BOOK REVIEWS

Ravi P. Agarwal, *Difference Equations and Inequalities: Theory, Methods and Applications,* Second Edition, Revised and Expanded, M. Dekker, Inc., Basel - New York 2000, xiii+971 pages, ISBN: 0-8247-9007-3.

Although difference equations manifest themselves as mathematical models describing real life situations in probability theory, queueing problems, statistics, stochastics time series, combinatorial analysis, number theory, electrical networks, quanta in radiation, genetics, economics, psychology, sociology, etc., these are sometimes considered as the discrete analogs of differential equations. It is an indisputable fact that difference equations appeared much earlier than differential equations and were very important for their development. It is only recently that difference equations have started receiving the attention they deserve. Perhaps this is largely due to the advent of computers, where differential equations are solved by using their approximate difference equation formulations.

The monograph under review is a virtual encyclopedia of results concerning difference equations. This second edition discusses solutions to linear and nonlinear difference equations, highlights discrete versions of Rolle's theorem, the mean value theorem, Taylor's formula, Hospital's rule, Kneser's theorem. The author investigates the stability and oscillatory properties of solutions of difference equations, explores a unified treatment of boundary value problems, introduces difference inequalities in several independent variables, explains Duffing's, van der Pol's, and Hill's equations (among other classical equations), evaluates Sturm-Liouville problems and related inequalities.

In this new edition, beside a new chapter on the qualitative properties of solutions of neutral difference equations, new material has been added in all existing chapters of the first edition. This includes a variety of interesting examples from real world applications, new theorems, over 200 additional problems and 400 further references.

J. Sándor

Jonathan M. Borwein and Adrian S. Lewis, *Convex Analysis and Nonlinear Optimization, Theory and Examples,* Canadian Mathematical Society (CMS) Books in Mathematics, Vol. 3, Springer-Verlag, New York Berlin Heidelberg, 2000, ISBN:0-387-98940-4.

The book is a concise account of convex analysis, its applications and extensions. It is aimed primarily at first-year graduate students, so that the treatment is restricted to Euclidean space, a framework equivalent, in fact, to the space $\mathbb{R}^n$, but the coordinate free notation, adopted by the authors, is more flexible and elegant. The proof techniques are chosen, whenever possible, in such a way that the extension to infinite dimensions be obvious for readers familiar with functional analysis (Banach space theory). Some of the challenges arising in infinite dimensions are discussed in Chapter 9, *Postscript: Infinite versus finite dimensions,* in which case the results involve deeper geometric properties of Banach spaces. The last section of this chapter contains notes on previous chapters, explaining which results extend to infinite dimension and which not, as well as sources where these extensions can be found.

The authors adopted a succint style, avoiding as much as possible complicated technical details, their goal being "to showcase a few memorable principles rather than to develop the theory to its limits". The book consists of short, self-contained sections, each followed by a rather extensive set of exercises grouped into three categories: examples that illustrate the ideas in the text or easy expansions of sketched proofs (no mark); important pieces of additional theory or more testing examples (marked by one asterisk); and longer, harder examples or peripheral theory (marked by two asterisks). Some bibliographical comments are also included along with these exercises, an approach which allow the authors to cover a large variety of topics. A good idea on the included material is given by the headings of the chapters and the presentation of some topics included in the main text or in exercises.

Ch. 1, *Background* - Euclidean spaces, symmetric matrices, in the main text, and Radstrom cancellation, recession cones, affine sets, inequalities for matrices, in exercises.

Ch. 2, *Inequality constraints* - optimality conditions, theorems of alternative, max-functions, in the main text, and nearest points, coercivity, Carathéodory's theorem, Kirchoff's law, Schur convexity, steepest descent, in exercises.

Ch. 3, *Fenchel duality* - subgradients and convex functions, the value function, the Fenchel conjugate, in the main text, and normal cones, Bregman distances, Log-convexity, Duffin's duality gap, Psenichnii-Rockafellar condition, order-convexity and order subgradients, symmetric Fenchel duality, in exercises.

Ch. 4, *Convex analysis* - continuity of convex functions, Fenchel biconjugation, Lagrangian duality, in the main text, and polars and polar calculus, extreme and exposed points, Pareto minimization, von Neumann minimax theorem, Kakutani's saddle point theorems, Fisher information function, in exercises.

Ch. 5, *Special cases* - polyhedral convex sets and functions, functions of eigenvalues, duality, convex process duality, in the main text, and polyhedral algebra, polyhedral cones, convex spectral functions, DC functions, normal cones, order epigraphs, multifunctions, in exercises.

Ch. 6, *Nonsmooth optimization* - generalized derivatives, regularity and strict differentiability, tangent cones, the limiting subdifferential, in the main text, and Dini derivatives and subdifferentials, mean value theorem, regularity and nonsmooth calculus, subdifferentials of eigenvalues, contingent and Clarke cones, Clarke's subdifferentials, in exercises.

Ch. 7, *Karush-Kuhn-Tucker theory* - metric regularity, the KKT theorem, metric regularity and the limiting subdifferential, second order conditions,in the main text, and Lipschitz extension, closure and Ekeland's principle, Liusternik theorem, Slater condition, Hadamard's inequality, Guignard optimality conditions, higher order conditions, in exercises.

Ch. 8, *Fixed points* - the Brouwer fixed point theorem, selection and the Kakutani-Fan fixed point theorem, variational inequalities, in the main text, and nonexpansive mappings and Browder-Kirk fixed point theorem, Knaster-Kuratowski-Mazurkiewicz principle, hairy ball theorem, hedgehod theorem, Borsuk-Ulam theorem, Michael's selection theorem, Hahn-Katetov-Dowker sandwich theorem, single-valuedness and maximal monotonicity, cuscos and variational inequalities, Fan minimax inequality, Nash equilibrium, Bolzano-Poincaré-Miranda intermediate value theorem, in exercises.

There is a chapter, Chapter 10, containing a list of named results and notation, organized by sections. Beside this, the book contains also an Index.

The bibliography counts 168 items.

Written by two experts in optimization theory and functional analysis, the book is an ideal introductory teaching text for first-year graduate students. By the wealth of highly non-trivial exercises, many of which are guided, it can serve for self-study too.

Ştefan Cobzaş

Function Spaces– The Fifth Conference, Edited by Henryk Hudzik and Leszek Skrzypczak, Lecture Notes in Pure and Applied Mathematics Vol. 213, Marcel Dekker, Inc New York-Basel 2000, xiv + 511 pp. ISBN: 0-8247-0419-3.

These are the proceedings, edited by H. Hudzyk and L. Skrzypczak, of the fifth conference **Function Spaces**, held in Poznan, Poland, one of the satellite conferences associated with of the International Congress of Mathematicians, Berlin 1998. The conference was attended by 121 mathematicians from Poland and abroad. During the conference two special sessions were organized: one dedicated to Wladislaw Orlicz (1903-1990), the founder of the Poznan school of functional analysis, and the other dedicated to Genadiĭ Lozanovsky (1937-1976), a member of the famous St. Petersburg school of lattice theory. The personality of W. Orlicz and his mathematical achievements are evoked in two papers; *Wladyslaw Orlicz: his life and contributions to mathematics* by L. Maligranda and W. Wnuk, and *Recent developments of some ideas and results of W. Orlicz on unconditional convergence* by L. Drewnowski. There are also two papers dedicated to G. Lozanovsky: *G. Ya. Lozanovsky: his life* by Rita

Lozanovskaya, and *G. Ya. Lozanowsky: his contributions to the theory of Banach lattices* by Y. A. Abaramovich and A. I. Veksler.

Beside these expository papers, the volume contains also 40 research papers, covering a large variety of topics from general theory to particular spaces, topological and geometrical properties, order structures and the interpolation of operators. A major theme of the volume is the geometry of Banach spaces, focusing on Orlicz spaces and fixed point theory. Other topics are disjointness preserving operators, integral operators, Hardy inequalities and Hardy operators, Hardy dyadic spaces, Köthe-Bochner function spaces, polynomial and multilinear properties of Banach spaces, and much more.

Among the contributors to the volume we mention: Y. A. Abramovich, A. K. Kitover, Bor-Luh Lin, Z. Cieselski, Shutao Chen, R. Urbanski, L. Maligranda, J. Kakol, G. Lewicki, Pei-Kee Lin, J. G. Llavona, R. Taberski, N. Popa, a.o.

The volume is a valuable addition to the existing literature on function spaces and will be an indispensable tool for researchers in functional analysis and its applications.

Ştefan Cobzaş

George Isac, *Topological Methods in Complementarity Problems,* Nonconvex Optimization and its Applications Vol. 41, Kluwer Academic Publishers, Dordrecht 2000, 704 pp, ISBN: 0-7923-6274-8

As far as we know, after the author's volume "Complementarity Problems" in Springer's Lecture Notes in Mathematics (Nr. 1528, 1992) this monography is the second one dedicated wholly to this subject. It is especially dedicated to the study of nonlinear complementarity problems in infinite dimensional spaces. Since the literature on this subject is very large, here only theoretical problems are considered, and first of all those which are related to topological methods.

The first chapter concerns on the notion of the cone in a topological vector space, on the order relation it introduces. The relation of the order and the topology is essential and in this respect some fundamental types of cones (and ordered topological vector spaces) such as normal, regular, completely regular, well based, polyedral cones are defined and their properties are studied.

In the second chapter the reader get the definition of the complementarity problem, the history of the term and a philosophy about the its importance. We remind the simplest form of the Nonlinear Complementarity Problem (NCP): Let $\langle E, E^* \rangle$ be a dual system of locally convex spaces and let $K \subset E$ be a a closed pointed convex cone, $K^* \subset E^*$ be its dual cone. Suppose that $f : E \to E^*$ is a function. Find $x_0 \in K$ such that $f(x_0) \in K^*$ and $\langle x_0, f(x_0) \rangle = 0$. Further a classification of the complementarity problems is given. Inside the two main class: Topological complementarity problems and Order complementarity problems, and especially in the first one a great variety of types are distinguished, such as various linear and nonlinear problems. The chapter ends with the list of the main problems which can be stated about complementarity problems (existence, unicity, dependence on parameters, sensitivity etc.).

Chapter 3 deals with the mode of appearance of the complementarity in mathematical programming, game theory, variational inequalities, etc. Besides purely mathematical formulations a lot of concrete economical, mechanical and technical problems are considered such as various equilibrium questions in the economy, in traffic flows, problem of maximizing oil production, problems in structural engineering, fluid flows, elasticity etc.

A short chapter is devoted to the equivalence of the complementarity problem with fixed point problems, with variational inequalities, minimization problems etc.

Chapter 5 deals with the solvability of the various types of NCP-s. Beginning with the classical existence and uniqueness theorems of Dorn, Cottle and Karamardian the chapter continues with the more recent results of existence and uniqueness results of NCP-s and their equivalent formulations. Global solvability, feasibility, boundendess of solution set are also considered.

The tietles of the following chapters are suggestive enough to reflect their special content: 6. Topological degree and complementarity. 7. Zero-epi mapping and complementarity. 8. Exceptional family of elements and complementarity. 9.Conditions $(S)_+$ and $S(S)^1_+$: application to complementarity theory. 10. Fixed points, coincidence equations on cones and complementarity. 11. Other topological results on complementarity theory.

Each chapter is followed by references, and a global reference list exists at the end of the volume. A Glossary of notations and an Index completes the monography.

The exposition implies the usage of a very large number of auxiliary results from topology and functional analysis. These results are only stated with bibliographic indications. Only the results strictly related to the subject are proved. The resulting text becomes this way a good reference book in the field, which can be used also as a guide for lectures or for the introduction in the subject. It is intended for mathematicians, engineers, economists and for anybody interested in the subject.

A.B. Németh