the path from applied mathematics to the numerical algorithm used for the software, critical information was lost since responsible people did not know enough mathematics.

As was mentioned before, here is a new challenge for applied mathematics: Every mathematical method that is used for the simulation of real life problems should nowadays also provide reliable information on the validity of the model and also reliable error inclusions since every mathematical model is based on some idealization that covers the true problem only partly and every computational, numerical method is incorrect to a certain extent. Of course, such worries are highly unwelcome in modern rushing life which unfortunately seems to be driven by the golden calf of financial profit.

"It is often not obvious how closely "pure" and "applied" mathematics are intertwined; this may be one of the reasons why the status of mathematical research is hugely undervalued in today's society. In addition, there is surely no other field in which the cultural lag is no enormous... One can state dispassionately that great segments of the population have never progressed beyond the mathematical level of the ancient Greeks. An equivalent backwardness in other fields - e.g. in medicine, or physics - would be perilous. This is very dangerous since never has a civilization been so infused with mathematical methology - right down to its everyday life - and so dependent on it as ours!" [2].

The danger for mathematics as a profession can rather clearly be seen nowadays. In my state which is booming in high technology, the State Commission for the Future Structural Development of the State Universities recommends reduction of the mathematical staff by 25% within the next 9 years, in spite of official state expert's recommendation in 1996 to expand scientific computing and numerical mathematics significantly. We currently discuss whether an open key professorship in this field should remain in the mathematical department or better be moved into some field of engineering applications or to computer science. "Der Spiegel" chose "Nobel prizes for nonsense" as headline of an article on the world congress of mathematics 1998 in Berlin. The number of freshmen and graduate students in mathematics dropped significantly during the last years in spite of their marvelous job chances.

Hence, it is just not enough to do good mathematics, pure and applied. We professional mathematicians must reform the education of school teachers in mathematics - it is simply improper to drill intelligent children with boring formalities. We also must reform our university teaching by showing our students from the beginning the power and beauty of applied mathematics, by treating difficult real life problems instead of agonizing them with ancient calculations. We must learn how to talk to our colleagues in other fields and must direct our research interests to challenging open problems of real life applications. The applied part of mathematics is livelier than ever but to me it is not so clear that professional mathematicians are really gripping the situation.

I hope that there will come a time when the following joke due to Professor Collatz will not be understood anymore: Two men in a balloon lost direction in fog.

Fortunately by the time they saw a man on the ground and asked him shouting: Where are we? It took a while before the man replied: You are in the gondola of a balloon! Disappointed, the balloon captain remarked to his friend: This was bad luck to meet a mathematician in our situation. - How did you know, he was a mathematician? - Firstly, replied the captain, before he made any statement he obviously was thinking hard. Secondly, his statement was clear and perfectly correct but - completely useless!

I am sure that applied mathematics is extremely useful!

Mulţumesc!

References

[1] Reuben Hersh, What is Mathematics, Really? Oxford University Press 1997.
[2] Hans Magnus Enzensberger, Drawbridge Up. Mathematics - A Cultural Anathema, A.K. Peters, LtD USA 1999.
[3] Hans Georg Bock et al., ENUMATH 97, 2nd European Conference on Numerical Mathematics and Advanced Applications. World Scientific, Singapore 1998.
[4] Helmut Neunzert, Progress in Industrial Mathematics at ECMI 94. Wiley/Teybner, Chichester/Stuttgart 1996.
[5] I.N. Sneddon, Encyclopaedic Dictionary of Mathematics for Engineers and Applied Scientists. Pergamon Press Ltd., Oxford 1976.
[6] Granino A. Korn, Theresa M. Korn, Mathematical Handbook for Scientists and Engineers. McGraw-Hill, New York 1961.
[7] A. Friedman, W. Littman, Industrial Mathematics. SIAM, Philadelphia 1994.
[8] Helmut Neunzert, Mathematik und Industrie: Von der Notwendigkeit eines Brückenschlags zwischen Theorie und Praxis. Wissenschfts-magazin der TU Berlin Heft 10(1987), 16-19.
[9] Karl-Heinz Hoffmann, Willi Jägger, Thomas Lehmann, Herrmann Schmuck, Mathematik, Schlüsseltechnologie für die Zukunft, Springer-Verlag, Berlin 1997.

UNIVERSITY OF STUTTGART