SYLLABUS

1. Information regarding the programme

1.1 Higher education institution	Babeş-Bolyai University, Cluj-Napoca
1.2 Faculty	Mathematics and Computer Science
1.3 Department	Mathematics
1.4 Field of study	Mathematics
1.5 Study cycle	Licence
1.6 Study programme / Qualification	Mathematics and Computer Science

2. Information regarding the discipline

2.1 Name of the discipline (en) (ro)	Calculus 2 (Differential and integral calculus in R^n) Analiză matematică 2 (Calcul diferenţial şi integral în R^n)						
2.2 Course coordinator			Trif Tiberiu-Vasile				
2.3 Seminar coordinator							
2.4 Year of study	1	2.5 Semester	2	2.6. Type of evaluation	Exam	2.7 Type of discipline	mandatory
2.8 Code of the discipline		MLE0071					

3. Total estimated time (hours/semester of didactic activities)

3.1 Hours per week	6	Of which: 3.2 course	3	3.3 seminar/laboratory	3
3.4 Total hours in the curriculum	84	Of which: 3.5 course	42	3.6 seminar/laboratory	42

Time allotment:	hours
Learning using manual, course support, bibliography, course notes	20
Additional documentation (in libraries, on electronic platforms, field documentation)	10
Preparation for seminars/labs, homework, papers, portfolios and essays	20
Tutorship	6
Evaluation	10
Other activities:	

3.7 Total individual study hours	66
3.8 Total hours per semester	150
3.9 Number of ECTS credits	6

4. Prerequisites (if necessary)

4.1 curriculum	• Calculus 1 (Calculus in R)
4.2 competencies	• Logical thinking abilities, problematisation

5. Conditions (if necessary)

5.1 For the course	• Classroom with adequate infrastrusture
5.2 For the seminar/lab activities	• Classroom with adequate infrastrusture

6. Specific competencies aquired

Professional competencies	• C1.4 Recognizing the main classes /types of mathematical problems and selecting the appropriate methods and techniques for their solving • C2.1 Identifying the basic notions used to describe some processes and phenomena
Transversal competencies	• CT1 Application of efficient and rigorous working rules, manifest responsible attitudes towards the scientific and didactic fields, respecting the professional and ethical principles

7. Objectives of the discipline (outcome of the acquired competencies)

7.1 General objective of the discipline	• Getting to know the topolgy of the Euclidean space $\mathbf{R}^n$, the differential calculus of functions of several variables, as well as the different types of integrals for functions of several variables (multiple integrals, line integrals, surface integrals)
7.2 Specific objectives of the discipline	• Presenting the basic notions and results concerning the topology of the Euclidean space $\mathbf{R}^n$ • Presenting the basic notions and results concerning the differential calculus of functions of several variables • Presenting the different types of integrals for functions of several variables (multiple integrals, line integrals, and surface integrals)

8. Content

8.1 Course	Teaching methods	Remarks
1. Topology in $\mathbf{R}^n$: the Euclidean space $\mathbf{R}^n$ (the inner product, the Euclidean norm, the Euclidean distance), the topological structure of $\mathbf{R}^n$ (balls, neighbourhoods, interior points, adherent points, boundary points, and limit points, open and closed sets). Sequences in $\mathbf{R}^n$: convergent and Cauchy sequences, characterization of adherent points, of limit points, and of closed sets by means of sequences.	Lecture, discussion, proof, problematisation	[3], pp. 110 – 132 pp. 159 – 185 [8], pp. 269 – 275
2. Compact sets in $\mathbf{R}^n$: definition of compact sets, examples of compact sets in $\mathbf{R}^n$, characterization of compact sets in $\mathbf{R}^n$. Limits of vector functions of vector variable: definition of the limit, characterization of the limit by means of sequences, operations with functions having a limit.	Lecture, discussion, proof, problematisation	[3], pp. 132 – 142 pp. 185 – 187 pp. 232 – 244 [8], pp. 290 – 301
3. Continuity of vector functions of vector variable: definition of the continuity at a point, characterization of the continuity by means of sequences, operations with continuous functions, the Weierstrass theorem. Linear mappings and their norm.	Lecture, discussion, proof, problematisation	[8], pp. 348 – 353
4. Differentiability in $\mathbf{R}^n$: the derivative of a vector function of a real variable, the mean value theorem for vector functions of a real variable. Differentiability of vector	Lecture, discussion, proof, problematisation	[5], pp. 393 – 404 pp. 413 – 417

functions of vector variable (definition of the Frechet differential, continuity of Frechet differentiable functions, derivative vs differential for vector functions of a real variable).		
5. Differentiability in $\mathbf{R}^n$: the directional derivative of a vector function of vector variable and its relationship with the Frechet differential, partial derivatives and their relationship with the Frechet differential. The chain rule, the differentiability of the inverse function.	Lecture, discussion, proof, problematisation	[5], pp. 343 – 350 pp. 417 – 422
6. Differentiabilty in $\mathbf{R}^n$: mean value theorems for functions of several variables. Functions of the class C^1. The local inversion theorem, the implicit function theorem.	Lecture, discussion, proof, problematisation	[5], pp. 422 – 441
7. Differentiability in $\mathbf{R}^n$: Lagrange multipliers, second order partial derivatives, the Schwarz and Young theorems concerning the mixed partial derivatives. Necessary and sufficient conditions for extrema. Higher order partial derivatives, Taylor's formula.	Lecture, discussion, proof, problematisation	[5], pp. 361 – 384 pp. 441 – 445
8. The Riemann integral on a compact interval in $\mathbf{R}^n$: definition of the Riemann integral on a compact interval in $\mathbf{R}^n$, Riemann integrability tests on a compact interval in $\mathbf{R}^n$ (the Heine, Cauchy, and Darboux tests). Computation of Riemann integrals on compact intervals by means of iterated integrals (the Fubini theorem).	Lecture, discussion, proof, problematisation	[6], pp. 192 – 198 [10]
9. The Riemann integral on bounded sets in $\mathbf{R}^n$: computation of Riemann integrals on bounded sets in $\mathbf{R}^n$ by means of iterated integrals (the Fubini theorem). Change of variables in multiple integrals. Applications in physics of multiple integrals: centres of gravity and moments of inertia.	Lecture, discussion, proof, problematisation	[6], pp. 224 – 234 pp. 329 – 335 [10]
10. Vector functions of bounded variation: definition, examples, properties of the total variation. Additivity of the total variation with respect to the interval, the Jordan reprezentation theorem, computation of the total variation for functions of the class C^1.	Lecture, discussion, proof, problematisation	[6], pp. 17, 21 [10], pp. 27 – 29 [11], pp. 114 – 115
11. Line integrals: paths, examples, equivalent paths, curves and oriented curves. First degree differential forms. Integration of first degree differential forms along a path (the line integral of the second kind), mechanical work.	Lecture, discussion, proof, problematisation	[6], pp. 135 – 145 [11], pp. 111 – 113 pp. 126 – 128 [10]
12. Line integrals: the Green formula, integration of exact differential forms, the Leibniz-Newton formula, the Poincaré theorem concerning the integration of exact differential forms, mechanical work in the gravitational field.	Lecture, discussion, proof, problematisation	[6], pp. 205 – 213 [11], pp. 128 – 133 [10]
13. Surface integrals: parametrized surfaces, examples. Differential forms of the second degree and their integrals over parametrized surfaces (surface integrals of the second kind).	Lecture, discussion, proof, problematisation	[6], pp. 306 – 314 [10]
14. The Stokes and the Gauss-Ostrogradski formulae.	Lecture, discussion, proof, problematisation	[10] [6], pp. 351 – 355

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12. RUDIN W.: Principles of Mathematical Analysis, 2nd Edition, McGraw-Hill, New York, 1964.

8.2 Seminar / laboratory	Teaching methods	Remarks
1. The Euclidean space $\mathbf{R}^n$: problems concerning the Euclidean space $\mathbf{R}^n$.	Discussion, problematisation	The coordinator's problem set
2. Compact sets in $\mathbf{R}^n$: problems concerning compact sets in $\mathbf{R}^n$.	Discussion, problematisation	[3], pp. 57 – 60
3. Limits of vector functions of vector variable, continuity of vector functions of vector variable. Linear mappings and their norm: computation of the norm for some concrete linear mappings.	Discussion, problematisation	[3], pp. 31 – 32 [8], pp. 45 – 46
4. Computation of directional derivatives, partial derivatives, and differentials for concrete functions.	Discussion, problematisation	[8], pp. 46 – 49
5. Differentials: study of the Frechet differentiability for concrete functions. Applications to the chain rule.	Discussion, problematisation	[8], pp. 50 – 56
6. Mean value theorems for functions of several variables. Difeomorphisms and implicit functions.	Discussion, problematisation	[8], pp. 56 – 69
7. Extrema for functions of several variables, higher order partial derivatives.	Discussion, problematisation	[8], pp. 73 – 79
8. Computation of double integrals over rectangles. Computation of triple integrals over parallelepipeds. Double and triple integrals over simple sets with respect to an axis.	Discussion, problematisation	[8], pp. 84 – 86 p. 91
9. Computation of double integrals by means of change of variables (polar coordinates).	Discussion, problematisation	[8], pp. 87 – 91
10. Computation of triple integrals by means of change of variables (spherical coordinates, cylindrical coordinates).	Discussion, problematisation	[8], pp. 92 – 94
11. Problems concerning functions of bounded variation. Line integrals of the first kind: definition, main theoretical results, computation of line integrals of the first kind along concrete paths.	Discussion, problematisation	[1], pp. 5 – 44 pp. 166 – 185 [2], pp. 44 – 48 [4], pp. 69 – 70 [5], pp. 10 – 15
12. Line integrals of the second kind: computation of the integrals of certaind first degree differential forms along concrete paths. Integration of some exact differential forms. Applications to the Green formula.	Discussion, problematisation	[1], pp. 185 – 228 [2], pp. 49 – 55 pp. 107 – 109 [4], pp. 70 – 73 p. 74
13. Computation of surface integrals of the first and of the second kind.	Discussion, problematisation	[2], pp. 91 – 96 pp. 101 – 104 [4], p. 87 – 88
14. Problems concerning the Stokes and the Gauss-Ostrogradski formulae.	Discussion, problematisation	[2], pp. 109 – 113

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9. Corroborating the content of the discipline with the expectations of the epistemic community, professional associations and representative employers within the field of the programme

The theme of this course (the topology of the Euclidian $\mathbf{R}^n$, the differential calculus of functions of several variables, functions of bounded variation, and various types of integrals for functions of several variables - multiple integrals, line integrals, and surface integrals) is provided in the study program of to all major universities in Romania and the world. It is an indispensable part of preparing future math teachers, future mathematics researchers, and those working in other fields that directly apply mathematical methods.

10. Evaluation

Type of activity	10.1 Evaluation criteria	10.2 Evaluation methods	10.3 Share in grade
10.4 Course	- knowing the main notions and results - knowing the proofs of the main theoretical results - applying the theoretical results in solving concrete problems	Midterm exam (mandatory) Written exam at the end of the semester	50% 50%
10.5 Seminar/lab	- solving concrete problems by using the theoretical results presented at the course		
10.6 Minimum performance standards			

- The definitions of the basic notions, the statements of the main theoretical results, and the ability to apply these results in solving simple problems
- Identifying and selecting methods to address simple concrete problems

Date | Signature of course coordinator | Signature of seminar coordinator

28.4.2024 ....

Date of approval | Signature of the head of departament

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